

1. Calculate the following

- (a) Rectangular coordinates of the following points given in terms of their polar coordinates (r, θ) : $(2\sqrt{3}, \frac{\pi}{3})$ and $(-5, \frac{3\pi}{2})$
- (b) All the polar representations of the following points given in terms of rectangular coordinates: $(\sqrt{6}, \sqrt{2})$ and $(-3, 3)$

2. Sketch the following curves given by polar equations. First plot the points corresponding to the following values of θ : $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, 2\pi$

(a) $r = \frac{12\theta}{\pi}$

(b) $r = \frac{\theta \sin \theta}{\pi}$

3. Convert the following equations. Can you guess what the curves are?

(a) Rectangular to polar: $x^2 + y^2 = 2xy$

(b) Polar to rectangular: $r = 2 \sin \theta$

4. Limits and continuity

(a) $\vec{r}(t) = \left\langle \frac{t}{e^t}, \cos t, \frac{\ln(1+t)}{t} \right\rangle$. For what values of t is \vec{r} continuous? Calculate $\lim_{t \rightarrow 0} \vec{r}(t)$.

(b) $\vec{r}(t) = \langle \tan t, \cot t, t \rangle$. For what values of t is \vec{r} continuous. Calculate $\lim_{t \rightarrow \pi} \vec{r}(t)$.

5. Sketch the following curves (plot at least 5 points on each curve)

(a) $\vec{r}(t) = \langle t, 2t \sin t, 3t \cos t \rangle, \quad t \geq 0$

(b) $\vec{r}(t) = \langle t^3, t^2 \rangle$

6. For a curve given by a vector function \vec{r} , the **tangent vector** at $\vec{r}(t)$ is the vector $\vec{r}'(t)$. The **unit tangent** at $\vec{r}(t)$ is the unit vector along $\vec{r}'(t)$, and the **tangent line** at $\vec{r}(t)$ is the line passing through $\vec{r}(t)$ and having $\vec{r}'(t)$ as the direction vector. Let

$$\vec{r}(t) = \left\langle (4 + \cos 4t) \cos t, (4 + \cos 4t) \sin t, \sin 4t \right\rangle, \quad 0 \leq t \leq 2\pi$$

(a) Calculate $\vec{r}'(t)$.

(b) What are the tangent and unit tangent vectors at $\vec{r}(\pi)$ and the point $\left\langle \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}, 0 \right\rangle$?

(c) What are the tangent lines at those two points to the curve?

7. Evaluate the following the following $\left(\frac{d}{dt} \vec{r}(t) = \vec{r}'(t)\right)$

(a) $\frac{d}{dt} \langle t^2, e^t, \ln t \rangle$

(b) $\frac{d}{d\theta} \langle 2(\theta - \sin \theta), 2(1 - \cos \theta) \rangle$

(c) $\int_0^{10} \langle t, t^2, t^3 \rangle dt$

(d) $\int_0^{2\pi} \langle 2(\theta - \sin \theta), 2(1 - \cos \theta) \rangle d\theta$

(e) $\int_{-1}^1 \langle 2t\sqrt{1+t^2}, e^t, 2t^2+1 \rangle dt$

8. **(Bonus)** Consider the sun located at the origin $O = (0, 0)$, the earth (E) is rotating around the sun at a distance $10r$ (at constant speed). The moon (M) is rotating around the earth at a distance r from the earth (again at constant speed relative to earth). During one complete rotation by earth around sun the moon rotates 10 times around earth.

(a) Find a parametric equation for the curve traced out by the moon.

(b) Sketch the curve that you obtained.

