Using AI for Theorem Proving

Project Midway Presentation
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Theorem Proving in Lean

```
variables x y z w : ℕ

example (h₁ : x = y) (h₂ : y = z)
(h₃ : z = w) : x = w :=
begin
  rw [h₁],
  rw [h₂],
  assumption
end
```

```
[{"goal": x y z w: nat
         h1: x = y
         h2: y = z
         h3: z = w
         |- x = w
  "policy": 0.33
  "tactic": "rw [h1]"},
 {"goal": x y z w: nat
         h1: x = y
         h2: y = z
         h3: z = w
         |- y = w
  "policy": 0.66
  "tactic": "rw [h2]"},
 {"goal": x y z w: nat
         h1: x = y
         h2: y = z
         h3: z = w
         |- z = w
  "policy": 1.0
  "tactic": "assumptions"}]
```

Tactic Generator Model

Model: ByT5-Small with 300M trainable parameters with 12 encoder and 4 decoder layers

Training objective:

```
Input:  '''x y z w: nat          Output: "rw [h1]"
(goal)      h1: x = y             (tactic)
            h2: y = z
            h3: z = w
            |- x = w'''
```

Policy Model

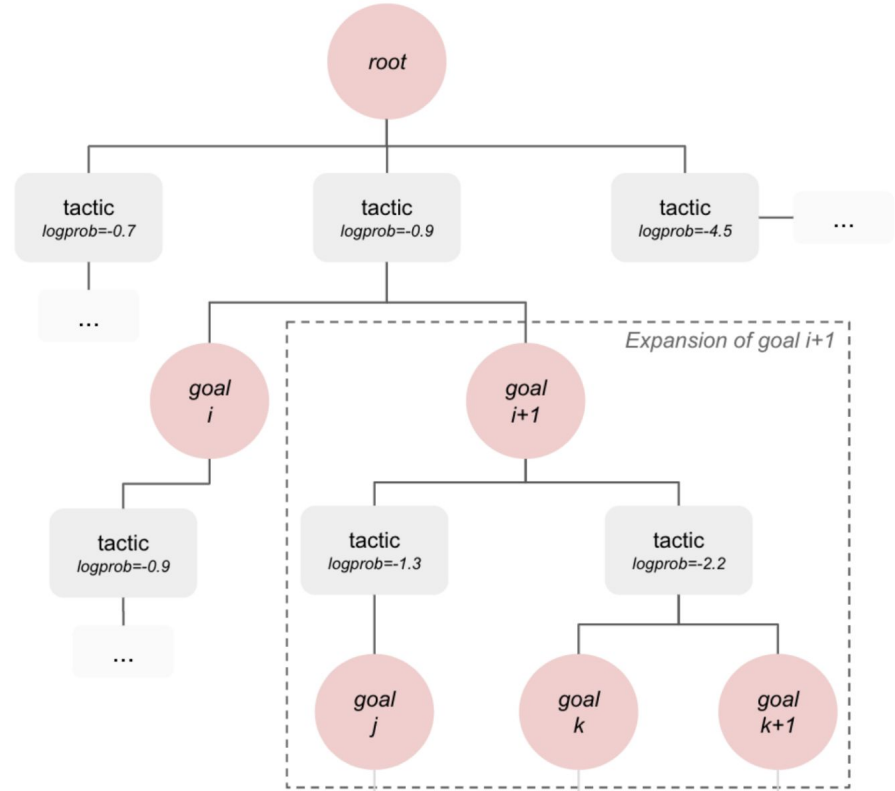**Model:** RoBERTa-base with 125M trainable parameters

Training Objective:
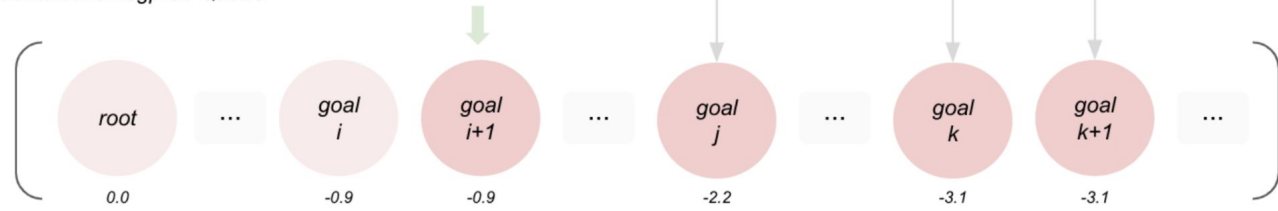
```
Input: '''x y z w: nat         Output: 0.33
(goal)    h1: x = y             (policy)
          h2: y = z
          h3: z = w
          |- x = w'''
```

Proof Search Tree

Proof Search Tree

root

tactic
logprob=-0.7

tactic
logprob=-0.9

tactic
logprob=-4.5

...

...

goal
i

goal
i+1

Expansion of goal i+1

tactic
logprob=-0.9

tactic
logprob=-1.3

tactic
logprob=-2.2

...

goal
j

goal
k

goal
k+1

Cumulative Logprob Queue

root

...

goal
i

goal
i+1

...

goal
j

...

goal
k

goal
k+1

...

0.0

-0.9

-0.9

-2.2

-3.1

-3.1

Results

Results (Pass@1) of proving theorems within a time limit of 10 minutes:

Method	Mathlib	MiniF2F
tactic log probability	55.0	28.6
tactic log probability + policy	**55.4**	**29.0**

Table 1. Table summarizing the effect of using tactic log probability & policy for guiding proof search.

Future Plan

- Integrate the policy model with Dynamic Sampling
- Limit on number of nodes to search instead of time limit
- Generate the data of success rate of goals and use it for training a new policy model

References:

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- Liu, Y., Ott, M., Goyal, N., Du, J., Joshi, M., Chen, D., Levy, O., Lewis, M., Zettlemoyer, L., and Stoyanov, V. Roberta: A robustly optimized bert pretraining approach. arXiv preprint arXiv:1907.11692, 2019
- Polu, S. and Sutskever, I. Generative language modeling for automated theorem proving. arXiv preprint arXiv:2009.03393, 2020

Thank You