Submission time 17:00. All problems are worth 20 points.

1. Let L/K be an extension of finite fields. Show that it is a Galois extension and the Galois group is cyclic.

2. Prove or disprove: There exists a positive integer n such that the cyclotomic extension $\mathbb{Q}(\zeta_n)$ contains a sub-field which is not Galois over $\mathbb{Q}$.

3. Let L/K be the splitting field of a separable polynomial $f \in K[x]$. If $\alpha_1, \ldots, \alpha_n$ are the roots of f in L show that
$$D_f = \pm \prod_{i=1}^{n} f'(\alpha_i)$$
where f' is the derivative of f and D_f is the discriminant of f. Show that L contains a square root of D_f.

4. Show that for any odd prime p, $\mathbb{Q}(\zeta_p)$ contains a unique quadratic extension of $\mathbb{Q}$ given by $\mathbb{Q}(\sqrt{\pm p})$. (Hint. Problem 3.)

5. Find the Galois group of the splitting field over $\mathbb{Q}$ of $x^4 + x^2 + 1 \in \mathbb{Q}[x]$.