1. Let $F = \mathbb{Q}(\zeta_n)$ and $a \in F$, let $a^{1/n} \in \mathbb{C}$ be a root of $x^n - a \in F[x]$. Consider the multiplicative group $F^\times$ and let $(F^\times)^n$ be the sub-group of the n-th powers of elements of $F^\times$. Show that $\mathrm{Gal}(F(a^{1/n})/F)$ is isomorphic to the cyclic sub-group of $F^\times/(F^\times)^n$ generated by a (we showed in class that it is a sub-group of $\mathbb{Z}/n\mathbb{Z}$). Determine the group $\mathrm{Gal}(F(a^{1/n})/F)$ when

   (a) $n = 12$ and $a = 16$,

   (b) $n = 5$ and $a = 4$.

2. Let K/F be a finite separable extension and L/F a finite Galois extension such that $K \subset L$. Choose a set of left coset representatives $\{g_1, \ldots, g_n\}$ for $\mathrm{Gal}(L/F)/\mathrm{Gal}(L/K)$. Show that the composite field $K' = (g_1K)\cdots(g_nK)$ is Galois over F. Prove this is the smallest Galois extension of F in L containing K (i.e. any other such extension must contain K').

3. Find the roots of the following cubic polynomials over $\mathbb{Q}$ using Cardano's formula:

   (a) $x^3 + x^2 - 2x - 1$,

   (b) $x^3 - 3x + 1$.

4. Let $f(x) \in \mathbb{Q}[x]$ be an irreducible cubic polynomial and E its splitting field over $\mathbb{Q}$. If D_f the discriminant of f is positive show that $E/\mathbb{Q}$ is not a radical extension.

5. Show that S_5 is generated by any 5-cycle and any transposition.

6. Let K/F be a radical extension and $\sigma : K \to K'$ an isomorphism, such that $\sigma(F) = F'$ show that K'/F' is also a radical extension.

7. Express $\cos \frac{2\pi}{17}$ in terms of radicals. (Hint. Dummit and Foote section on solvabity by radicals).