Quiz 2

Physics of Atoms and Molecules

1. Derive the relation between Einsteins's coefficients $A_{ab}$ and $B_{ab}$ for a given number density of photon $n(\omega)d\omega = C\omega^2 d\omega$, $C$ being a constant.[4]

Hint : (Bose factor)$/(\pi^2 c^3)$ has been set to C

2. Derive selection rule for the magnetic quantum number $m$ for $\vec{A} = \vec{A}_0 zt$, $t$ being time and $\vec{A}_0 \parallel \hat{x}$. Use the choice of gauge we have adopted so far in class.[4]

Hint : $\nabla.\vec{A}$ is zero but $\langle\phi_a|\vec{A}.\nabla|\phi_b\rangle$ will lead to $\langle\phi_a|z\hat{p}_x|\phi_b\rangle$ which you have seen in the electric quadrupole transition.

3. Consider a system with only three states available to it, as you would get if you diagonalize a 3x3 Hamiltonian describing the system. Suppose the system interacts with its environment and as a result undergoes down transition from a level 'a' at 5 eV to two levels 'b' and 'c' receptively at 3eV and 2eV with natural line widths 0.01 eV and 0.001 eV (in class we wrote it in terms of $\omega$).
(In real experiments you get absorption or emission peaks which are Lorentzian in shape with a FWHM rendering the natural line width at low temperature and starts becoming Gaussian with increasing temperature. Now you know why. )
Within the three states description of the system (which is of course approximately valid in the interacting scenario) find the ratio of tradition dipole elements $|\vec{r}_{ba}|$ and $|\vec{r}_{ca}|$. [3]

Hint : Note $W_{ab}^{sp} = (1/\tau) \propto |M_{ba}|^2/\omega_{ba}$ and $M_{ba} \propto \omega_{ba}|\vec{r}_{ba}|$. You can find ratios of $\omega$ and $\tau$ from the given quantities

Calculate the oscillator strengths $f_{ba}$ and $f_{ca}$ assuming the three state description of the system to be complete. [2]

Hint: $f_{ba} \propto \omega_{ba}|\vec{r}_{ba}|^2$ and $\sum_b f_{ba} = 1$

4. If the pure thermal broadening of transition 'a' to 'b' is 0.005 eV then how much will it be for the transition from 'a' to 'c' ? [2]

Hint : FWHM $\Delta\omega_{ab} \propto \omega_{ab}$