1. Calculate the arc lengths of the following curves between the given values of the parameter
   (a) $\overrightarrow{r}(t) = \langle t\sin t + \cos t,\ t\cos t - \sin t,\ t^2\rangle, \quad 0 \le t \le 2\pi$
   (b) $\overrightarrow{r}(t) = \langle t^2, t^3, t^3\rangle, \quad 0 \le t \le 1$

2. For the curve $\overrightarrow{r}(t) = \langle e^t\cos t,\ e^t\sin t,\ \sqrt{2}e^t\rangle$
   (a) Find the arc length function with respect to the point where $t = 0$ and in the direction of increasing t.
   (b) Parametrize the curve with respect to the arc leangth as measured in (a).
   (c) Find the curvature of the curve at the point where $t = 0$.

3. Find the unit tangent, normal and binormal vectors, and the curvature of the following curve at the points $(2, 0, 0)$ and $(0, 3, 2\pi)$. (Hint. use the cross product formula for curvature)

$$\overrightarrow{r}(t) = \langle 2\cos t,\ 3\sin t,\ 4t\rangle$$

4. Given a curve $\overrightarrow{r}(t)$, we shall prove the folowing formula for curvature using the steps below

$$\kappa(t) = \frac{||\overrightarrow{r}'(t) \times \overrightarrow{r}''(t)||}{||\overrightarrow{r}'(t)||^3}$$

   (a) Show that $\overrightarrow{T}(t) \perp \overrightarrow{T}'(t)$ and that $||\overrightarrow{T}(t) \times \overrightarrow{T}'(t)|| = ||\overrightarrow{T}'(t)||$, where $\overrightarrow{T}(t)$ is the unit tangent.
   (b) Note that $\overrightarrow{r}'(t) = ||\overrightarrow{r}'(t)||\overrightarrow{T}(t)$, use this to show that

$$\overrightarrow{r}''(t) = \frac{d||\overrightarrow{r}'(t)||}{dt}\overrightarrow{T}(t) + ||\overrightarrow{r}'(t)||\overrightarrow{T}'(t)$$

   (c) Calculate $\overrightarrow{r}'(t) \times \overrightarrow{r}''(t)$ and show that $||\overrightarrow{r}'(t) \times \overrightarrow{r}''(t)|| = ||\overrightarrow{r}'(t)||^2||\overrightarrow{T}'(t)||$.
   (d) Now infer that the above formula for curvature holds. (Remember curvature was $\kappa(t) = \frac{||\overrightarrow{T}'(t)||}{||\overrightarrow{r}'(t)||}$)

5. Find the position function of the particle moving in space with
   (a) innitial velocity $\langle -2, 1, -1\rangle$, innitial position $< 1, 1, 1 >$ and has acceleration $\overrightarrow{a}(t) = \langle 1, t, t^2\rangle$.
   (b) mass 2kg, innitial velocity $\langle 0, 0, 1\rangle$, innitial position $\langle 1, 0, 0\rangle$, and is acted upon by a force $\langle -2\cos t, -2\sin t, 0\rangle$ in Newtons.

6. Find the velocity and acceleration of the following particles whose position functions are given
   (a) $\vec{r}(t) = \langle t,\ e^t,\ e^{2t} \rangle$ (b) $\vec{r}(t) = \left\langle \frac{1}{1+t^2},\ \cos 2t,\ \sin 2t \right\rangle$

7. Two particles are moving in space and their positions functions are given by the

$$\vec{r}(t) = \langle e^t,\ \ln(1+t),\ t \rangle \qquad \vec{q}(t) = \langle \cos t, \sin t,\ 5t \rangle$$

   Do the particles collide? Do their paths intersect?

8. (Bonus) Michael Jordan shoots the basketball from a spot that is 18m south and 15m east of the basket. He shoots the ball from a height of 2m, and the basket is at a height of 3m from the ground. As expected MJ makes the basket and the ball takes just 3s to travel to the basket. If there is an air drag of 2N towards the west and the ball weighs 1kg what innitial velocity did MJ shoot the ball with? (Take acceleration due to gravity to be $10m/s^2$)