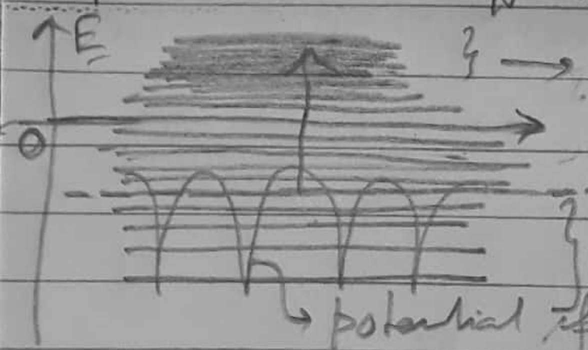
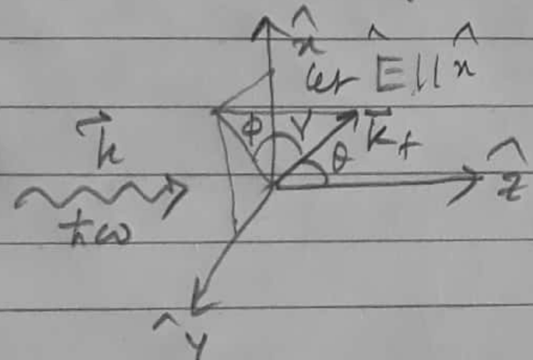
Photoelectric effect: An extreme example of stimulated process.



The up arrow indicates transition characteristic of photoelectric

Incident photons excite electron from valence (say, 1s state) to continuum of states leading to ejection of electron from matter.

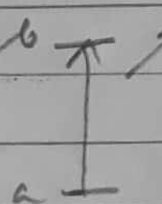


The ejected electron goes to a free electron state $E_f = \dfrac{\hbar^2 |\vec{k}_f|^2}{2m} \simeq \hbar\omega$

$$\phi_b \equiv \phi_f = V^{-1/2} \exp(-i \vec{k}_f \cdot \vec{r})$$

$$\phi_a \equiv \phi_{1s}$$

We assume here that $E_f \gg E_{1s}$



Recall $\sigma_{ba} = \dfrac{W^{st}_{ba}\, \hbar\omega_{ba}}{I(\omega_{ba})} = \left[\left(\dfrac{e^2}{4\pi\epsilon_0}\right) \dfrac{\pi^2}{m^2 c} \dfrac{I(\omega_{ba})}{\omega_{ba}^2} |M_{ba}|^2 \right] \dfrac{\hbar\omega_{ba}}{I(\omega_{ba})}$

$$= \left(\frac{e^2}{4\pi\epsilon_0}\right) \frac{\hbar \pi^2}{m^2 \omega_{ba} c} |M_{ba}|^2$$

$$= \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right) \frac{\hbar^2 \pi^2}{m^2 \omega_{ba}} |M_{ba}|^2 = \frac{\hbar^2 \pi^2 \alpha}{m^2 \omega_{ba}} |M_{ba}|^2$$

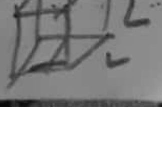
$= \frac{1}{137}$ fine structure constant.

In our case $|M_{ba}|^2$ is a function of $\vec{k}_f$ assuming "a" is 1s and "b" is ϕ_f

$$\sigma_{tot} = \frac{\pi^2 \alpha \hbar^2}{m^2} \int_b d\vec{k}_f \frac{|M_{ba}|^2}{\omega_{ba}} D(\vec{k}_f);$$

Note $D(\vec{k}_f)\, d\vec{k}_f = \dfrac{k_f^2\, dk_f\, d\Omega}{\left(\frac{2\pi}{L}\right)^3}$

Assuming $V = L^3$



$$\Delta(k_f)\,dk_f = \frac{V k_f^2\, dk_f\, d\Omega}{(2\pi)^3} = \frac{V}{(2\pi)^3} k_f \frac{m\, d\omega}{\hbar} \quad \Big| \because \hbar\omega = \frac{\hbar^2 k_f^2}{2m}$$

$$\therefore \sigma_{tot} = \frac{\pi^2 \alpha \hbar^2}{m^2} \int \frac{V}{(2\pi)^3} \frac{k_f\, m}{\omega(k_f)} |M(\vec{k}_f)|^2\, d\Omega\, d\omega(k_f)$$

$$\therefore \sigma_{tot}(\omega) = \frac{\pi^2 \alpha \hbar^2}{m^2 \omega} \frac{V m}{\hbar (2\pi)^3} k_f \int |M(\vec{k}_f)|^2 d\Omega$$

$$\therefore \frac{d\sigma(\omega)}{d\Omega} = (2\pi)^{-3} \frac{\pi^2 \alpha \hbar}{m} \left(\frac{k_f}{\omega}\right) |V^{1/2} M(\vec{k}_f)|^2,$$ Note k_f is fn of ω

$$V^{1/2} M(\vec{k}_f) = V^{-1/2} \int e^{-i\vec{k}_f\cdot\vec{r}} e^{i\vec{k}\cdot\vec{r}}\, \hat{E}\cdot\nabla \phi_{1s}(\vec{r})\, d\vec{r}$$

$$\nabla \int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}} \phi_{1s}(\vec{r})\, dr = \int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}} \nabla \phi_{1s}(r)\, dr + \int \left(\nabla e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}}\right) \phi_{1s}(r)\, dr$$

$\parallel$

0

$\because \phi_{1s}(\vec{r})$ is bound so $\to 0$ at ∞ $\Rightarrow \int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}}\, \hat{E}\cdot\nabla\phi_{1s}(\vec{r})\, d\vec{r}$

$$= -i\hat{E}\cdot(\vec{k}-\vec{k}_f) \int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}} \phi_{1s}(\vec{r})\, d\vec{r}$$

Recall the directions chosen (prev. page)

$$\hat{E}\cdot(\vec{k}-\vec{k}_f) = -\hat{E}\cdot\vec{k}_f = -|\vec{k}_f|\cos\theta = -|\vec{k}_f| \sin\theta\cos\phi$$

$$\int e^{i(\vec{k}-\vec{k}_f)\cdot\vec{r}} \phi_{1s}\, d\vec{r} = \int e^{i\vec{K}\cdot\vec{r}} \phi_{1s}(\vec{r})\, d\vec{r} \underset{\text{FT of } \phi_{1s}}{=} 8\pi \left(\frac{Z^3}{\pi a_0^3}\right)^{1/2} \frac{(Z/a_0)}{[(Z/a_0)^2 + K^2]^2} = V^{1/2} M$$

$$K^2 = k^2 + k_f^2 - 2kk_f\cos\theta = k_f^2\left[\frac{k^2}{k_f^2} + 1 - 2\frac{k}{k_f}\cos\theta\right]$$

Now $\hbar\omega \simeq \frac{\hbar^2 k_f^2}{2m} \Rightarrow \hbar kc \simeq \frac{\hbar^2 k_f^2}{2m} \Rightarrow \frac{k}{k_f} = \frac{\hbar k_f}{2mc} = \frac{v_f}{2c}$ which is much less than 1

$$\therefore K^2 \simeq k_f^2\left[1 - \frac{v_f\cos\theta}{c}\right]$$

Also, since $\hbar\omega \simeq \frac{\hbar^2 k_f^2}{2m} \gg |E_{1s}|$; $E_{1s} = -Z^2(e^2/4\pi\epsilon_0)/2a_0$

$$\Rightarrow k_f^2 \gg \frac{2m|E_{1s}|}{\hbar^2} \Rightarrow k_f^2 a_0^2 \gg \frac{2m|E_{1s}|}{\hbar^2}a_0^2 \; ; \; a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}$$

$$k_f^2 a_0^2 \gg \frac{2m}{\hbar^2}\frac{Z^2}{2}\left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{4\pi\epsilon_0\hbar^2}{me^2}\right)$$

$$k_f^2 a_0^2 \gg Z^2$$

$$Z^2 + k^2 a_0^2 = Z^2 + a_0^2 k_f^2 \left(1 - \frac{v_f}{c}\cos\theta\right) \approx a_0^2 k_f^2 \left(1 - \frac{v_f}{c}\cos\theta\right)$$

$\because Z \ll k_f^2 a_0^2$ shown in prev page

$$\frac{d\sigma(\omega)}{d\Omega} = (2\pi)^{-3} \frac{\pi^2 \alpha \hbar}{m}\left(\frac{k_f}{\omega}\right)\left[8\pi\left(\frac{Z^3}{\pi a_0^3}\right)^{1/2} \frac{(Z/a_0)(-k_f \sin\theta\cos\phi)}{\left[(Z/a_0)^2 + k^2\right]^2}\right]^2$$

$$= \frac{64}{8}\alpha\frac{\hbar}{m}\left(\frac{k_f^3}{\omega}\right)\left(\frac{Z}{a_0}\right)^5 \frac{a_0^2 \sin^2\theta\cos^2\phi}{\left[Z^2 + a^2 k^2\right]^4}$$

$$= 8\alpha\frac{\hbar}{m}\left(\frac{k_f^3}{\omega}\right)\left(\frac{Z}{a_0}\right)^5 \frac{a_0^8 \sin^2\theta \cos^2\phi}{a_0^8 k_f^8 \left(1 - \frac{v_f}{c}\cos\theta\right)^4}$$

→ average of $\cos^2\phi$ over ϕ for all incident angles of polarisation $\approx \frac{1}{2}$ (see figure)

$$= 4\alpha\left(\frac{\hbar}{m}\right)\frac{1}{\omega}\left(\frac{Z}{a_0 k_f}\right)^5 \sin^2\theta\left(1 + 4\frac{v_f}{c}\cos\theta\right)$$

for all (θ, ϕ) direction: $\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \int \left[\frac{d\sigma}{d\Omega}\right] \sin\theta\, d\theta\, d\phi$

for all emitted directions.

$$\sigma(\omega) = 4\alpha\left(\frac{\hbar}{m}\right)\frac{1}{\omega}\left(\frac{Z}{a_0 k_f}\right)^5 \left(\frac{4}{3} \times 2\pi\right)$$

$$= \frac{32}{3}\pi\alpha\left(\frac{\hbar}{m}\right)\frac{1}{\omega}\left(\frac{Z}{a_0 k_f}\right)^5$$

Note $\hbar\omega \approx \frac{\hbar^2 k_f^2}{2m}$

$k_f = \sqrt{\frac{2m\omega}{\hbar}}$

$$= \frac{32\pi}{3}\alpha\frac{1}{m}\left(\frac{\hbar}{\omega}\right)\frac{Z^5}{a_0^5}\left(\frac{2m\omega}{\hbar}\right)^{-5/2}$$

$$= \frac{128\sqrt{2}}{3}\pi\alpha\frac{1}{m^{7/2}}\frac{\hbar^{7/2}}{a_0^5} Z^5 \omega^{-7/2}$$

$\therefore \sigma \propto \omega^{-7/2}$ and $\sigma \propto Z^5$ → Applicable for ionization of one electron atoms and ions and ejection of electron from inner core levels by X ray although Dipole approx may not be valid