Landau Diamagnetism

Diamagnetic response is given by all atoms and molecules constituting matter since the orbital motion of electron in them will always get modified in a way to oppose the change in the magnetic field they are exposed to.
— Lenz's Law.

For free electrons (neglect spin) : $\frac{(\hat{p} + e\vec{A})\cdot(\hat{p}+e\vec{A})}{2m}\psi = E\psi$

We choose $B \parallel \hat{z}$. Landau gauge : $A_x = -yB$, $A_y = A_z = 0$

Since the $\hat{H}$ commutes with $\hat{p}_x$ and $\hat{p}_z$ with such choice of $\vec{A}$ we seek a seperable sol^n.

$$\psi(x y z) = e^{ik_x x} e^{ik_z z}\, \phi(y)$$

$$\Rightarrow \left[(\hbar k_x - eyB)^2/2m + p_y^2/2m + \hbar^2 k_z^2/2m \right] \phi(y) = E\phi(y)$$

$$\Rightarrow \left[\frac{1}{2} m \omega_c^2 (y - y_0) + p_y^2/2m + \hbar^2 k_z^2/2m \right] \cdot \phi(y) = E\phi(y)$$

$$\omega_c = eB/m, \quad y_0 = -\hbar k_x / eB$$

Recall
$\frac{mv^2}{r} = qvB$
$\Rightarrow \omega = \frac{v}{r} = \frac{qB}{m}$
↓
Cyclotron freq

$$\Rightarrow \left[\underbrace{\left\{ \tfrac{1}{2} m\omega^2 (y - y_0) + p_y^2/2m \right\}}_{\text{QHO}} + \hbar^2 k_z^2/2m \right] \phi(y) = E\,\phi(y)$$

$$\Rightarrow E \equiv E_{n k_z} = \hbar\omega_c\left(n + \tfrac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m},$$

Note $\omega_c \to \omega_c^* = eB/m^*$ for electron and hole.

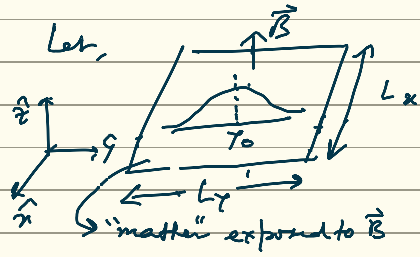
Note that another choice of gauge $A_x = 0$, $A_y = xB$, $A_z = 0$ would lead us to a sol^n of form. $\phi(x)\, e^{ik_y y} e^{ik_z z}$

$\Rightarrow k_z$ sustains as a good quantum number but x and y degrees of freedom of the problem are interchangeable and thus pertains to a single quantum number n.

How to choose k_x to be able to write $\psi(\vec{r})$?

Note y_0 depends on k_x.
↓
Centre of $\phi(y)$

Let,



∴ States which will be substantially occupied, must have sufficient normalization within the "matter" exposed to $\vec{B}$.

$\Rightarrow y_0$ must be well inside $\Rightarrow -\frac{L_y}{2} < y_0 < L_y/2$ (kind of a liberal estimate)

$\Rightarrow -L_y/2 < \frac{\hbar k_x}{m\omega_c} < L_y/2 \Rightarrow -\frac{m\omega_c L_y}{2\hbar} < k_x < \frac{m\omega_c L_y}{2\hbar}$

$\Rightarrow$ Range of $k_x = \frac{m\omega_c L_y}{\hbar}$

Now let us consider the "matter" as a BVK cell : $\Delta k_x = \frac{2\pi}{L_x}$. With BVK approx the range of y_0 become more appropriate.

$\Rightarrow$ Number of possible k_x values :

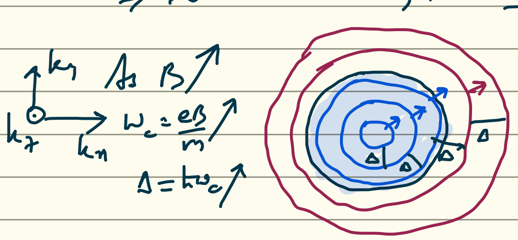
$$\left(\frac{m\omega_c L_y}{\hbar}\right)\Big/\Delta k_x = \left(m\omega_c L_x L_y\right)/h = \frac{m\omega_c A}{h}$$

$\Rightarrow$ Degeneracy of E_{nk_z} per unit area : $m\omega_c/h = eB/h$

∴ For a Landau level (LL) to exist : $\frac{eBA}{h} > 1 \Rightarrow B > h/eA \sim$ Tesla (high!)

Thus the moment $\vec{B}$ is switched on, sets of (eBA/h) number of free electron states would collapse into degenerate Landau levels of energy E_{nk_z} with their $\langle p_z \rangle$ preserved.

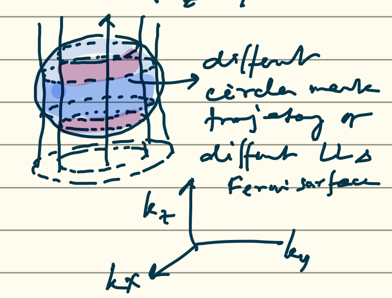
$\Rightarrow$ Semiclassically : $\frac{\langle p_x^2 \rangle}{2m} + \frac{\langle p_y^2 \rangle}{2m} = \hbar\omega_c\left(n + \frac{1}{2}\right) \Rightarrow$

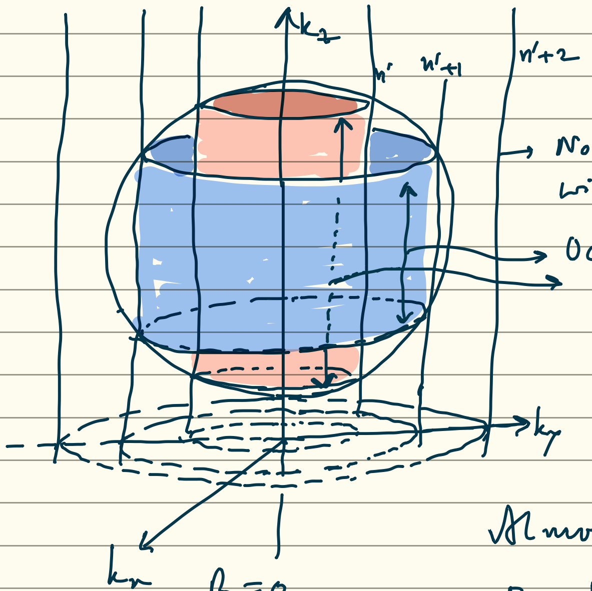


∴ As B↗ : Radius of cylinder of LL states ↗

∴ LLs pops out of the Fermi Sphere.

(LL → Landau level).





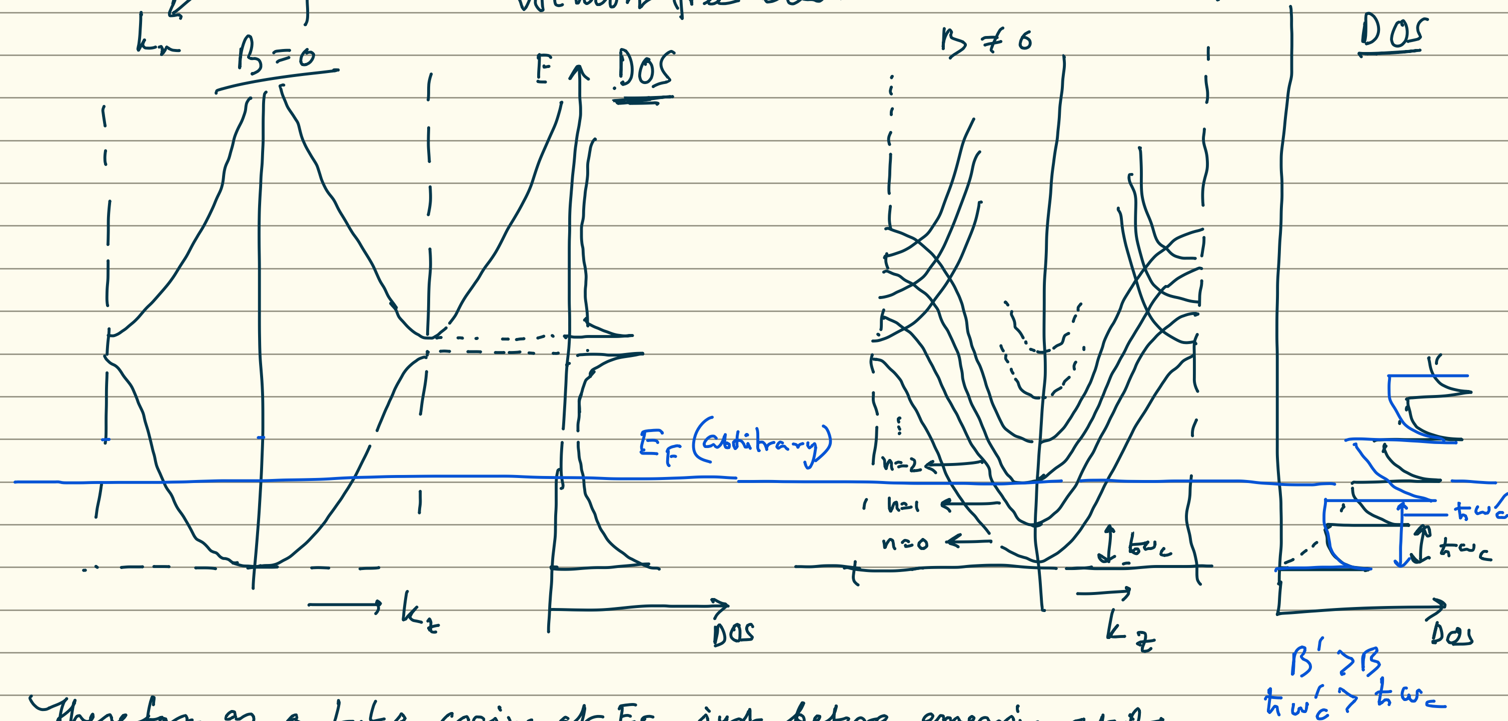
→ No intercept with F. Sp. (no occupation)

Occupied $(n'+1, k_z)$
" (n', k_z)

Landau considered free electron. The same is extendable to nearly free electron.
"Free" nature of electron assumes ballistic motion: large relaxation time τ.
$\therefore \tau\omega_c >> 1$ for Landau levels to be observed.
$\Rightarrow \tau \downarrow \Rightarrow$ low T high B.

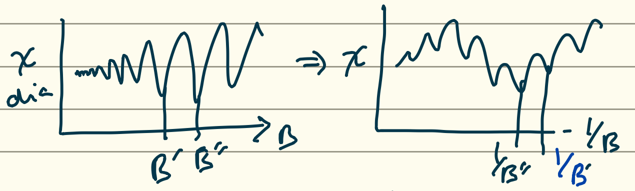
Energy bands:

Almost free electron



Therefore as a L.L. arrive at E_F just before emerging out of Fermi Sph, the DOS peaks at E_F ⇒ A peak in all properties dependent on DOS at Fermi Level.

Magnetization
Diamagnetic response
Optical absorption
Conductivity ....
Th. Ex.



→ De haas Van Alphen oscillation of →

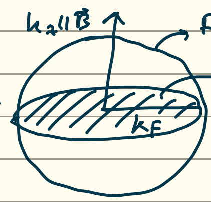
Let l # of LL occupied at B' $\Rightarrow E_F = \hbar\omega'(l+\frac{1}{2}) \Rightarrow \frac{E_F}{\hbar\omega'} = (l+\frac{1}{2}) \Rightarrow \frac{mE_F}{\hbar e B'} = (l+\frac{1}{2})$

$\therefore (l-1)$ # of LL " " $B'' \Rightarrow$

$$\therefore E_F = \hbar\omega''\left[(l-1)+\frac{1}{2}\right] \Rightarrow \frac{mE_F}{\hbar e B''} = (l-\frac{1}{2})$$

$$\therefore \frac{1}{B'} - \frac{1}{B''} = \frac{\hbar e}{mE_F}\left(l+\frac{1}{2}-(l-\frac{1}{2})\right) = \frac{e\hbar}{mE_F}$$

Recall, $E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\frac{1}{\pi}(\pi k_F^2) = \frac{\hbar^2\Omega_F}{2m\pi}$ ⇒

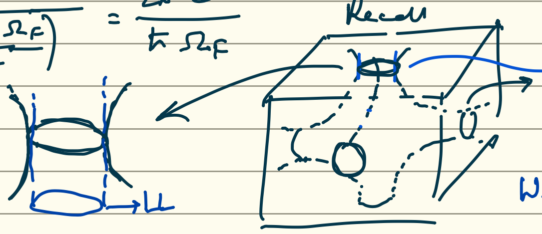


→ Area Ω_F
↓
Cross section of Fermi Sph.
⊥ to B

$$\therefore \frac{1}{B'} - \frac{1}{B''} = \frac{e\hbar}{m\left(\frac{\hbar^2\Omega_F}{2m\pi}\right)} = \frac{2\pi e}{\hbar\Omega_F}$$

When B' and B'' are field intensities associated with subsequent peaks/dips.

Recall



Oscillation would happen when LL cylinder crosses the "neck".

We get cross section of Fermi surface!