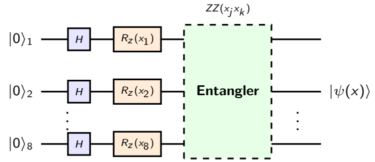
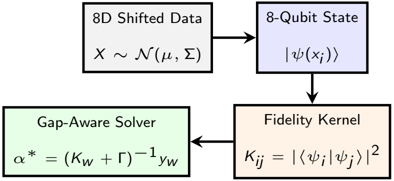
Title, base paper and problem setup

- **Title of the Project:** Spectral Instability in Quantum Kernel Ridge Regression
- **Base Papers & Usage:**
  - **Quantum Separation:** *Quantum Speed-up in ML* (Liu et al., 2021) & *Quantum Feature Spaces* (Havlíček et al., 2019) — Provable classical separation via IQP/Discrete-Log concepts.
  - **Kernel Geometry:** *Power of Data in QML* (Huang et al., 2021) — Geometric difference & classical kernel bounds.
  - **Eigengap Collapse ($\delta \approx 0$):** *Exponential Concentration* (Thanasilp et al., 2024) & *Inductive Bias* (Kübler et al., 2021) — Quantum spectral decay.
  - **Subspace Rotation:** *Rotation of Eigenvectors* (Davis-Kahan, 1970) & *Invariant Subspaces* (Gil', 2016) — $\|\sin\Theta\| \leq \frac{\|V\|}{\delta}$.
  - **Risk Bounds (Step 1):** *Optimal Rates for RLS* (Caponnetto & De Vito, 2007) — RKHS out-of-sample excess risk under shift.
  - **Gap Damping & Shift (Steps 2–3):** *Differential Spectral Damping* (Vashishtha, 2026) & *Covariate Shift* (Sugiyama et al., 2007).
- **Input, Output & Dataset:**
  - **Input:** $x \in [0, 2\pi)^8$ (8 features mapped to 8 qubits); **Output:** $y = f_0(x) + \epsilon \in \mathbb{R}$.
  - **Kernel:** $K_{ij} = |\langle\psi(x_i)|\psi(x_j)\rangle|^2 = |\langle 0|^{\otimes 8} U^\dagger(x_j)U(x_i)|0\rangle^{\otimes 8}|^2 \in [0, 1]$.
  - **Benchmark:** 8-qubit IQP/Discrete-Log Concept Class under controlled domain shift ($P_{\text{train}} \to P_{\text{test}}$).

8-Qubit IQP Feature Map ($U(x)|0\rangle^{\otimes 8}$)



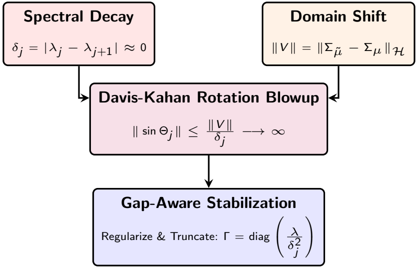
Data Pipeline & Dual Solver



Research question and tentative timeline

- **What Problem Are You Asking?**
  - Why does provable quantum advantage in QKRR collapse out-of-sample under covariate shift?
  - **Mechanism:** Distribution shift $\mu \to \tilde{\mu} \implies \Sigma_{\tilde{\mu}} = \Sigma_{\mu} + V$ on RKHS. Exponential spectral concentration forces $\delta \approx 0$, causing Davis-Kahan rotation $\frac{\|V\|}{\delta} \to \infty$ to scramble coordinate axes and destroy out-of-sample risk.
- **Scope, Assumptions & Contributions:**
  - **Assumptions:** Covariate shift ($P_{\text{train}}(x) \neq P_{\text{test}}(x)$), self-adjoint compact covariance operators $\Sigma_{\mu}, \Sigma_{\tilde{\mu}} \in \mathcal{S}_{\infty}(\mathcal{H})$.
  - **Theory (Step 1):** Caponnetto–De Vito RKHS excess risk bound $\|f^* - f_0\|^2_{L^2(\tilde{\mu})}$ explicitly parameterized by δ and $\|V\|$.
  - **Algorithm (Steps 2–3):** Gap-Aware Regularizer $\Gamma = \text{diag}(\lambda/\delta_j^2)$ + density reweighting $w(x) = \frac{P_{\text{test}}(x)}{P_{\text{train}}(x)}$ to suppress unstable tail modes.

Subspace Rotation Failure Mechanism



Deliverables & Timeline (Autumn 2026)

14–15 Sep (Pres. 2): Analytical excess risk bound derivation parameterized by $(\delta, \|V\|)$; baseline QKRR replication.
26–27 Oct (Pres. 3): Dual Gap-Aware solver implementation; 8-qubit IQP benchmark verification.
16–17 Nov (Pres. 4): Multi-model benchmarking vs Classical RBF/XGBoost & final 2-page report.