1. Determine the Galois groups of the folowing polynomials:
(a) $x^3 - x^2 - 4$,
(b) $x^3 - x + 1$,

2. Determine the Galois group of $x^4 + 8x + 12$.

3. Determine the Galois group of $x^4 + 2x^2 + x + 3$.

4. Show that $\mathbb{Q}(\sqrt[3]{2})$ is not a sub-field of any cyclotomic extension of $\mathbb{Q}$.

5. Let $\pm\alpha, \pm\beta$ be the roots of $f(x) = x^4 + ax^2 + b \in \mathbb{Z}[x]$. Prove that $f(x)$ is irreducible if and only if $\alpha^2, \alpha \pm \beta \notin \mathbb{Q}$. If $f(x)$ is irreducible and G is the Galois group of the splitting field of f over $\mathbb{Q}$ then show that
(a) $G \cong V$ the Klein 4 group if and only if $\alpha\beta \in \mathbb{Q}$,
(b) $G \cong \mathbb{Z}/4\mathbb{Z}$ if and only if $b(a^2 - 4b)$ is a square in $\mathbb{Q}$,
(c) $G \cong D_8$ the dihedral group if and only if b and $b(a^2 - 4b)$ are not squares in $\mathbb{Q}$.

6. Prove that $x^4 + px + p \in \mathbb{Q}[x]$ is irreducible for all primes $p \in \mathbb{Z}$. If $p \neq 3, 5$ then show that the Galois group G of the splitting field over $\mathbb{Q}$ is S_4. For $p = 3$ show that $G \cong D_8$ and when $p = 5$, $G \cong \mathbb{Z}/4\mathbb{Z}$.

7. Show that any finite extension of $\mathbb{F}_{p^n}$ is Galois. What is the Galois group?

8. Find all the monic irreducible poynomials of degree 4 over $\mathbb{F}_2$.