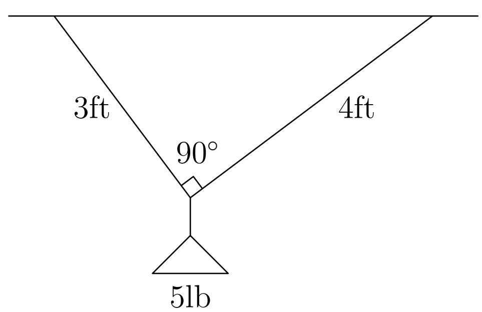
Math 230 **Assignment 1** Due Monday, June 25 2012

Please write your answers neatly with explanation and show all your calculations. Stapling your papers will prevent them from being misplaced. Late homework will have a 20% penalty.

1. Consider the points $P = (5, -4, -5)$, $Q = (2, -3, -3)$, and $R = (4, -5, -3)$. Let $\triangle PQR$, be the triangle with vertices P, Q, R.
   (a) Find the length of the sides of $\triangle PQR$.
   (b) Show that it is a right angled triangle. Which vertex has the right angle, and which side is the hypotenuse?
   (c) Let S be the midpoint of the hypotenuse. Find the equation of the sphere that has center S and passes through R. Do Q and P lie on the sphere too?

2. Let $A = (2, -1, 4)$, $B = (-2, -3, 8)$ and $C = (0, 3, 5)$.
   (a) Express the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of the standard basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
   (b) Find the point D such that the points A, B, C, D form a parallelogram with D, the vertex opposite to A. (Hint. Use vector addition)
   (c) Find the unit vector in the direction of $\overrightarrow{AD}$.

3. Ropes 3 feet and 4 feet in length are fastened to a weight of 5 pounds. The ropes are attached to the roof at points so that they make a right angle at the top of the weight. Find the tension in each rope.



4. Consider the regular tetrahedron formed by the points $P = (1, 0, 0), Q = (0, 1, 0), R = (0, 0, 1), S = (1, 1, 1)$. Their centroid is the point $C = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. This is the configuration of the methane molecule CH_4, with hydrogen atoms (H) at the vertices of the tetrahedron and the carbon atom (C) at the centroid. (See problem 39 section 10.3 of textbook for a diagram.)
   (a) Find the angle between the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$. (use calculator if necessary)
   (b) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. What type of triangle is $\triangle PQR$?
   (c) Find the projections of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$ and $\overrightarrow{PC}$ onto $\overrightarrow{PR}$.

5. Let $\overrightarrow{u} = \langle 1,1,1\rangle$, $\overrightarrow{v} = \langle -1,1,1\rangle$. Calculate the following,
   (a) $\overrightarrow{u} \cdot \overrightarrow{v}$,
   (b) $\overrightarrow{w} = \overrightarrow{v} - \text{proj}_{\overrightarrow{u}} \overrightarrow{v}$,
   (c) angle between $\overrightarrow{w}$ and $\overrightarrow{u}$,
   (d) $\overrightarrow{x} = \overrightarrow{u} \times \overrightarrow{v}$,
   (e) volume of the parallelepiped formed by $\overrightarrow{u}, \overrightarrow{v}$ and $\overrightarrow{x} + \overrightarrow{w}$.

6. Let $\overrightarrow{a}$, $\overrightarrow{b}$, be vectors
   (a) If $\overrightarrow{v} = s\overrightarrow{a} + t\overrightarrow{b}$, where s, t are scalars. What is $\overrightarrow{v} \cdot (\overrightarrow{a} \times \overrightarrow{b})$?
   (b) If $\overrightarrow{c} \cdot (\overrightarrow{a} \times \overrightarrow{b}) = 0$ what can you say about the vector $\overrightarrow{c}$?
   (c) Show that there are non-zero vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ such that $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$, but $\overrightarrow{b} \neq \overrightarrow{c}$.
   (d) Show that there are non-zero vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{c}$, but $\overrightarrow{b} \neq \overrightarrow{c}$.
   (e) Show that if $\overrightarrow{a} \neq \overrightarrow{0}$, and $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{c}$, then $\overrightarrow{b} = \overrightarrow{c}$.

7. (**Bonus**) Platonic solids are regular polyhedrons which have congruent regular polygons for all its faces, and same number of faces meeting at each vertex. See `http://en.wikipedia.org/wiki/Platonic_solid`.
   Consider the octahedron with vertices $A = (s,0,0), B = (0,s,0), C = (-s,0,0), D = (0,-s,0), E = (0,0,s), F = (0,0,-s)$. It has 6 vertices, 8 faces each an equilateral triangle, and 12 edges.
   (a) Calculate the angle between $\overrightarrow{u} = \overrightarrow{AE} \times \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{AD} \times \overrightarrow{AE}$. This is the Dihedral angle, the angle between any two faces. (use a calculator)
   (b) Calculate the radius and center of the inscribed sphere, the sphere which is tangent to each face at its center. (Hint. use symmetry to locate the center)
   (c) Calculate the surface area. (Hint. first compute the area of a face)