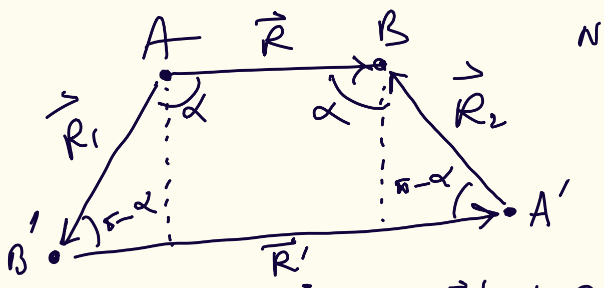
Crystalographic restriction theorem:

If a Bravais lattice has n-fold rotational symmetry about an axis then it means that if we rotate the lattice by angle $\alpha = \frac{360}{n}$ about the axis clockwise or anticlockwise then the rotated lattice would exactly coinside with the original unrotated lattice.

$\Rightarrow$



Note $|\vec{R}_1| = |\vec{R}_2| = |\vec{R}|$

$\vec{R}_1, \vec{R}_2, \vec{R}, \vec{R}'$ all are lattice vectors

$\vec{R} \parallel \vec{R}'$ $\therefore$ $\vec{R}' = m\vec{R}$, $m = 0, \pm 1, \pm 2, \pm 3 \ldots$

Note $|\vec{R}'| = |\vec{R}| + 2|\vec{R}|\cos\alpha$

$|m||\vec{R}| = |\vec{R}| + 2|\vec{R}|\cos\alpha$

$\Rightarrow \cos\alpha = \dfrac{1-|m|}{2}$

Since $|\cos\alpha| \leq 1$, allowed $m = 0, \pm 1, \pm 2, \pm 3$

$m=0 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = 60^\circ \Rightarrow n = 6$

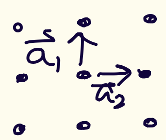
$m = \pm 1 \Rightarrow \cos\alpha = 0 \Rightarrow \alpha = 90^\circ \Rightarrow n = 4$

$m = \pm 2 \Rightarrow \cos\alpha = -\frac{1}{2} \Rightarrow \alpha = 120^\circ \Rightarrow n = 3$

$m = \pm 3 \Rightarrow \cos\alpha = -1 \Rightarrow \alpha = 180^\circ \Rightarrow n = 2$

Example

4 fold symmetry:

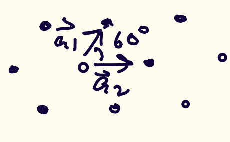


$|\vec{a}_1| = |\vec{a}_2|$; $\vec{a}_1 \perp \vec{a}_2$

Square lattice in 2D

Cubic lattice in 3D

3, 6 fold sym:

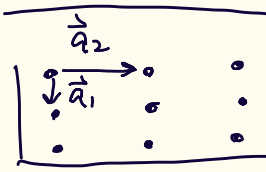


$|\vec{a}_1| = |\vec{a}_2|$; $\angle \vec{a}_1, \vec{a}_2 = 60^\circ$

Triangular lattice in 2D

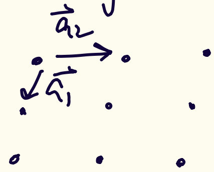
Hexagonal lattice in 3D.

2 fold sym:



$\vec{a}_1 \perp \vec{a}_2$ $|\vec{a}_1| \neq |\vec{a}_2|$

Rectangular lattice



Oblique lattice $\rightarrow$ $\angle \vec{a}_1 \vec{a}_2 \neq (60, 90)$