Math 230 **Assignment 2** Due Monday, July 2 2012

Please write your answers neatly with explanation and show all your calculations. Stapling your papers will prevent them from being misplaced. Late homework will have a 20% penalty.

1. Find equations for the following (write equations for lines in both parametric and symmetric forms):
   (a) Line L passing through $(-2, 1, -3)$ and $(3, -2, -1)$.
   (b) Plane P passing through $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 3)$.
   (c) Line perpendicular to P and passing through $(0, 0, 0)$.
   (d) Line of intersection of the planes P and $x - 3y + 7z + 4 = 0$.
   (e) Plane containing the line L and perpendicular to plane P.

2. For each pair of lines among L_1, L_2 and L_3, check whether they are parallel, intersecting or skew. For intersecting pairs find the point of intersection and for parallel pairs find the distance between them.
$$L_1 : \ \vec{r} = \langle 1, -1, 2\rangle + t\langle 1, 1, 1\rangle$$
$$L_2 : \ x = 2 - s, \ y = -2 + s, \ z = 3 + s$$
$$L_3 : \ x = -y = 3 - z$$

3. For the pairs of planes below, determine if they are parallel or intersecting. If intersecting, find the angle between them and if parallel, the distance between them.
   (a) $P_1 : \ x + 2y + 3z = 4$ and $P_2 :$ the plane of points that are equidistant from the points $(5, 7, 11)$ and $(-3, 5, -7)$
   (b) $P_1 : \ x + y + z = 0$ amd $P_2 :$ plane containing the points $(1, 1, 1)$, $(3, 0, 0)$ and $(1, 0, 2)$.

4. Consider the sphere
$$S : \ 2x^2 + 2y^2 + 2z^2 - 2x + 6y + 2z + 1 = 0$$
   Find the equation of the tangent plane to S at $\left(1, -\frac{1}{2}, \frac{1}{2}\right)$. (Hint. The tangent plane to a sphere at a point, passes through that point and is perpendicular to the line joining the center of the sphere to that point.)

5. Consider the skew lines $L_1 : \ \vec{r} = t\langle 2, 2, 1\rangle$ and $L_2 : \ \vec{r} = \langle 1, -3, 3\rangle + s\langle -1, 0, 1\rangle$.
   (a) Find planes P_1 containing L_1 and P_2 containing L_2 that are parallel to each other.
   (b) Find the distance between the skew lines, which is the distance between the parallel planes.

6. Sketch the following surfaces. Show the traces for each of the planes $x = k, y = k$ and $z = k$, for atleast 2 values of k. Identify each surface.

   (a) $z = \sin y$ (b) $x = 4y^2 - z^2$

   (c) $4x^2 - 16y^2 + z^2 = 16$ (d) $x^2 - y^2 + z^2 - 4x + 4y + 6z + 17 = 0$

7. Find the equation of the following surfaces and sketch them.

   (a) The surface of points that are equidistant from the point $(0, 0, 2)$ and the plane $z + 2 = 0$.

   (b) The surface obtained by taking the curve $x^2 - 4z^2 = 4$, in the $y = 0$ plane and rotating it about the z-axis.

8. (**Bonus**) Answer the following.

   (a) Let the parametric equation of two parallel lines be $\vec{r} = \vec{a} + s\,\vec{b}$, and $\vec{r} = \vec{c} + t\,\vec{b}$. Show that the distance between them is

$$\left|\left|(\vec{c} - \vec{a}) - \frac{(\vec{c} - \vec{a}) \cdot \vec{b}}{||\vec{b}||^2}\,\vec{b}\right|\right|$$

   (b) Show that the distance between two parallel planes $ax+by+cz+d = 0$ and $ax+by+cz+d' = 0$ is given by

$$\frac{|d - d'|}{\sqrt{a^2 + b^2 + c^2}}$$

   (c) Find the point of intersection of the following three planes: $2x-2y+2z-1 = 0$, $x+2y+z = 0$ and $3x - 3z + 4 = 0$.