

1. Suppose $z = x^2 + y^3$, where $x = st$, and y is an unknown function of s and t . But we know that when $(s, t) = (2, 1)$, then $\frac{\partial y}{\partial t} = 0$. Find $\frac{\partial z}{\partial t}(2, 1)$.

2. Suppose the following function is used to model the monthly demand for bicycles:

$$P(x, y) = 200 + 20\sqrt{0.1x + 10} - 12\sqrt[3]{y}$$

In this formula, x represents the price (in dollars per gallon) of automobile gasoline and y represents the selling price (in dollars) of each bicycle. Furthermore, suppose the price of gasoline t months from now will be

$$x = 1 + 0.1t - \cos \frac{\pi t}{6}$$

and the price of each bicycle will be

$$y = 200 + 2t \sin \frac{\pi t}{6}$$

At what rate will the monthly demand for bicycles be changing in 6 months from now?

3. Let $z = x^3 + y^3 + 3xy$, where $x = e^r \cos \theta$, and $y = e^r \sin \theta$. Calculate $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$, at $x = \frac{e}{2}$, and $y = \frac{e\sqrt{3}}{2}$.

4. Consider the curve given by the equation

$$5x^2 + 5y^2 + 6xy = 4$$

Find the points on the curve where the tangent line is parallel to x axis or the line $x + y = 0$. (Hint. use implicit differentiation)

5. Find the tangent plane to the surface $9z^2 - x^2 + 9y^2 = 9$ at $(1, 1, \frac{1}{3})$, and $(4, 1, \frac{4}{3})$.

6. Suppose a rain drop falls on the surface given by the equation $z = 2x^2 + 5y^2$, at the point $(-2, 1, 13)$. In which direction will the rain drop start falling? (Hint. It will fall in the direction of steepest **downward** slope at that point.)

7. Find and analyze all the critical points of $f(x, y) = 2x^3y - y^2 - 3xy$, using the second derivative test.

8. Find the global extrema for the function $g(x, y) = x^3 + y^3 - 3x - 3y$, in the region $\{(x, y) \mid -2 \leq x \leq 2, -2 \leq y \leq 2\}$.