Submit problems 2, 7 and 11 by Thursday, August 24. Concepts covered: Irreducibility criteria, characteristic of a field, field extensions. Reading: Rotman section ?.

1. If K is a finite field of characteristic p then show that $\phi_p : K \to K$ given by $\phi_p(a) = a^p$ is a field automorphism. This is called the **Frobenius automorphism**. What is this automorphism for $\mathbb{Z}/p\mathbb{Z}$?

2. Problem 64, page 43 of Rotman.

3. Problem 65, of Rotman.

4. Let R be an integral domain containing a field F. If R is a finite dimensional vector space over F show that R is a field. Is this true without the finite dimensional assumption? (Artin)

5. (Artin) If F is a field containing exactly 8 elements what is the characteristic of F, prove your statement.

6. Show that any finite field has to be of order p^n for some prime integer p.

7. (Artin) Determine the irreducible polynomial of $\sqrt{3} + \sqrt{5}$ over the following fields
   (a) $\mathbb{Q}$ (b) $\mathbb{Q}(\sqrt{5})$ (c) $\mathbb{Q}(\sqrt{10})$ (d) $\mathbb{Q}(\sqrt{15})$.

8. (Artin) Let $\alpha = \sqrt[3]{2}$ the real cube root of 2 compute the irreducible polynomial of $1 + \alpha^2$ over $\mathbb{Q}$.

9. Show that $p(x) = x^3 - 3x + 3$ is irreducible in $\mathbb{Q}[x]$. Let $a \in \mathbb{C}$ be a root of p, find the inverse of $a^2 + a + 1$ explicitly in terms of powers of a.

10. (Artin) Let K/F be an algebraic extension. If $\alpha \in K$ and $[F(\alpha) : F] = 5$ show that $F(\alpha) = F(\alpha^2)$. In other words α can be expressed as a linear combination of even powers of α. Give an example of this situation when $F = \mathbb{Q}$ writing down the explicit expression.

11. (Artin) Let a be a positive integer that is not a perfect square. Show that $[\mathbb{Q}(\sqrt[4]{a}) : \mathbb{Q}] = 4$.

12. (Artin) Determine whether $i = \sqrt{-1}$ is in the fields
    (a) $\mathbb{Q}(\sqrt{-2})$ (b) $\mathbb{Q}(\sqrt[4]{-2})$ (c) $\mathbb{Q}(\alpha)$ where $\alpha \in \mathbb{C}$ and $\alpha^3 + \alpha + 1 = 0$.