Radial Basis Functions

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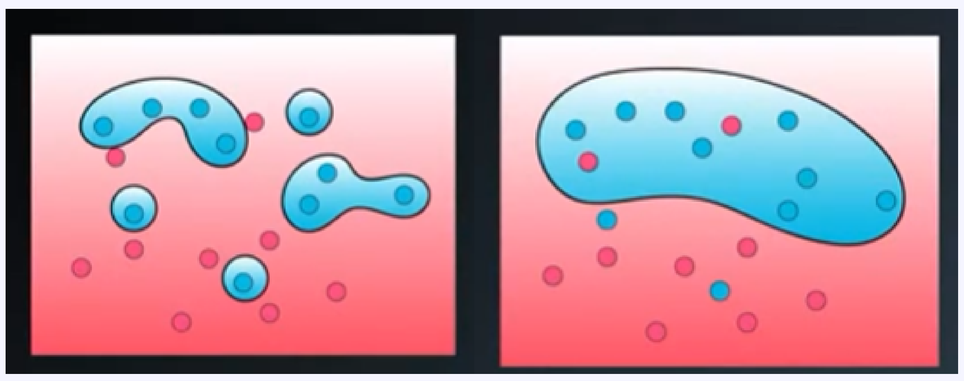
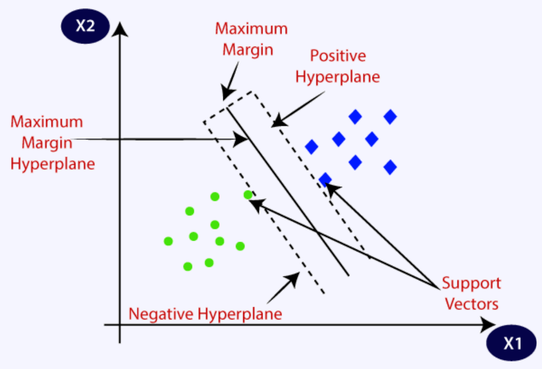
Introduction

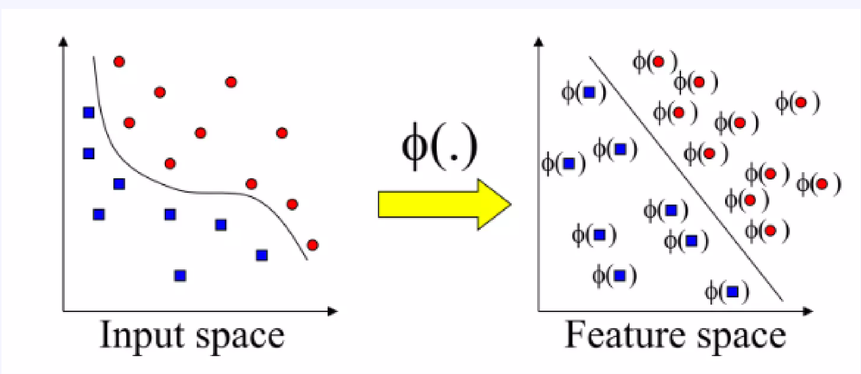
Classification



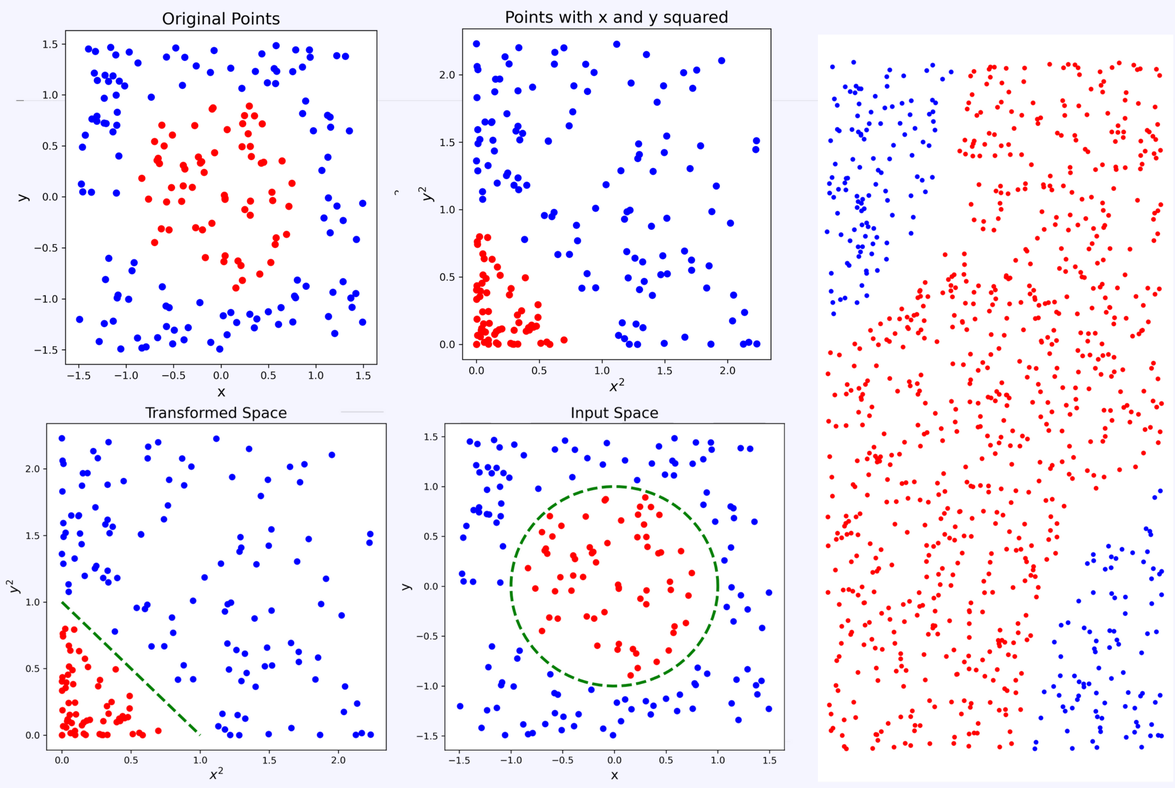
Linear SVM

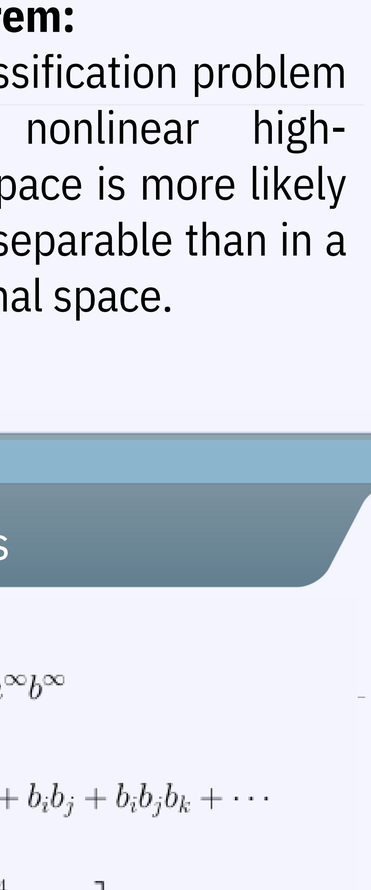


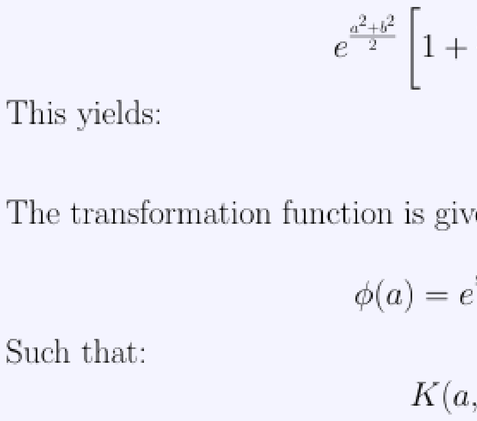
Transformations



Separability of Patterns







Cover's Theorem:

A pattern classification problem cast in a nonlinear high-dimensional space is more likely to be linearly separable than in a low-dimensional space.

Radial Basis Functions

Consider a projection in infinite dimensions:

$$ab + a^2b^2 + a^3b^3 + a^4b^4 + \cdots a^\infty b^\infty$$

The transformation function is given by:

$$\phi(a) = a_i + a_i a_j + a_i a_j a_k + \cdots \; ; \quad \phi(b) = b_i + b_i b_j + b_i b_j b_k + \cdots$$

After some substitutions:

$$e^{\frac{a^2+b^2}{2}} \left[1 + \frac{ab}{1!} + \frac{a^2b^2}{2!} + \frac{a^3b^3}{3!} + \frac{a^4b^4}{4!} + \cdots\right]$$

This yields:

$$= e^{\frac{a^2+b^2}{2}} e^{ab} = e^{-\frac{(a-b)^2}{2}}$$

The transformation function is given by:

$$\phi(a) = e^{\frac{a^2}{2}} + \frac{e^{\frac{a^2}{2}} a}{1!} + \frac{e^{\frac{a^2}{2}} a^2}{2!} + \frac{e^{\frac{a^2}{2}} a^3}{3!} + \cdots$$

Such that:

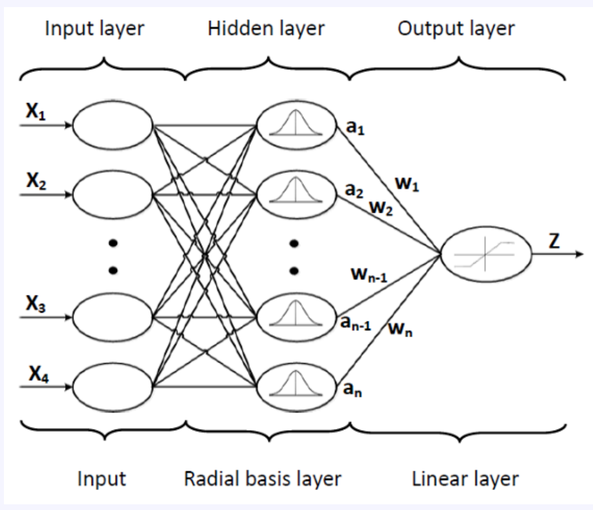
$$K(a,b) = \phi(a) \cdot \phi(b) = e^{-\frac{(a-b)^2}{2}}$$

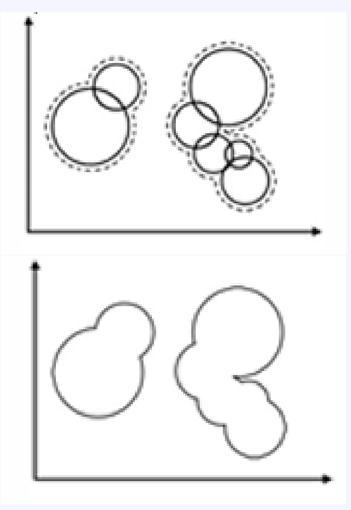
Introducing γ as a scaling factor (a hyperparameter):

$$exp\left[-\frac{\gamma}{2}(a_i - b_j)^2\right]$$

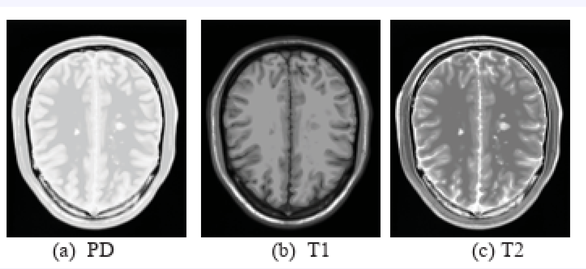
Medical Applications

RBF Network





Classification of Brain MRI Scans



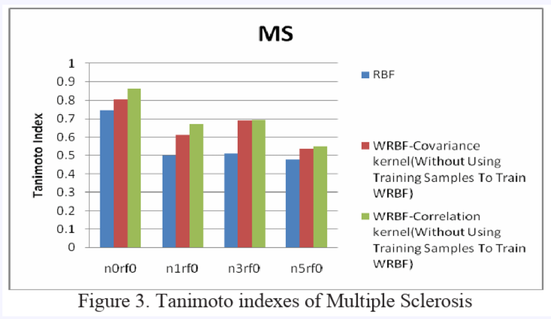
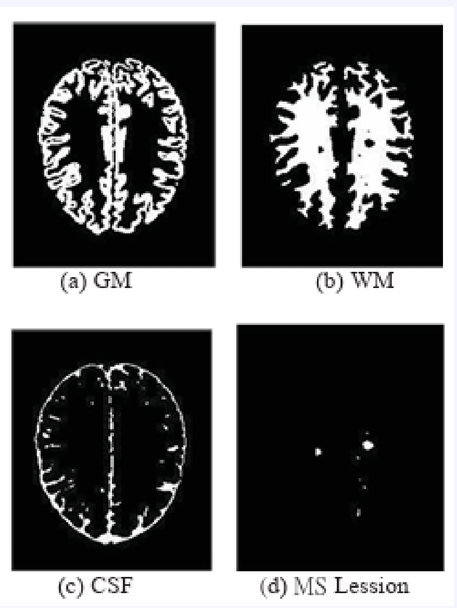
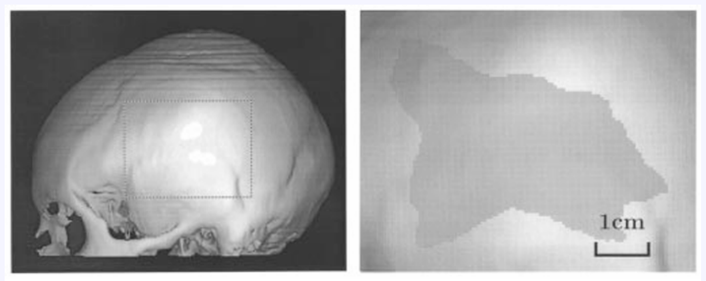
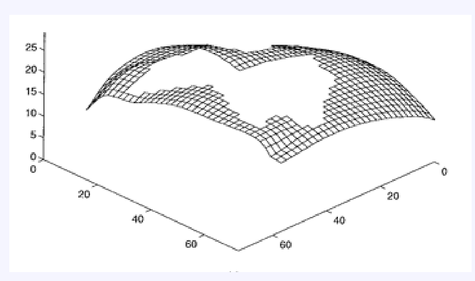


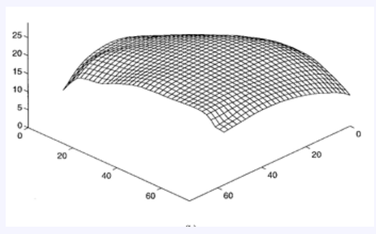
Figure 3. Tanimoto indexes of Multiple Sclerosis

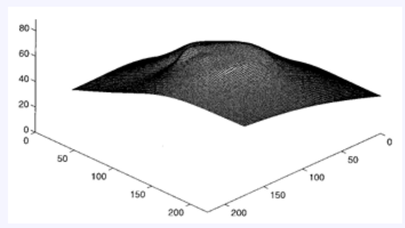


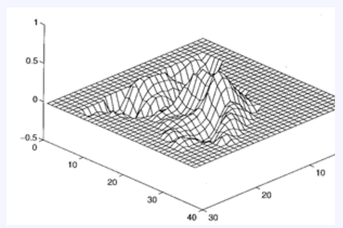
Surface Interpolation for Medical Imaging











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