Please answer all questions. Total time 50 minutes.

1. Prove or disprove:
   (a) Any finite integral domain is a field. (5)

   (b) Any prime ideal in a finite ring is maximal. (5)

2. Prove the following:
   (a) Let $0 \to M' \xrightarrow{f} M \xrightarrow{g} M'' \to 0$ be an exact sequence of R modules. If M' and M'' are both finitely generated, then so is M. (5)

   (b) Let M and N be submodules of an R module L. If $M + N$ and $M \cap N$ are both finitely generated then M and N are also finitely generated. (5)

3. Let R be a ring and N a flat R module. If

$$M' \xrightarrow{f} M_i \xrightarrow{g} M''$$

is an exact sequence of R modules, show that

$$M' \otimes N \xrightarrow{f \otimes \mathrm{Id}} M \otimes N \xrightarrow{g \otimes \mathrm{Id}} M'' \otimes N$$

is also exact. (Hint. Look at Ker f and Im g.) (10)