ON THE NUMBER OF REPRESENTATIONS BY CERTAIN OCTONARY QUADRATIC FORMS WITH COEFFICIENTS 1, 2, 3, 4 AND 6

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ABSTRACT. In this paper, we find formulas for the number of representations of certain diagonal octonary quadratic forms with coefficients $1, 2, 3, 4$ and 6. We obtain these formulas by constructing explicit bases of the space of modular forms of weight 4 on $\Gamma_0(48)$ with character.

1. INTRODUCTION

Let $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$ denote the set of positive integers, non-negative integers and integers respectively. For $a_1, \ldots, a_8 \in \mathbb{N}$ and $n \in \mathbb{N}_0$, we define

$$N(a_1, \ldots, a_8; n) := \text{card}\left\{(x_1, \ldots, x_8) \in \mathbb{Z}^8 | n = a_1 x_1^2 + \cdots + a_8 x_8^2\right\}.$$

Note that $N(a_1, \ldots, a_8; 0) = 1$. Without loss of generality we may assume that

$$a_1 \leq a_2 \leq \cdots \leq a_8 \text{ and } \gcd(a_1, \ldots, a_8) = 1.$$

Formulae for $N(a_1, \ldots, a_8; n)$ for the octonary quadratic forms

$$\sum_{r=1}^{i} x_r^2 + 2\sum_{r=i+1}^{i+j} x_r^2 + 3\sum_{r=i+j+1}^{i+j+k} x_r^2 + 6\sum_{r=i+j+k+1}^{i+j+k+l} x_r^2 \tag{1}$$

for all the partitions $i + j + k + l = 8$, $i, j, k, l \geq 0$ appeared in the literature. When all of them (i, j, k, l) are even (there are 26 cases), it was obtained by several authors (see [1, 4, 6, 11, 13, 14, 20]). When all of them are odd (there are 10 cases), it was obtained in [5]. Recently, in [3], the authors considered the rest of the cases (mixed parity) and this completed all the cases for the coefficients $1, 2, 3$ or 6. In [15] the author considers one octonary quadratic form with coefficients $2, 3, 6$ and 12. A few cases of the coefficients $1, 2, 3, 6$ are also considered in this paper. It is to be noted that various methods were used to obtain these formulas such as elementary evaluations, using the method of convolution sums of the divisor functions and the theory of modular forms. However, in most of the cases modular forms techniques were used.

Formulae for $N(a_1, \ldots, a_8; n)$ for the octonary quadratic forms

$$\sum_{r=1}^{i} x_r^2 + 2\sum_{r=i+1}^{i+j} x_r^2 + 4\sum_{r=i+j+1}^{i+j+k} x_r^2, \tag{2}$$

where $i + j + k = 8$, $(i, j, k) \in \{(7, 0, 1), (1, 0, 7), (6, 0, 2), (2, 0, 6), (5, 2, 1), (1, 2, 5), (5, 0, 3), (3, 0, 5), (4, 2, 2), (2, 2, 4), (2, 4, 2), (4, 0, 4), (3, 4, 1), (3, 2, 3), (1, 6, 1), (1, 4, 3)\}$ were obtained in [1, 2]. There are a total of 38 cases with coefficients $1, 2, 4$ and out of which 10 of them are obtainable from the previous works. The remaining 12 cases are $\{(1, 1, 6), (1, 3, 4), (1, 5, 2), (2, 1, 5), (2, 3, 3), (2, 5, 1), (3, 1, 4), (3, 3, 2), (4, 1, 3), (4, 3, 1), (5, 1, 2), (6, 1, 1)\}$.

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In this paper, we first complete the 12 cases remaining for the coefficients $1, 2, 4$ and then extend the above works to find formulae for the diagonal octonary quadratic forms with coefficients $1, 2, 3, 4$ and 6 by using the theory of modular forms. More precisely, we consider the following octonary quadratic forms (with coefficients $1, 2, 3, 4, 6$):

$$\sum_{r=1}^{i} x_r^2 + 2\sum_{r=i+1}^{i+j} x_r^2 + 3\sum_{r=i+j+1}^{i+j+k} x_r^2 + 4\sum_{r=i+j+k+1}^{i+j+k+l} x_r^2 + 6\sum_{r=i+j+k+l+1}^{i+j+k+l+m} x_r^2, \tag{3}$$

where $i+j+k+l+m = 8$ and find their number of representations $N(a_1, \ldots, a_8; n)$. The corresponding theta series will be a modular form of weight 4 on $\Gamma_1(48)$. The case $l = 0$ is the case with coefficients $1, 2, 3, 6$, which was done earlier (as mentioned above) and so we take $l \neq 0$. When $m = 0$, there are 84 cases (with $k, l \neq 0$) which is given in Table 1. (Here we have not included the cases $k = 0$, which corresponds to the coefficients $1, 2, 4$ and $l = 0$, which corresponds to the coefficients $1, 2, 3$.) When $m \neq 0$, there are 210 cases which is given in Table 2. Out of these 210 cases, the following 7 cases $((0,0,0,1,7), (0,0,0,2,6), (0,0,0,3,5), (0,0,0,4,4), (0,0,0,5,3), (0,0,0,6,2), (0,0,0,7,1))$ can be obtained from the earlier results with coefficients $2, 3$ only. So, here we consider only the remaining 203 cases. In both the tables, we indicate the corresponding modular forms spaces. We also remark that the 21 cases $\{(0,4,0,2,2), (0,2,0,4,2), (0,2,0,2,4), (0,5,0,1,2), (0,1,0,3,4), (0,3,0,3,2), (0,1,0,5,2), (0,3,0,1,4), (0,1,0,1,6), (0,3,0,2,3), (0,1,0,4,3), (0,3,0,4,1), (0,5,0,2,1), (0,1,0,2,5), (0,1,0,6,1), (0,4,0,1,3), (0,2,0,3,3), (0,6,0,1,1), (0,2,0,1,5), (0,4,0,3,1), (0,2,0,5,1)\}$ can also be obtained from the results arising from the coefficients $1, 2, 3$. However, these cases are also kept in the respective tables. Since we consider different bases for the modular forms spaces, our formulas in these cases are also different from the previous formulas obtained by Alaca and Kesicioğlu [3, 4].

2. Statement of the Results

Let us first consider the 12 remaining cases of the quadratic forms given by (2) with $i+j+k = 8$ as mentioned in the introduction. We denote the number of representations of an integer n by these quadratic forms as $N(1^i, 2^j, 4^k; n)$. Then, the following theorem gives the representation numbers $N(1^i, 2^j, 4^k; n)$ for the 12 cases $(i,j,k) \in \{(1,1,6), (1,3,4), (1,5,2), (2,1,5), (2,3,3), (2,5,1), (3,1,4), (3,3,2), (4,1,3), (4,3,1), (5,1,2), (6,1,1)\}$.

Theorem 2.1. *Let $n \in \mathbb{N}$. Then*

$$\begin{aligned}
\text{(i)}\ N(1^1, 2^1, 4^6; n) &= \frac{2}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{6}{11}a_{4,8,\chi_8;1}(n) + \frac{14}{11}a_{4,8,\chi_8;2}(n) \\
&\quad + \frac{48}{11}a_{4,8,\chi_8;1}(n/2) - \frac{28}{11}a_{4,8,\chi_8;2}(n/2), \\
\text{(ii)}\ N(1^1, 2^3, 4^4; n) &= \frac{4}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{1}{11}a_{4,8,\chi_8;1}(n) + \frac{17}{11}a_{4,8,\chi_8;2}(n) \\
&\quad + \frac{48}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2), \\
\text{(iii)}\ N(1^1, 2^5, 4^2; n) &= \frac{8}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{2}{11}a_{4,8,\chi_8;1}(n) + \frac{12}{11}a_{4,8,\chi_8;2}(n) \\
&\quad + \frac{48}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2), \\
\text{(iv)}\ N(1^2, 2^1, 4^5; n) &= \frac{4}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{12}{11}a_{4,8,\chi_8;1}(n) + \frac{28}{11}a_{4,8,\chi_8;2}(n) \\
&\quad + \frac{92}{11}a_{4,8,\chi_8;1}(n/2) - \frac{28}{11}a_{4,8,\chi_8;2}(n/2),
\end{aligned}$$

$$\text{(v) } N(1^2,2^3,4^3;n) = \frac{8}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{2}{11}a_{4,8,\chi_8;1}(n) + \frac{34}{11}a_{4,8,\chi_8;2}(n) + \frac{92}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(vi) } N(1^2,2^5,4^1;n) = \frac{16}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{4}{11}a_{4,8,\chi_8;1}(n) + \frac{24}{11}a_{4,8,\chi_8;2}(n) + \frac{48}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(vii) } N(1^3,2^1,4^4;n) = \frac{8}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{13}{11}a_{4,8,\chi_8;1}(n) + \frac{45}{11}a_{4,8,\chi_8;2}(n) + \frac{136}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(viii) } N(1^3,2^3,4^2;n) = \frac{16}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{4}{11}a_{4,8,\chi_8;1}(n) + \frac{46}{11}a_{4,8,\chi_8;2}(n) + \frac{136}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(ix) } N(1^4,2^1,4^3;n) = \frac{16}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{4}{11}a_{4,8,\chi_8;1}(n) + \frac{68}{11}a_{4,8,\chi_8;2}(n) + \frac{180}{11}a_{4,8,\chi_8;1}(n/2) + \frac{104}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(x) } N(1^4,2^3,4^1;n) = \frac{32}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{8}{11}a_{4,8,\chi_8;1}(n) + \frac{48}{11}a_{4,8,\chi_8;2}(n) + \frac{136}{11}a_{4,8,\chi_8;1}(n/2) + \frac{16}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(xi) } N(1^5,2^1,4^2;n) = \frac{32}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) - \frac{14}{11}a_{4,8,\chi_8;1}(n) + \frac{92}{11}a_{4,8,\chi_8;2}(n) + \frac{224}{11}a_{4,8,\chi_8;1}(n/2) + \frac{192}{11}a_{4,8,\chi_8;2}(n/2),$$

$$\text{(xii) } N(1^6,2^1,4^1;n) = \frac{64}{11}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{2}{11}\sigma_{3;\mathbf{1},\chi_8}(n/2) - \frac{28}{11}a_{4,8,\chi_8;1}(n) + \frac{96}{11}a_{4,8,\chi_8;2}(n) + \frac{224}{11}a_{4,8,\chi_8;1}(n/2) + \frac{192}{11}a_{4,8,\chi_8;2}(n/2).$$

The terms appearing on the right-hand side of the above formulas are defined in §4.1.

Next we shall state the formulae for the quadratic forms with coefficients $1,2,3,4$ given in Table 1. We state them as four statements in the theorem, each statement corresponds to the four modular forms spaces that appear in the table.

Theorem 2.2. *Let $n \in \mathbb{N}$ and i,j,k,l be non-negative integers such that $i+j+k+l=8$.*
(i) *For each entry (i,j,k,l) in Table 1 corresponding to the space $M_4(\Gamma_0(48))$, we have*

$$N(1^i,2^j,3^k,4^l;n) = \sum_{\alpha=1}^{30} a_\alpha A_\alpha(n), \tag{4}$$

where $A_\alpha(n)$ are the Fourier coefficients of the basis elements f_α defined in §4.2 and the values of the constants a_α are given in Table 3.
(ii) *For each entry (i,j,k,l) in Table 1 corresponding to the space $M_4(\Gamma_0(48),\chi_8)$, we have*

$$N(1^i,2^j,3^k,4^l;n) = \sum_{\alpha=1}^{28} b_\alpha B_\alpha(n), \tag{5}$$

where $B_\alpha(n)$ are the Fourier coefficients of the basis elements g_α defined in §4.3 *and the values of the constants b_α are given in Table* 4.
(iii) *For each entry (i,j,k,l) in Table* 1 *corresponding to the space $M_4(\Gamma_0(48),\chi_{12})$, we have*

$$N(1^i,2^j,3^k,4^l;n) = \sum_{\alpha=1}^{30} c_\alpha C_\alpha(n), \tag{6}$$

where $C_\alpha(n)$ are the Fourier coefficients of the basis elements h_α defined in §4.4 *and the values of the constants c_α are given in Table* 5.
(iv) *For each entry (i,j,k,l) in Table* 1 *corresponding to the space $M_4(\Gamma_0(48),\chi_{24})$, we have*

$$N(1^i,2^j,3^k,4^l;n) = \sum_{\alpha=1}^{28} d_\alpha D_\alpha(n), \tag{7}$$

where $D_\alpha(n)$ are the Fourier coefficients of the basis elements F_α defined in §4.5 *and the values of the constants d_α are given in Table* 6.

In the following theorem we list the formulas for the octonary quadratic forms with coefficients $1,2,3,4,6$ corresponding to Table 2. The proof is similar to the proof of Theorem 2.2 and so we omit the details.

Theorem 2.3. *Let $n\in\mathbb{N}$ and i,j,k,l,m be non-negative integers such that $i+j+k+l+m=8$.*
(i) *For each entry (i,j,k,l,m) in Table* 2 *corresponding to the space $M_4(\Gamma_0(48))$, we have*

$$N(1^i,2^j,3^k,4^l,6^m;n) = \sum_{\alpha=1}^{30} a'_\alpha A_\alpha(n), \tag{8}$$

where $A_\alpha(n)$ are the Fourier coefficients of the basis elements f_α defined in §4.2 *and the values of the constants a'_α are given in Table* 7.
(ii) *For each entry (i,j,k,l,m) in Table* 2 *corresponding to the space $M_4(\Gamma_0(48),\chi_8)$, we have*

$$N(1^i,2^j,3^k,4^l,6^m;n) = \sum_{\alpha=1}^{28} b'_\alpha B_\alpha(n), \tag{9}$$

where $B_\alpha(n)$ are the Fourier coefficients of the basis elements g_α defined in §4.3 *and the values of the constants b'_α are given in Table* 8.
(iii) *For each entry (i,j,k,l,m) in Table* 2 *corresponding to the space $M_4(\Gamma_0(48),\chi_{12})$, we have*

$$N(1^i,2^j,3^k,4^l,6^m;n) = \sum_{\alpha=1}^{30} c'_\alpha C_\alpha(n), \tag{10}$$

where $C_\alpha(n)$ are the Fourier coefficients of the basis elements h_α defined in §4.4 *and the values of the constants c'_α are given in Table* 9.
(iv) *For each entry (i,j,k,l,m) in Table* 2 *corresponding to the space $M_4(\Gamma_0(48),\chi_{24})$, we have*

$$N(1^i,2^j,3^k,4^l,6^m;n) = \sum_{\alpha=1}^{28} d'_\alpha D_\alpha(n), \tag{11}$$

where $D_\alpha(n)$ are the Fourier coefficients of the basis elements F_α defined in §4.5 *and the values of the constants d'_α are given in Table* 10.

3. Preliminaries

As we use the theory of modular forms, we shall first present some preliminary facts on modular forms. For $k \in \frac{1}{2}\mathbb{Z}$, let $M_k(\Gamma_0(N), \chi)$ denote the space of modular forms of weight k for the congruence subgroup $\Gamma_0(N)$ with character χ and $S_k(\Gamma_0(N), \chi)$ be the subspace of cusp forms of weight k for $\Gamma_0(N)$ with character χ. We assume $4|N$ when k is not an integer and in that case, the character χ which is a Dirichlet character modulo N, is an even character. When χ is the trivial (principal) character modulo N, we shall denote the spaces by $M_k(\Gamma_0(N))$ and $S_k(\Gamma_0(N))$ respectively. Further, when $k \geq 4$ is an integer and $N = 1$, we shall denote these vector spaces by M_k and S_k respectively.

For an integer $k \geq 4$, let E_k denote the normalized Eisenstein series of weight k in M_k given by

$$E_k(z) = 1 - \frac{2k}{B_k} \sum_{n \geq 1} \sigma_{k-1}(n) q^n,$$

where $q = e^{2i\pi z}$, $z \in \mathcal{H}$, the complex upper half-plane, $\sigma_r(n)$ is the sum of the rth powers of the positive divisors of n, and B_k is the k-th Bernoulli number defined by $\displaystyle\frac{x}{e^x - 1} = \sum_{m=0}^{\infty} \frac{B_m}{m!} x^m$.

The classical theta function which is fundamental to the theory of modular forms of half-integral weight is defined by

$$\Theta(z) = \sum_{n \in \mathbb{Z}} q^{n^2}, \tag{12}$$

and is a modular form in the space $M_{1/2}(\Gamma_0(4))$. Another function which is mainly used in our work is the Dedekind eta function $\eta(z)$ and it is given by

$$\eta(z) = q^{1/24} \prod_{n \geq 1} (1 - q^n). \tag{13}$$

An eta-quotient is a finite product of integer powers of $\eta(z)$ and we denote it as follows:

$$\prod_{i=1}^{s} \eta^{r_i}(d_i z) := d_1^{r_1} d_2^{r_2} \cdots d_s^{r_s}, \tag{14}$$

where d_i's are positive integers and r_i's are non-zero integers.

In the following we shall present some facts about modular forms of integral and half-integral weights, which we shall be using in our proof.

Fact I.
We give a fact about certain duplication of modular forms, which follow from the two results: Chapter 3, Propostion 17 of [12] and Proposition 1.3 of [23]. If f is a modular form in $M_k(\Gamma_0(N), \chi)$, then for a positive integer d, the function $f(dz)$ is a modular form in $M_k(\Gamma_0(dN), \chi)$, if k is an integer and it belongs to the space $M_k(\Gamma_0(dN), \chi\chi_d)$, if k is a half-integer. Here, we have used the notation $\chi_m := \left(\frac{m}{\cdot}\right)$, the Kronecker symbol, where m is a non-zero integer, which is a character modulo $|m|$.

Fact II.
For positive integers r, r_1, r_2, d_1, d_2, we have

$$\Theta^r(d_1 z) \in \begin{cases} M_{r/2}(\Gamma_0(4d_1), \chi_{d_1}) & \text{if } r \text{ is odd}, \\ M_{r/2}(\Gamma_0(4d_1), \chi_{-4}) & \text{if } r \equiv 2 \pmod 4, \\ M_{r/2}(\Gamma_0(4d_1)) & \text{if } r \equiv 0 \pmod 4. \end{cases} \tag{15}$$

$$\Theta^{r_1}(d_1z)\cdot\Theta^{r_2}(d_2z)\in\begin{cases}M_{\frac{r_1+r_2}{2}}(\Gamma_0(4[d_1,d_2]),\chi_{(-d_1d_2)}) & \text{if } r_1r_2 \text{ is odd }, r_1+r_2\equiv 2 \pmod 4,\\ M_{\frac{r_1+r_2}{2}}(\Gamma_0(4[d_1,d_2]),\chi_{(d_1d_2)}) & \text{if } r_1r_2 \text{ is odd }, r_1+r_2\equiv 0 \pmod 4.\end{cases} \tag{16}$$

In order to get the above fact we use the following properties. By Fact I, if $f\in M_k(\Gamma_0(N),\chi)$, then $f(dz)$ belongs to $M_k(\Gamma_0(dN),\chi')$, where $\chi'=\chi$ if k is an integer and $\chi'=\chi\chi_d$, if k is a half-integer. Next, if $f_i\in M_{k_i}(\Gamma_0(4N_i),\chi_i)$, $i=1,2$ are two modular forms of weight k_i (where k_1 and k_2 are integers or half-integers). Then, it follows that the product f_1f_2 is a modular form in $M_{k_1+k_2}(\Gamma_0(4[N_1,N_2]),\psi)$, where ψ is a character modulo $4[N_1,N_2]$. If both the weights k_1 and k_2 are integers, then the resulting form is of weight k_1+k_2 (which is an integer) and so $\psi(-1)=(-1)^{k_1+k_2}=\chi_1(-1)\chi_2(-1)$, implying $\psi=\chi_1\chi_2$. If k_1+k_2 is half-integer (say) with k_1 integer and k_2 half-integer, then $\chi_1(-1)=(-1)^{k_1}$ and χ_2 is an even character modulo $4N_2$. Since the resulting form is of half-integral weight we must have ψ an even character. Therefore, $\psi=\chi_1\chi_2$ if k_1 is even and $\psi=\chi_1\chi_2\chi_{-4}$ if k_1 is odd. If both k_1 and k_2 are half-integer, then the resulting form is of integer weight k_1+k_2. Since $\chi_1\chi_2$ is an even character, if k_1+k_2 is an odd integer, then the character of the space should be $\chi_1\chi_2\chi_{-4}$ instead of $\chi_1\chi_2$. Using these facts, we get the special cases as mentioned in (15) and (16). For details we refer to [12, Chap. 4, Proposition 3] and [23, Proposition 1.3].

Fact III.

It is a fact that the vector space $M_k(\Gamma_1(N))$ is decomposed into modular forms space with character as follows. For this fact we refer to [12, Proposition 28, p. 137].

$$M_k(\Gamma_1(N))=\oplus_\chi M_k(\Gamma_0(N),\chi), \tag{17}$$

where the direct sum varies over all Dirichlet characters modulo N if the weight k is a positive integer and varies over all even Dirichlet characters modulo N, $4|N$, if the weight k is half-integer. Further, if k is an integer, one has $M_k(\Gamma_0(N),\chi)=\{0\}$, if $\chi(-1)\neq(-1)^k$. We also have the following decomposition of the space into subspaces of Eisenstein series and cusp forms:

$$M_k(\Gamma_0(N),\chi)=\mathcal{E}_k(\Gamma_0(N),\chi)\oplus S_k(\Gamma_0(N),\chi), \tag{18}$$

where $\mathcal{E}_k(\Gamma_0(N),\chi)$ is the space generated by the Eisenstein series of weight k on $\Gamma_0(N)$ with character χ.

Fact IV.

For this fact we refer to the works of Atkin-Lehner and Li [7, 17]. By the Atkin-Lehner theory of newforms, the space $S_k(\Gamma_0(N),\chi)$ can be decomposed into the space of newforms and oldforms:

$$S_k(\Gamma_0(N),\chi)=S_k^{new}(\Gamma_0(N),\chi)\oplus S_k^{old}(\Gamma_0(N),\chi), \tag{19}$$

where the above is an orthogonal direct sum (with respect to the Petersson scalar product) and

$$S_k^{old}(\Gamma_0(N),\chi)=\bigoplus_{\substack{r|N,r<N\\ rd|N}} S_k^{new}(\Gamma_0(r),\chi)|B(d). \tag{20}$$

In the above, $S_k^{new}(\Gamma_0(N),\chi)$ is the space of newforms and $S_k^{old}(\Gamma_0(N),\chi)$ is the space of oldforms and the operator $B(d)$ is given by $f(z)\mapsto f(dz)$.

Fact V:

Suppose that χ and ψ are primitive Dirichlet characters with conductors M and N, respectively. For a positive integer k, let

$$E_{k,\chi,\psi}(z):=c_0+\sum_{n\geq 1}\left(\sum_{d|n}\psi(d)\cdot\chi(n/d)d^{k-1}\right)q^n, \tag{21}$$

where

$$c_0 = \begin{cases} 0 & \text{if } M > 1, \\ -\frac{B_{k,\psi}}{2k} & \text{if } M = 1, \end{cases}$$

and $B_{k,\psi}$ denotes generalized Bernoulli number with respect to the character ψ. Then, the Eisenstein series $E_{k,\chi,\psi}(z)$ belongs to the space $M_k(\Gamma_0(MN), \chi/\psi)$, provided $\chi(-1)\psi(-1) = (-1)^k$ and $MN \neq 1$. When $\chi = \psi = 1$ (i.e., when $M = N = 1$) and $k \geq 4$, we have $E_{k,\chi,\psi}(z) = -\frac{B_k}{2k} E_k(z)$, where E_k is the normalized Eisenstein series of weight k as defined before. For more details we refer to [19, Chapter 7] and [24, Section 5.3].

We give a notation to the inner sum in (21):

$$\sigma_{k-1;\chi,\psi}(n) := \sum_{d|n} \psi(d) \cdot \chi(n/d) d^{k-1}. \tag{22}$$

For more details on the theory of modular forms of integral and half-integral weights, we refer to [7, 12, 17, 19, 23].

4. Proofs

In this section, we shall give a proof of our results. As mentioned in the introduction, we shall be using the theory of modular forms. Using Fact II, it is easy to see that the theta series associated to the quadratic forms with coefficients $1, 2, 4$ belong to the space $M_4(\Gamma_0(16), \chi)$, where χ is a Dirichlet character modulo 16 and the theta series associated to the quadratic forms with coefficients $1, 2, 3, 4, 6$ belong to the space $M_4(\Gamma_0(48), \psi)$, where ψ is a Dirichlet character modulo 48. Therefore, in order to get the required formulae for $N(a_1, \ldots, a_8; n)$, we need a basis for these spaces. We shall give explict bases for the following spaces of modular forms of weight 4:

$$M_4(\Gamma_0(16), \chi_8), M_4(\Gamma_0(48)), M_4(\Gamma_0(48), \chi_8), M_4(\Gamma_0(48), \chi_{12}), M_4(\Gamma_0(48), \chi_{24}).$$

(We have used the L-functions and modular forms database [16] to get some of the cusp forms of weight 4.)

4.1. **A basis for $M_4(\Gamma_0(16), \chi_8)$ and proof of Theorem 2.1.** The vector space $M_4(\Gamma_0(16), \chi_8)$ has dimension 8 and the cusp forms space $S_4(\Gamma_0(16), \chi_8)$ has dimension 4. Moreover, we have $S_4^{new}(\Gamma_0(16), \chi_8) = \{0\}$ and $S_4^{new}(\Gamma_0(8), \chi_8)$ is 2-dimensional. Let $\mathbf{1}$ denote the trivial character with conductor 1. Then by Fact V, the Eisenstein series $E_{4,\mathbf{1},\chi_8}(z)$ and $E_{4,\chi_8,\mathbf{1}}(z)$ span the space $\mathcal{E}_4(\Gamma_0(8), \chi_8)$. They are given by

$$E_{4,\mathbf{1},\chi_8}(z) \;=\; \frac{11}{2} + \sum_{n\geq 1} \sigma_{3;\mathbf{1},\chi_8}(n) q^n, \quad E_{4,\chi_8,\mathbf{1}}(z) \;=\; \sum_{n\geq 1} \sigma_{3;\chi_8,\mathbf{1}}(n) q^n. \tag{23}$$

The space $S_4^{new}(\Gamma_0(8), \chi_8)$ is spanned by the following two eta-quotients (from now onwards we will be using the notation given in (14)):

$$f_{4,8,\chi_8;1}(z) = 1^{-2}2^{11}4^{-3}8^2 = \sum_{n\geq 1} a_{4,8,\chi_8;1}(n) q^n, \; f_{4,8,\chi_8;2}(z) = 1^2 2^{-3} 4^{11} 8^{-2} = \sum_{n\geq 1} a_{4,8,\chi_8;2}(n) q^n. \tag{24}$$

In the following proposition, we shall give a basis of $M_4(\Gamma_0(16), \chi_8)$.

Proposition 4.1. *A basis of $M_4(\Gamma_0(16), \chi_8)$ is given by*

$$\left\{ E_{4,\mathbf{1},\chi_8}(tz), E_{4,\chi_8,\mathbf{1}}(tz), f_{4,8,\chi_8;1}(tz), f_{4,8,\chi_8;2}(tz) \middle| t \mid 2 \right\}. \tag{25}$$

We are now ready to prove Theorem 2.1.

In these cases, the quadratic forms have coefficients $1,2,4$. Therefore, by Fact II, all of them belong to the space of modular forms of weight 4 on $\Gamma_0(16)$. Since the power of theta function corresponding to the coefficient 2 is odd in all these cases, the modular forms spaces will have charcter χ_8. Therefore, expressing each of the theta series corresponding to these 12 quadratic forms as a linear combination of the basis elements given in Proposition 4.1. Now, by comparing the nth Fourier coefficients, we obtain the formulas listed in Theorem 2.1.

4.2. **A basis for $M_4(\Gamma_0(48))$ and proof of Theorem 2.2(i).** The vector space $M_4(\Gamma_0(48))$ has dimension 30 and we have $\dim_{\mathbb{C}} \mathcal{E}_4(\Gamma_0(48)) = 12$ and $\dim_{\mathbb{C}} S_4(\Gamma_0(48)) = 18$. For $d = 6, 8, 12, 16$ and 24, $S_4^{new}(\Gamma_0(d))$ is one-dimensional and $\dim_{\mathbb{C}} S_4^{new}(\Gamma_0(48)) = 3$. Let us define some eta-quotients and use them to give an explicit basis for $S_4(\Gamma_0(48))$. Let

$$f_{4,6}(z) = 1^2 2^2 3^2 6^2 := \sum_{n\geq 1} a_{4,6}(n) q^n, \quad f_{4,8}(z) = 2^4 4^4 := \sum_{n\geq 1} a_{4,8}(n) q^n, \tag{26}$$

$$f_{4,12}(z) = 1^{-1} 2^2 3^3 4^3 6^2 12^{-1} - 1^3 2^2 3^{-1} 4^{-1} 6^2 12^3 := \sum_{n\geq 1} a_{4,12}(n) q^n, \tag{27}$$

$$f_{4,16}(z) = 2^{-4} 4^{16} 8^{-4} := \sum_{n\geq 1} a_{4,16}(n) q^n, \ f_{4,24}(z) = 1^{-4} 2^{11} 3^{-4} 4^{-3} 6^{11} 12^{-3} := \sum_{n\geq 1} a_{4,24}(n) q^n. \tag{28}$$

Let χ_{-4} be the primitive odd character modulo 4. Then the following new Eisenstein series belongs to $\mathcal{E}_4(\Gamma_0(16))$:

$$E_{4,\chi_{-4},\chi_{-4}}(z) = \sum_{n\geq 1} \sigma_{3,\chi_{-4},\chi_{-4}}(n) q^n = \sum_{n\geq 1} \left(\frac{-4}{n}\right) \sigma_3(n) q^n. \tag{29}$$

We use the following notation in the sequel. For a Dirichlet character χ and a function f with Fourier expansion $f(z) = \sum_{n\geq 1} a(n) q^n$, we define the twisted function $f \otimes \chi(z)$ as follows.

$$f \otimes \chi(z) = \sum_{n\geq 1} \chi(n) a(n) q^n. \tag{30}$$

A basis for the space $M_4(\Gamma_0(48))$ is given in the following proposition.

Proposition 4.2. *A basis for the Eisenstein series space $\mathcal{E}_4(\Gamma_0(48))$ is given by*

$$\left\{E_{4,\chi_{-4},\chi_{-4}}(z), E_{4,\chi_{-4},\chi_{-4}}(3z), E_4(tz), t|48\right\} \tag{31}$$

and a basis for the space of cusp forms $S_4(\Gamma_0(48))$ is given by

$$\begin{aligned}\{f_{4,6}(t_1 z), t_1|8; f_{4,8}(t_2 z), t_2|6; f_{4,12}(t_3 z), t_3|4; f_{4,16}(t_4 z), t_4|3;\\ f_{4,24}\otimes\chi_4(t_5 z), t_5|2; f_{4,6}\otimes\chi_{-4}(z), f_{4,12}\otimes\chi_{-4}(z), f_{4,24}\otimes\chi_{-4}(z),\}\end{aligned} \tag{32}$$

Together they form a basis for $M_4(\Gamma_0(48))$.

For the sake of simplicity in the formulae, we list these basis elements as $\{f_\alpha(z) | 1 \leq \alpha \leq 30\}$, where $f_1(z) = E_4(z)$, $f_2(z) = E_4(2z)$, $f_3(z) = E_4(3z)$, $f_4(z) = E_4(4z)$, $f_5(z) = E_4(6z)$, $f_6(z) = E_4(8z)$, $f_7(z) = E_4(12z)$, $f_8(z) = E_4(16z)$, $f_9(z) = E_4(24z)$, $f_{10}(z) = E_4(48z)$, $f_{11}(z) = E_{4,\chi_{-4},\chi_{-4}}(z)$, $f_{12}(z) = E_{4,\chi_{-4},\chi_{-4}}(3z)$, $f_{13}(z) = f_{4,6}(z)$, $f_{14}(z) = f_{4,6}(2z)$, $f_{15}(z) = f_{4,6}(4z)$, $f_{16}(z) = f_{4,6}(8z)$, $f_{17}(z) = f_{4,8}(z)$, $f_{18}(z) = f_{4,8}(2z)$, $f_{19}(z) = f_{4,8}(3z)$, $f_{20}(z) = f_{4,8}(6z)$, $f_{21}(z) = f_{4,12}(z)$, $f_{22}(z) = f_{4,12}(2z)$, $f_{23}(z) = f_{4,12}(4z)$, $f_{24}(z) = f_{4,16}(z)$, $f_{25}(z) = f_{4,16}(3z)$, $f_{26}(z) = f_{4,24}\otimes\chi_4(z)$, $f_{27}(z) = f_{4,24}\otimes\chi_4(2z)$, $f_{28}(z) = f_{4,6}\otimes\chi_{-4}(z)$, $f_{29}(z) = f_{4,12}\otimes\chi_{-4}(z)$, $f_{30}(z) = f_{4,24}\otimes\chi_{-4}(z)$.
We also express the Fourier coefficients of the function $f_\alpha(z)$ as $\sum_{n\geq 1} A_\alpha(n) q^n$, $1 \leq \alpha \leq 30$.

We are now ready to prove the theorem. Noting that all the 10 cases corresponding to the trivial character space (in Table 1) have the property that the powers of the theta functions corresponding to the coefficients 2 and 3 are even. Therefore, the resulting functions belong to the space of modular forms of weight 4 on $\Gamma_0(48)$ with trivial character (we use Fact II to prove this). So, we can express these theta functions as a linear combination of the basis given in Proposition 4.2 as follows.

$$\Theta^i(z)\Theta^j(2z)\Theta^k(3z)\Theta^l(4z) = \sum_{\alpha=1}^{30} a_\alpha f_\alpha(z), \tag{33}$$

where a_α's some constants. Comparing the n-th Fourier coefficients on both the sides, we get

$$N(1^i, 2^j, 3^k, 4^l; n) = \sum_{\alpha=1}^{30} a_\alpha A_\alpha(n).$$

Explicit values for the constants $a_\alpha, 1 \le \alpha \le 30$ corresponding to the 10 cases are given in Table 3.

4.3. **A basis for $M_4(\Gamma_0(48), \chi_8)$ and proof of Theorem 2.2(ii).** The space $M_4(\Gamma_0(48), \chi_8)$ is 28 dimensional and the cusp forms space has dimension 20. For the space of Eisenstein series we use the basis elements of $\mathcal{E}_4(\Gamma_0(8), \chi_8)$ given in (23). For the space of cusp forms, there are no newforms and the oldforms classes are $S_4^{new}(\Gamma_0(8), \chi_8)$ and $S_4^{new}(\Gamma_0(24), \chi_8)$. A basis for $S_4^{new}(\Gamma_0(8), \chi_8)$ is given in (24). The following six eta-quotients span the space $S_4^{new}(\Gamma_0(24), \chi_8)$.

$$\begin{aligned}
f_{4,24,\chi_8;1}(z) &= 1^2 2^1 3^{-4} 4^1 6^{10} 8^2 12^{-4} := \sum_{n\ge1} a_{4,24,\chi_8;1}(n) q^n,\\
f_{4,24,\chi_8;2}(z) &= 1^1 2^3 3^{-1} 4^1 6^4 8^{-1} 24^1 := \sum_{n\ge1} a_{4,24,\chi_8;2}(n) q^n,\\
f_{4,24,\chi_8;3}(z) &= 1^{-1} 2^4 3^1 6^3 8^1 12^1 24^{-1} := \sum_{n\ge1} a_{4,24,\chi_8;3}(n) q^n,\\
f_{4,24,\chi_8;4}(z) &= 1^{-2} 2^4 4^2 6^1 8^2 12^1 := \sum_{n\ge1} a_{4,24,\chi_8;4}(n) q^n,\\
f_{4,24,\chi_8;5}(z) &= 2^1 3^{-2} 4^1 6^4 12^2 24^2 := \sum_{n\ge1} a_{4,24,\chi_8;5}(n) q^n,\\
f_{4,24,\chi_8;6}(z) &= 1^{-6} 2^{14} 6^1 8^{-2} 12^1 := \sum_{n\ge1} a_{4,24,\chi_8;6}(n) q^n
\end{aligned} \tag{34}$$

A basis for the space $M_4(\Gamma_0(48), \chi_8)$ is given in the following proposition.

Proposition 4.3. *A basis for the space $M_4(\Gamma_0(48), \chi_8)$ is given by*

$$\begin{aligned}
&\{E_{4,\mathbf{1},\chi_8}(tz),\ E_{4,\chi_8,\mathbf{1}}(tz), t|6; f_{4,8,\chi_8;1}(t_1z), f_{4,8,\chi_8;2}(t_1z), t_1|6; f_{4,24,\chi_8;1}(t_2z),\\
&f_{4,24,\chi_8;2}(t_2z), f_{4,24,\chi_8;3}(t_2z), f_{4,24,\chi_8;4}(t_2z), f_{4,24,\chi_8;5}(t_2z), f_{4,24,\chi_8;6}(t_2z), t_2|2\},
\end{aligned} \tag{35}$$

where $E_{4,\mathbf{1},\chi_8}(z)$ and $E_{4,\chi_8,\mathbf{1}}(z)$ are defined in (23)*, $f_{4,8,\chi_8;i}(z)$, $i = 1,2$ are defined in* (24) *and $f_{4,24,\chi_8;j}(z)$, $1 \le j \le 6$ are defined by* (34)

For the sake of simplifying the notation, we shall list the basis in Proposition 4.3 as $\{g_\alpha(z) | 1 \le \alpha \le 28\}$, where $g_1(z) = E_{4,\mathbf{1},\chi_8}(z)$, $g_2(z) = E_{4,\mathbf{1},\chi_8}(2z)$, $g_3(z) = E_{4,\mathbf{1},\chi_8}(3z)$, $g_4(z) = E_{4,\mathbf{1},\chi_8}(6z)$, $g_5(z) = E_{4,\chi_8,\mathbf{1}}(z)$, $g_6(z) = E_{4,\chi_8,\mathbf{1}}(2z)$, $g_7(z) = E_{4,\chi_8,\mathbf{1}}(3z)$, $g_8(z) = E_{4,\chi_8,\mathbf{1}}(6z)$, $g_9(z) = f_{4,8,\chi_8;1}(z)$, $g_{10}(z) = f_{4,8,\chi_8;1}(2z)$, $g_{11}(z) = f_{4,8,\chi_8;1}(3z)$, $g_{12}(z) = f_{4,8,\chi_8;1}(6z)$, $g_{13}(z) = f_{4,8,\chi_8;2}(z)$, $g_{14}(z) = f_{4,8,\chi_8;2}(2z)$, $g_{15}(z) = f_{4,8,\chi_8;2}(3z)$, $g_{16}(z) = f_{4,8,\chi_8;2}(6z)$, $g_{17}(z) = f_{4,24,\chi_8;1}(z)$, $g_{18}(z) = f_{4,24,\chi_8;1}(2z)$, $g_{19}(z) = f_{4,24,\chi_8;2}(z)$, $g_{20}(z) = f_{4,24,\chi_8;2}(2z)$, $g_{21}(z) = f_{4,24,\chi_8;3}(z)$, $g_{22}(z) =$

$f_{4,24,\chi_8;3}(2z)$, $g_{23}(z) = f_{4,24,\chi_8;4}(z)$, $g_{24}(z) = f_{4,24,\chi_8;4}(2z)$, $g_{25}(z) = f_{4,24,\chi_8;5}(z)$, $g_{26}(z) = f_{4,24,\chi_8;5}(2z)$, $g_{27}(z) = f_{4,24,\chi_8;6}(z)$, $g_{28}(z) = f_{4,24,\chi_8;6}(2z)$.
As before, we also write the Fourier expansions of these basis elements as $g_\alpha(z) = \sum_{n\geq 1} B_\alpha(n)q^n$, $1 \leq \alpha \leq 28$.

In this case, all the 10 quadruples corresponding to the χ_8 character space (in Table 1) have the property that the powers of the theta functions corresponding to the coefficients 2 are odd and corresponding to 3 are even. Therefore, the resulting products of theta functions are modular forms of weight 4 on $\Gamma_0(48)$ with character χ_8 (we use Fact II to prove this). So, we can express these products of theta functions as a linear combination of the basis given in Proposition 4.3 as follows.

$$\Theta^i(z)\Theta^j(2z)\Theta^k(3z)\Theta^l(4z) = \sum_{\alpha=1}^{28} b_\alpha g_\alpha(z). \tag{36}$$

Comparing the n-th Fourier coefficients on both the sides, we get

$$N(1^i, 2^j, 3^k, 4^l; n) = \sum_{\alpha=1}^{28} b_\alpha B_\alpha(n).$$

Explicit values for the constants b_α, $1 \leq \alpha \leq 28$ corresponding to these 10 cases are given in Table 4.

4.4. **A basis for $M_4(\Gamma_0(48), \chi_{12})$ and proof of Theorem 2.2(iii).** The dimension of the space in this case is 30, with $\dim_{\mathbb{C}} \mathcal{E}_4(\Gamma_0(48), \chi_{12}) = 12$ and $\dim_{\mathbb{C}} S_4(\Gamma_0(48), \chi_{12}) = 18$. Regarding the old class, the space $S_4^{new}(\Gamma_0(12), \chi_{12})$ has dimension 4 and is spanned by the following four eta-quotients:

$$\begin{aligned} f_{4,12,\chi_{12};1}(z) &= 2^{-1}3^4 4^2 6^5 12^{-2}, \quad f_{4,12,\chi_{12};2}(z) = 3^4 4^3 6^{-2} 12^3, \\ f_{4,12,\chi_{12};3}(z) &= 2^2 3^4 4^{-1} 6^{-4} 12^7, \quad f_{4,12,\chi_{12};4}(z) = 1^4 4^{-1} 6^{-2} 12^7. \end{aligned} \tag{37}$$

We write the Fourier expansions of these forms as $f_{4,12,\chi_{12};j}(z) = \sum_{n\geq 1} a_{4,12,\chi_{12};j}(n)q^n$, $1 \leq j \leq 4$. In order to get the span of the newforms space of level 48, we need the following three eta-quotients of level 48 and character χ_{12}.

$$\begin{aligned} &1^{-4}2^7 4^5 6^{-3} 8^{-3} 12^9 24^{-3} := \sum_{n\geq 1} a_{4,48,\chi_{12};1}(n)q^n, \ 2^{-3}3^4 4^9 6^7 8^{-3} 12^5 24^{-3} := \sum_{n\geq 1} a_{4,48,\chi_{12};2}(n)q^n, \\ &1^{-2}2^2 3^2 4^2 8^1 12^2 24^1 := \sum_{n\geq 1} a_{4,48,\chi_{12};3}(n)q^n. \end{aligned} \tag{38}$$

Using the above three eta-quotients, we obtain the span of the space $S_4^{new}(\Gamma_0(48), \chi_{12})$ given by the following six forms.

$$\begin{aligned} f_{4,48,\chi_{12};1}(z) &= \sum_{\substack{n\geq 1\\ n\equiv 1 \pmod 4}} a_{4,48,\chi_{12};1}(n)q^n, \quad f_{4,48,\chi_{12};2}(z) = \sum_{\substack{n\geq 1\\ n\equiv 3 \pmod 4}} a_{4,48,\chi_{12};1}(n)q^n, \\ f_{4,48,\chi_{12};3}(z) &= \sum_{\substack{n\geq 1\\ n\equiv 1 \pmod 4}} a_{4,48,\chi_{12};2}(n)q^n, \quad f_{4,48,\chi_{12};4}(z) = \sum_{\substack{n\geq 1\\ n\equiv 3 \pmod 4}} a_{4,48,\chi_{12};2}(n)q^n, \\ f_{4,48,\chi_{12};5}(z) &= \sum_{\substack{n\geq 1\\ n\equiv 1 \pmod 4}} a_{4,48,\chi_{12};3}(n)q^n, \quad f_{4,48,\chi_{12};6}(z) = \sum_{\substack{n\geq 1\\ n\equiv 3 \pmod 4}} a_{4,48,\chi_{12};3}(n)q^n, \end{aligned} \tag{39}$$

A basis for the space $(M_4(\Gamma_0(48)), \chi_{12})$ is given in the following proposition.

Proposition 4.4. *A basis for the space* $M_4(\Gamma_0(48),\chi_{12})$ *is given by*

$$\begin{aligned}\{E_{4,\mathbf{1},\chi_{12}}(tz), E_{4,\chi_{12},\mathbf{1}}(tz), E_{4,\chi_{-4},\chi_{-3}}(tz), E_{4,\chi_{-3},\chi_{-4}}(tz), t|4; f_{4,12,\chi_{12};j}(t_1 z), t_1|4, 1\le j\le 4;\\ f_{4,48,\chi_{12};1}(z), f_{4,48,\chi_{12};2}(z), f_{4,48,\chi_{12};3}(z), f_{4,48,\chi_{12};4}(z), f_{4,48,\chi_{12};5}(z), f_{4,48,\chi_{12};6}(z)\},\end{aligned} \tag{40}$$

where the Eisenstein series in the basis are defined by (21).

Let us denote the 30 basis elements in the above proposition as follows. $\{h_\alpha(z)|1\le\alpha\le 30\}$, where $h_1(z)=E_{4,\mathbf{1},\chi_{12}}(z)$, $h_2(z)=E_{4,\chi_{12},\mathbf{1}}(z)$, $h_3(z)=E_{4,\chi_{-4},\chi_{-3}}(z)$, $h_4(z)=E_{4,\chi_{-3},\chi_{-4}}(z)$, $h_5(z)=E_{4,\mathbf{1},\chi_{12}}(2z)$, $h_6(z)=E_{4,\chi_{12},\mathbf{1}}(2z)$, $h_7(z)=E_{4,\chi_{-4},\chi_{-3}}(2z)$, $h_8(z)=E_{4,\chi_{-3},\chi_{-4}}(2z)$, $h_9(z)=E_{4,\mathbf{1},\chi_{12}}(4z)$, $h_{10}(z)=E_{4,\chi_{12},\mathbf{1}}(4z)$, $h_{11}(z)=E_{4,\chi_{-4},\chi_{-3}}(4z)$, $h_{12}(z)=E_{4,\chi_{-3},\chi_{-4}}(4z)$, $h_{12+j}(z)=f_{4,12,\chi_{12};j}(z)$, $1\le j\le 4$, $h_{16+j}(z)=f_{4,12,\chi_{12};j}(2z)$, $1\le j\le 4$, $h_{20+j}(z)=f_{4,12,\chi_{12};j}(4z)$, $1\le j\le 4$, $h_{24+j}(z)=f_{4,48,\chi_{12};j}(z)$, $1\le j\le 6$.

To prove Theorem 2.2(iii), we consider the case of 10 quadruples corresponding to the χ_{12} character space (in Table 1). Now the roles of coefficients have interchanged and they have the property that the powers of the theta functions corresponding to the coefficients 2 are even and corresponding to 3 are odd. Therefore, the resulting products of theta functions are modular forms of weight 4 on $\Gamma_0(48)$ with character χ_{12} (once again we use Fact II to get this). So, we can express these products of theta functions as a linear combination of the basis given in Proposition 4.4 as follows.

$$\Theta^i(z)\Theta^j(2z)\Theta^k(3z)\Theta^l(4z)=\sum_{\alpha=1}^{30}c_\alpha h_\alpha(z), \tag{41}$$

where c_α's are some constants. By comparing the n-th Fourier coefficients on both the sides, we get

$$N(1^i,2^j,3^k,4^l;n)=\sum_{\alpha=1}^{30}c_\alpha C_\alpha(n),$$

where $h_\alpha(z)=\sum_{n\ge1}C_\alpha(n)q^n$, $1\le\alpha\le30$. Explicit values for the constants c_α, $1\le\alpha\le30$ corresponding to these 10 cases are given in Table 5.

4.5. **A basis for $M_4(\Gamma_0(48),\chi_{24})$ and proof of Theorem 2.2(iv).** To get the span of the Eisenstein series space $\mathcal{E}_4(\Gamma_0(48),\chi_{24})$, we use the Eisenstein series $E_{4,\chi,\psi}(z)$ defined in (21), where $\chi,\psi\in\{\mathbf{1},\chi_{-8},\chi_{-12},\chi_{24}\}$. Note that $S_4^{new}(\Gamma_0(48),\chi_{24})=\{0\}$. The space $S_4^{new}(\Gamma_0(24),\chi_{24})$ is spanned by the following ten eta-quotients (notation as in (14)):

$$\begin{aligned}&f_{4,24,\chi_{24};1}(z)=3^{-2}6^78^312^324^{-3},\ f_{4,24,\chi_{24};2}(z)=3^24^76^{-3}8^{-2}12^4,\ f_{4,24,\chi_{24};3}(z)=3^24^{-3}6^18^612^2,\\&f_{4,24,\chi_{24};4}(z)=3^26^{-3}8^312^524^1,\quad f_{4,24,\chi_{24};5}(z)=3^24^26^{-3}8^{-1}12^324^5,\\&f_{4,24,\chi_{24};6}(z)=3^24^16^18^{-2}12^{-2}24^8,\quad f_{4,24,\chi_{24};7}(z)=3^24^16^18^{-2}12^{-2}24^8,\\&f_{4,24,\chi_{24};8}(z)=1^13^{-1}6^18^{-2}12^124^8, f_{4,24,\chi_{24};9}(z)=2^23^64^16^{-3}8^2,\ f_{4,24,\chi_{24};10}(z)=3^24^36^512^{-4}24^2.\end{aligned} \tag{42}$$

We write the Fourier expansions as $f_{4,24,\chi_{24};j}(z)=\sum_{n\ge1}a_{4,24,\chi_{24};j}(n)q^n$. We now give a basis for the space $M_4(\Gamma_0(48),\chi_{24})$ in the following proposition.

Proposition 4.5. *The following functions span the space* $M_4(\Gamma_0(48),\chi_{24})$.

$$\begin{aligned}\{E_{4,\mathbf{1},\chi_{24}}(tz), E_{4,\chi_{24},\mathbf{1}}(tz), E_{4,\chi_{-8},\chi_{-3}}(tz), E_{4,\chi_{-3},\chi_{-8},}(tz), t|2;\\ f_{4,24,\chi_{24};j}(z), 1\le j\le 10; f_{4,24,\chi_{24};j}(2z), 1\le j\le 10\}.\end{aligned} \tag{43}$$

We list these basis elements as $\{F_\alpha(z)|1\le\alpha\le28\}$, where $F_1(z)=E_{4,\mathbf{1},\chi_{24}}(z)$, $F_2(z)=E_{4,\mathbf{1},\chi_{24}}(2z)$, $F_3(z)=E_{4,\chi_{-8},\chi_{-3}}(z)$, $F_4(z)=E_{4,\chi_{-8},\chi_{-3}}(2z)$, $F_5(z)=E_{4,\chi_{24},\mathbf{1}}(z)$, $F_6(z)=E_{4,\chi_{24},\mathbf{1}}(2z)$, $F_7(z)=E_{4,\chi_{-3},\chi_{-8},}(z)$, $F_8(z)=E_{4,\chi_{-3},\chi_{-8},}(2z)$, $F_9(z)=f_{4,24,\chi_{24};1}(z)$, $F_{10}(z)=f_{4,24,\chi_{24};1}(2z)$,

$F_{11}(z) = f_{4,24,\chi_{24};2}(z)$, $F_{12}(z) = f_{4,24,\chi_{24};2}(2z)$, $F_{13}(z) = f_{4,24,\chi_{24};3}(z)$, $F_{14}(z) = f_{4,24,\chi_{24};3}(2z)$, $F_{15}(z) = f_{4,24,\chi_{24};4}(z)$, $F_{16}(z) = f_{4,24,\chi_{24};4}(2z)$, $F_{17}(z) = f_{4,24,\chi_{24};5}(z)$, $F_{18}(z) = f_{4,24,\chi_{24};5}(2z)$, $F_{19}(z) = f_{4,24,\chi_{24};6}(z)$, $F_{20}(z) = f_{4,24,\chi_{24};6}(2z)$, $F_{21}(z) = f_{4,24,\chi_{24};7}(z)$, $F_{22}(z) = f_{4,24,\chi_{24};7}(2z)$, $F_{23}(z) = f_{4,24,\chi_{24};8}(z)$, $F_{24}(z) = f_{4,24,\chi_{24};8}(2z)$, $F_{25}(z) = f_{4,24,\chi_{24};9}(z)$, $F_{26}(z) = f_{4,24,\chi_{24};9}(2z)$, $F_{27}(z) = f_{4,24,\chi_{24};10}(z)$, $F_{28}(z) = f_{4,24,\chi_{24};10}(2z)$.
We express the Fourier coefficients of the function $F_\alpha(z)$ as $\sum_{n\geq 1} D_\alpha(n)q^n$, $1 \leq \alpha \leq 28$. In this case all the 10 quadruples have the property that the powers of the theta functions corresponding to the coefficients 2 and 3 are both odd. Therefore, the resulting functions belong to the space of modular forms of weight 4 on $\Gamma_0(48)$ with character χ_{24}. As before, we use Fact II to arrive at this result. So, one can express these theta functions as a linear combination of the basis elements:

$$\Theta^i(z)\Theta^j(2z)\Theta^k(3z)\Theta^l(4z) = \sum_{\alpha=1}^{28} d_\alpha F_\alpha(z), \tag{44}$$

where d_α's are some constants. Comparing the n-th Fourier coefficients on both the sides, we get

$$N(1^i, 2^j, 3^k, 4^l; n) = \sum_{\alpha=1}^{28} d_\alpha D_\alpha(n).$$

The values for the constants d_α, $1 \leq \alpha \leq 28$ corresponding to the 10 cases are given in Table 6.

Remark 4.1. As we see in the proofs of Theorems 2.1 and 2.2, the main part of the proof is the construction of a basis of modular forms of weight 4 on $\Gamma_0(48)$ with some character. In the case of Theorem 2.3 we need to consider the quadratic forms given by Table 2. Since all these quadratic forms correspond to modular forms of weight 4 on $\Gamma_0(48)$ with character (as specified in the table), we can follow the proof of Theorem 2.2 and complete the proof. The corresponding coefficients are given in Tables 7 to 10. So, we omit the details here.

Remark 4.2. To prove our theorems, we determined explicit bases for the spaces of modular forms $M_4(\Gamma_0(48), \chi)$, where χ is either trivial character modulo 48 or $\chi = \chi_m$, $m = 8, 12$ or 24. For our construction of bases we used the newforms theory which involves old classes of levels $6, 8, 12, 16$ and 24. Therefore, from our construction it is easy to get bases for the modular form spaces $M_4(\Gamma_0(24), \chi)$, where χ is either the trivial character modulo 24 or $\chi = \chi_m$, $m = 8, 12$ or 24. In our work [22], we used the above bases of $M_4(\Gamma_0(24), \chi)$ to find formulas for the representation numbers of a positive integer by certain classes of quadratic forms in 8 variables.

Remark 4.3 We now make a remark about the number of coefficients of the products of the theta functions that are needed to get the linear combination constants a_α, b_α, c_α, d_α (and a'_α, b'_α, c'_α, d'_α) that are listed in the tables (3 to 6) (resp. 7 to 10). The Sturm bound for the determination of a non-zero modular form of level N is $k\nu/12$, where ν is the index of the congruence subgroup $\Gamma_0(N)$ in $SL_2(\mathbb{Z})$ (see [25]). In our case $N = 48$, $k = 4$, the index $\nu = 96$ and so the Sturm bound is 32. In the table below, we list the number of coefficients needed (to get the constants), the Sturm bound and the dimension of the respective spaces of modular forms.

Space	no. of coefficients needed	Sturm bound	dimension	Theorem
$M_4(\Gamma_0(48))$	33	32	30	2.2 (i) and 2.3 (i)
$M_4(\Gamma_0(48), \chi_8)$	28	32	28	2.2 (ii) and 2.3 (ii)
$M_4(\Gamma_0(48), \chi_{12})$	31	32	30	2.2 (iii) and 2.3 (iii)
$M_4(\Gamma_0(48), \chi_{24})$	28	32	28	2.2 (iv) and 2.3 (iv)

Remark 4.4 In a recent result, Z. S. Aygin [8, 9] proved an orthogonal relation and used it to compute the coefficients of Eisenstein series part of a given modular form f of weight $2k$ on $\Gamma_0(N)$ (N is odd and square-free) in terms of sum of divisors function. In particular for the modular form

given by the products of the theta function $T(z) = \prod_{\delta|N} \Theta(\delta z)^{r_\delta}$, his result yields explicit constants of the Eisenstein series part of the function $T(z)$ which involve the exponents r_δ. For the interested reader, we refer to [8, Theorem 7.1.2], [9, Theorem 5.2]. We also refer to [10, Theorem 2.1].

4.6. **Sample formulas.** In this section we shall give explicit formulas for Theorems 2.2 and 2.3 for a few cases.

Formulas for the cases $(5,0,2,1),(3,2,2,1)$ **of Theorem 2.2(i).**

$$N(1^5,3^2,4^1;n) =$$
$$\frac{14}{5}\sigma_3(n) - \frac{54}{5}\sigma_3(n/3) - \frac{238}{5}\sigma_3(n/4) + \frac{252}{5}\sigma_3(n/8) + \frac{918}{5}\sigma_3(n/12) - \frac{448}{5}\sigma_3(n/16) -$$
$$\frac{972}{5}\sigma_3(n/24) + \frac{1728}{5}\sigma_3(n/48) + \frac{7}{20}\sigma_{3;\chi_{-4},\chi_{-4}}(n) + \frac{27}{20}\sigma_{3;\chi_{-4},\chi_{-4}}(n/3) - \frac{4}{5}a_{4,6}(n) + \frac{16}{5}a_{4,6}(n/2) -$$
$$\frac{176}{5}a_{4,6}(n/4) - \frac{1408}{5}a_{4,6}(n/8) + a_{4,8}(n/2) + 27a_{4,8}(n/6) + 4a_{4,12}(n) + 20a_{4,12}(n/4) + \frac{1}{4}a_{4,16}(n) -$$
$$\frac{27}{4}a_{4,16}(n/3) + 9a_{4,24}(n/2)\left(\frac{4}{n/2}\right) + \frac{22}{5}a_{4,6}(n)\left(\frac{-4}{n}\right) + \frac{5}{4}a_{4,12}(n)\left(\frac{-4}{n}\right) - \frac{9}{4}a_{4,24}(n)\left(\frac{-4}{n}\right),$$

$$N(1^3,2^2,3^2,4^1;n) =$$
$$\frac{7}{5}\sigma_3(n) - \frac{7}{5}\sigma_3(n/2) - \frac{27}{5}\sigma_3(n/3) + \frac{27}{5}\sigma_3(n/6) + \frac{28}{5}\sigma_3(n/8) - \frac{448}{5}\sigma_3(n/16) -$$
$$\frac{108}{5}\sigma_3(n/24) + \frac{1728}{5}\sigma_3(n/48) - \frac{2}{5}a_{4,6}(n) - 24a_{4,6}(n/4) - \frac{448}{5}a_{4,6}(n/8) - \frac{1}{2}a_{4,8}(n) - a_{4,8}(n/2) -$$
$$\frac{27}{2}a_{4,8}(n/3) - 27a_{4,8}(n/6) + 2a_{4,12}(n) + 2a_{4,12}(n/2) + 20a_{4,12}(n/4) + \frac{1}{2}a_{4,16}(n) -$$
$$\frac{27}{2}a_{4,16}(n/3) + \frac{3}{2}a_{4,24}(n)\left(\frac{4}{n}\right) + 3a_{4,24}(n/2)\left(\frac{4}{n/2}\right) + 3a_{4,6}(n)\left(\frac{-4}{n}\right) - \frac{3}{2}a_{4,24}(n)\left(\frac{-4}{n}\right).$$

Formulas for the cases $(4,1,2,1),(2,3,2,1)$ **of Theorem 2.2(ii).**

$$N(1^4,2^1,3^2,4^1;n) =$$
$$-\frac{26}{451}\sigma_{3;\mathbf{1},\chi_8}(n/2) + \frac{108}{451}\sigma_{3;\mathbf{1},\chi_8}(n/6) + \frac{832}{451}\sigma_{3;\chi_8,\mathbf{1}}(n) + \frac{3456}{451}\sigma_{3;\chi_8,\mathbf{1}}(n/3) - \frac{12448}{451}a_{4,8,\chi_8;1}(n) +$$
$$\frac{38416}{451}a_{4,8,\chi_8;1}(n/2) + \frac{1296}{451}a_{4,8,\chi_8;1}(n/3) + \frac{98496}{451}a_{4,8,\chi_8;1}(n/6) + \frac{14016}{451}a_{4,8,\chi_8;2}(n) - \frac{28032}{451}a_{4,8,\chi_8;2}(n/2) -$$
$$\frac{1728}{451}a_{4,8,\chi_8;2}(n/3) - \frac{3456}{451}a_{4,8,\chi_8;2}(n/6) + \frac{668}{41}a_{4,24,\chi_8;1}(n) - \frac{2368}{41}a_{4,24,\chi_8;1}(n/2) + \frac{3240}{41}a_{4,24,\chi_8;2}(n) -$$
$$\frac{11208}{41}a_{4,24,\chi_8;2}(n/2) - \frac{216}{41}a_{4,24,\chi_8;3}(n) + \frac{2856}{41}a_{4,24,\chi_8;3}(n/2) - \frac{6240}{41}a_{4,24,\chi_8;4}(n) + \frac{20672}{41}a_{4,24,\chi_8;4}(n/2) -$$
$$\frac{11232}{41}a_{4,24,\chi_8;5}(n) + \frac{46656}{41}a_{4,24,\chi_8;5}(n/2) + \frac{932}{41}a_{4,24,\chi_8;6}(n) - \frac{3568}{41}a_{4,24,\chi_8;6}(n/2),$$

$$N(1^2,2^3,3^2,4^1;n) =$$
$$-\frac{26}{451}\sigma_{3;\mathbf{1},\chi_8}(n/2)+\frac{108}{451}\sigma_{3;\mathbf{1},\chi_8}(n/6)+\frac{416}{451}\sigma_{3;\chi_8,\mathbf{1}}(n)+\frac{1728}{451}\sigma_{3;\chi_8,\mathbf{1}}(n/3)-\frac{6224}{451}a_{4,8,\chi_8;1}(n)+$$
$$\frac{16768}{451}a_{4,8,\chi_8;1}(n/2)+\frac{648}{451}a_{4,8,\chi_8;1}(n/3)+\frac{98496}{451}a_{4,8,\chi_8;1}(n/6)+\frac{7008}{451}a_{4,8,\chi_8;2}(n)+\frac{832}{451}a_{4,8,\chi_8;2}(n/2)-$$
$$\frac{864}{451}a_{4,8,\chi_8;2}(n/3)-\frac{3456}{451}a_{4,8,\chi_8;2}(n/6)+\frac{416}{41}a_{4,24,\chi_8;1}(n)-\frac{1384}{41}a_{4,24,\chi_8;1}(n/2)+\frac{1620}{41}a_{4,24,\chi_8;2}(n)-$$
$$\frac{6288}{41}a_{4,24,\chi_8;2}(n/2)-\frac{108}{41}a_{4,24,\chi_8;3}(n)+\frac{1872}{41}a_{4,24,\chi_8;3}(n/2)-\frac{2464}{41}a_{4,24,\chi_8;4}(n)+\frac{10176}{41}a_{4,24,\chi_8;4}(n/2)-$$
$$\frac{5616}{41}a_{4,24,\chi_8;5}(n)+\frac{23040}{41}a_{4,24,\chi_8;5}(n/2)+\frac{384}{41}a_{4,24,\chi_8;6}(n)-\frac{1928}{41}a_{4,24,\chi_8;6}(n/2).$$

Formulas for the cases $(5,0,1,2),(3,2,1,2)$ of Theorem 2.2(iii).

$$N(1^5,3^1,4^2;n) =$$
$$\frac{54}{23}\sigma_{3;\chi_{12},\mathbf{1},}(n)-\frac{2}{23}\sigma_{3;\chi_{-4},\chi_{-3}}(n)+\frac{108}{23}\sigma_{3;\chi_{12},\mathbf{1},}(n/2)+\frac{4}{23}\sigma_{3;\chi_{-4},\chi_{-3}}(n/2)+\frac{1}{23}\sigma_{3;\mathbf{1},\chi_{12}}(n/4)-$$
$$\frac{1728}{23}\sigma_{3;\chi_{12},\mathbf{1},}(n/4)+\frac{64}{23}\sigma_{3;\chi_{-4},\chi_{-3}}(n/4)-\frac{27}{23}\sigma_{3;\chi_{-3},\chi_{-4}}(n/4)+\frac{63}{23}a_{4,12,\chi_{12};1}(n)+\frac{12}{23}a_{4,12,\chi_{12};2}(n)+$$
$$\frac{54}{23}a_{4,12,\chi_{12};3}(n)-\frac{174}{23}a_{4,12,\chi_{12};4}(n)+\frac{348}{23}a_{4,12,\chi_{12};1}(n/2)-\frac{924}{23}a_{4,12,\chi_{12};2}(n/2)-\frac{12}{23}a_{4,12,\chi_{12};3}(n/2)-$$
$$\frac{120}{23}a_{4,12,\chi_{12};4}(n/2)+\frac{632}{23}a_{4,12,\chi_{12};1}(n/4)-\frac{2528}{23}a_{4,12,\chi_{12};2}(n/4)+\frac{2528}{23}a_{4,12,\chi_{12};3}(n/4)+$$
$$\frac{-5056}{23}a_{4,12,\chi_{12};4}(n/4)+9a_{4,48,\chi_{12};1}(n)+3a_{4,48,\chi_{12};2}(n)-4a_{4,48,\chi_{12};3}(n)-64a_{4,48,\chi_{12};5}(n),$$

$$N(1^3,2^2,3^1,4^2;n) =$$
$$\frac{27}{23}\sigma_{3;\chi_{12},\mathbf{1},}(n)-\frac{1}{23}\sigma_{3;\chi_{-4},\chi_{-3}}(n)+\frac{1}{23}\sigma_{3;\mathbf{1},\chi_{12}}(n/4)-\frac{27}{23}\sigma_{3;\chi_{-3},\chi_{-4}}(n/4)+\frac{43}{23}a_{4,12,\chi_{12};1}(n)-\frac{40}{23}a_{4,12,\chi_{12};2}(n)+$$
$$\frac{73}{23}a_{4,12,\chi_{12};3}(n)-\frac{179}{23}a_{4,12,\chi_{12};4}(n)+8a_{4,12,\chi_{12};1}(n/2)-32a_{4,12,\chi_{12};2}(n/2)+32a_{4,12,\chi_{12};3}(n/2)+$$
$$8a_{4,12,\chi_{12};4}(n/2)+\frac{256}{23}a_{4,12,\chi_{12};1}(n/4)-\frac{1408}{23}a_{4,12,\chi_{12};2}(n/4)+\frac{1312}{23}a_{4,12,\chi_{12};3}(n/4)-\frac{2528}{23}a_{4,12,\chi_{12};4}(n/4)+$$
$$\frac{9}{2}a_{4,48,\chi_{12};1}(n)+\frac{7}{2}a_{4,48,\chi_{12};2}(n)-\frac{3}{2}a_{4,48,\chi_{12};3}(n)+\frac{3}{2}a_{4,48,\chi_{12};4}(n)-24a_{4,48,\chi_{12};5}(n)-24a_{4,48,\chi_{12};6}(n).$$

Formulas for the cases $(4,1,1,2),(2,3,1,2)$ of Theorem 2.2(iv).

$$N(1^4,2^1,3^1,4^2;n)=$$
$$\frac{1}{261}\sigma_{3;\mathbf{1},\chi_{24}}(n/2)+\frac{16}{261}\sigma_{3;\chi_{-8},\chi_{-3}}(n)+\frac{48}{29}\sigma_{3;\chi_{24},\mathbf{1}}(n)-\frac{3}{29}\sigma_{3;\chi_{-3},\chi_{-8}}(n/2)+\frac{7432}{261}a_{4,24,\chi_{24};1}(n)+$$
$$\frac{45424}{261}a_{4,24,\chi_{24};1}(n/2)+\frac{1378}{87}a_{4,24,\chi_{24};2}(n)+\frac{13576}{87}a_{4,24,\chi_{24};2}(n/2)-\frac{10214}{261}a_{4,24,\chi_{24};3}(n)-$$
$$\frac{80672}{261}a_{4,24,\chi_{24};3}(n/2)-\frac{17048}{87}a_{4,24,\chi_{24};4}(n)-\frac{123680}{87}a_{4,24,\chi_{24};4}(n/2)-\frac{5804}{87}a_{4,24,\chi_{24};5}(n)-$$
$$\frac{42944}{87}a_{4,24,\chi_{24};5}(n/2)+\frac{57448}{261}a_{4,24,\chi_{24};6}(n)+\frac{624352}{261}a_{4,24,\chi_{24};6}(n/2)+\frac{14218}{261}a_{4,24,\chi_{24};7}(n)+$$
$$\frac{252928}{261}a_{4,24,\chi_{24};7}(n/2)+\frac{25472}{261}a_{4,24,\chi_{24};8}(n)-\frac{252928}{261}a_{4,24,\chi_{24};8}(n/2)-\frac{5792}{261}a_{4,24,\chi_{24};9}(n)-$$
$$\frac{39656}{261}a_{4,24,\chi_{24};9}(n/2)-\frac{6418}{261}a_{4,24,\chi_{24};10}(n)-\frac{43888}{261}a_{4,24,\chi_{24};10}(n/2),$$

$$N(1^2,2^3,3^1,4^2;n)=$$
$$\frac{1}{261}\sigma_{3;\mathbf{1},\chi_{24}}(n/2)+\frac{8}{261}\sigma_{3;\chi_{-8},\chi_{-3}}(n)+\frac{24}{29}\sigma_{3;\chi_{24},\mathbf{1}}(n)-\frac{3}{29}\sigma_{3;\chi_{-3},\chi_{-8}}(n/2)+\frac{5804}{261}a_{4,24,\chi_{24};1}(n)+$$
$$\frac{24544}{261}a_{4,24,\chi_{24};1}(n/2)+\frac{1472}{87}a_{4,24,\chi_{24};2}(n)+\frac{9400}{87}a_{4,24,\chi_{24};2}(n/2)-\frac{9022}{261}a_{4,24,\chi_{24};3}(n)-$$
$$\frac{51440}{261}a_{4,24,\chi_{24};3}(n/2)-\frac{15136}{87}a_{4,24,\chi_{24};4}(n)-\frac{72176}{87}a_{4,24,\chi_{24};4}(n/2)-\frac{4468}{87}a_{4,24,\chi_{24};5}(n)-$$
$$\frac{13712}{87}a_{4,24,\chi_{24};5}(n/2)+\frac{61088}{261}a_{4,24,\chi_{24};6}(n)+\frac{377968}{261}a_{4,24,\chi_{24};6}(n/2)+\frac{19898}{261}a_{4,24,\chi_{24};7}(n)+$$
$$\frac{119296}{261}a_{4,24,\chi_{24};7}(n/2)+\frac{21088}{261}a_{4,24,\chi_{24};8}(n)-\frac{119296}{261}a_{4,24,\chi_{24};8}(n/2)-\frac{22952}{261}a_{4,24,\chi_{24};9}(n/2)-$$
$$\frac{5036}{261}a_{4,24,\chi_{24};10}(n)-\frac{25096}{261}a_{4,24,\chi_{24};10}(n/2).$$

Formulas for the cases $(0,0,4,2,2),(1,1,1,2,3)$ of Theorem 2.3(i).

$$N(3^4,4^2,6^2;n)=$$
$$\frac{1}{10}\sigma_3(n)-\frac{1}{10}\sigma_3(n/2)-\frac{21}{10}\sigma_3(n/3)+\frac{21}{10}\sigma_3(n/6)+\frac{8}{10}\sigma_3(n/8)-\frac{64}{5}\sigma_3(n/16)-\frac{84}{5}\sigma_3(n/24)+$$
$$\frac{1344}{25}\sigma_3(n/48)-\frac{4}{15}a_{4,6}(n)-\frac{4}{3}a_{4,6}(n/2)-16a_{4,6}(n/4)-\frac{384}{5}a_{4,6}(n/8)-\frac{1}{2}a_{4,8}(n)-5a_{4,8}(n/2)-$$
$$\frac{3}{2}a_{4,8}(n/3)-15a_{4,8}(n/6)+\frac{1}{6}a_{4,12}(n)+2a_{4,12}(n/2)+12a_{4,12}(n/4)-a_{4,16}(n)+3a_{4,16}(n/3)+$$
$$\frac{1}{2}a_{4,24}(n)\left(\frac{4}{n}\right)+3a_{4,24}(n/2)\left(\frac{4}{n/2}\right)+\frac{4}{3}a_{4,6}(n)\left(\frac{-4}{n}\right)+\frac{2}{3}a_{4,12}(n)\left(\frac{-4}{n}\right)-a_{4,24}(n)\left(\frac{-4}{n}\right)$$

$N(1^1,2^1,3^1,4^2,6^3;n) =$
$\frac{1}{8}\sigma_3(n)-\frac{1}{8}\sigma_3(n/2)-\frac{9}{8}\sigma_3(n/3)+\frac{9}{8}\sigma_3(n/6)+2\sigma_3(n/8)-32\sigma_3(n/16)-18\sigma_3(n/24)+288\sigma_3(n/48)-$
$4a_{4,6}(n/4)-32a_{4,6}(n/8)-\frac{1}{8}a_{4,8}(n)-\frac{5}{2}a_{4,8}(n/2)+\frac{9}{8}a_{4,8}(n/3)-\frac{27}{2}a_{4,8}(n/6)+\frac{3}{8}a_{4,12}(n)+$
$2a_{4,12}(n/2)+6a_{4,12}(n/4)-\frac{1}{4}a_{4,16}(n)-\frac{9}{4}a_{4,16}(n/3)+\frac{5}{8}a_{4,24}(n)\left(\frac{4}{n}\right)+\frac{3}{2}a_{4,24}(n/2)\left(\frac{4}{n/2}\right)+$
$a_{4,6}(n)\left(\frac{-4}{n}\right)+\frac{1}{2}a_{4,12}(n)\left(\frac{-4}{n}\right)-\frac{1}{4}a_{4,24}(n)\left(\frac{-4}{n}\right)$

Formulas for the cases $(0,0,3,2,3),(0,0,1,4,3)$ **of Theorem 2.3(ii).**

$N(3^3,4^2,6^3;n) =$
$\frac{4}{451}\sigma_{3;\mathbf{1},\chi_8}(n/2)+\frac{78}{451}\sigma_{3;\mathbf{1},\chi_8}(n/6)+\frac{32}{451}\sigma_{3;\chi_8,\mathbf{1}}(n)-\frac{624}{451}\sigma_{3;\chi_8,\mathbf{1}}(n/3)+\frac{44746}{4059}a_{4,8,\chi_8;1}(n)-$
$\frac{17944}{4059}a_{4,8,\chi_8;1}(n/2)-\frac{234}{451}a_{4,8,\chi_8;1}(n/3)-\frac{15456}{451}a_{4,8,\chi_8;1}(n/6)-\frac{7232}{451}a_{4,8,\chi_8;2}(n)-\frac{128}{451}a_{4,8,\chi_8;2}(n/2)+$
$\frac{312}{451}a_{4,8,\chi_8;2}(n/3)-\frac{2496}{451}a_{4,8,\chi_8;2}(n/6)-\frac{2582}{369}a_{4,24,\chi_8;1}(n)+\frac{1664}{369}a_{4,24,\chi_8;1}(n/2)-\frac{3682}{123}a_{4,24,\chi_8;2}(n)+$
$\frac{2776}{123}a_{4,24,\chi_8;2}(n/2)-\frac{650}{123}a_{4,24,\chi_8;3}(n)-\frac{1520}{123}a_{4,24,\chi_8;3}(n/2)+\frac{23752}{369}a_{4,24,\chi_8;4}(n)-\frac{13888}{369}a_{4,24,\chi_8;4}(n/2)+$
$\frac{4488}{41}a_{4,24,\chi_8;5}(n)-\frac{6976}{41}a_{4,24,\chi_8;5}(n/2)-\frac{1154}{41}a_{4,24,\chi_8;6}(n)+\frac{1016}{41}a_{4,24,\chi_8;6}(n/2),$

$N(3^1,4^4,6^3;n) =$
$-\frac{8}{451}\sigma_{3;\mathbf{1},\chi_8}(n/2)+\frac{90}{451}\sigma_{3;\mathbf{1},\chi_8}(n/6)+\frac{16}{451}\sigma_{3;\chi_8,\mathbf{1}}(n)+\frac{180}{451}\sigma_{3;\chi_8,\mathbf{1}}(n/3)-\frac{8177}{1353}a_{4,8,\chi_8;1}(n)+$
$\frac{3544}{1353}a_{4,8,\chi_8;1}(n/2)-\frac{1962}{451}a_{4,8,\chi_8;1}(n/3)+\frac{17136}{451}a_{4,8,\chi_8;1}(n/6)+\frac{3600}{451}a_{4,8,\chi_8;2}(n)+\frac{256}{451}a_{4,8,\chi_8;2}(n/2)+$
$\frac{8028}{451}a_{4,8,\chi_8;2}(n/3)-\frac{2880}{451}a_{4,8,\chi_8;2}(n/6)+\frac{499}{123}a_{4,24,\chi_8;1}(n)-\frac{344}{123}a_{4,24,\chi_8;1}(n/2)+\frac{589}{41}a_{4,24,\chi_8;2}(n)-$
$\frac{648}{41}a_{4,24,\chi_8;2}(n/2)-\frac{83}{41}a_{4,24,\chi_8;3}(n)+\frac{248}{41}a_{4,24,\chi_8;3}(n/2)-\frac{2984}{123}a_{4,24,\chi_8;4}(n)+\frac{3136}{123}a_{4,24,\chi_8;4}(n/2)-$
$\frac{2184}{41}a_{4,24,\chi_8;5}(n)+\frac{3456}{41}a_{4,24,\chi_8;5}(n/2)+\frac{163}{41}a_{4,24,\chi_8;6}(n)-\frac{240}{41}a_{4,24,\chi_8;6}(n/2).$

Formulas for the cases $(0,0,5,1,2),(0,0,1,3,4)$ **of Theorem 2.3(iii).**

$N(3^5,4^1,6^2;n) =$
$\frac{2}{23}\sigma_{3;\chi_{12},\mathbf{1},}(n)+\frac{2}{23}\sigma_{3;\chi_{-4},\chi_{-3}}(n)+\frac{1}{23}\sigma_{3;\mathbf{1},\chi_{12}}(n/4)+\frac{1}{23}\sigma_{3;\chi_{-3},\chi_{-4}}(n/4)+\frac{19}{23}a_{4,12,\chi_{12};1}(n)+$
$\frac{22}{23}a_{4,12,\chi_{12};3}(n)+\frac{22}{23}a_{4,12,\chi_{12};4}(n)+16a_{4,12,\chi_{12};2}(n/2)+24a_{4,12,\chi_{12};3}(n/2)-\frac{416}{23}a_{4,12,\chi_{12};1}(n/4)+$
$\frac{2912}{23}a_{4,12,\chi_{12};3}(n/4)+\frac{2912}{23}a_{4,12,\chi_{12};4}(n/4)-\frac{5}{2}a_{4,48,\chi_{12};1}(n)+\frac{7}{6}a_{4,48,\chi_{12};2}(n)+$
$\frac{3}{2}a_{4,48,\chi_{12};3}(n)+\frac{31}{6}a_{4,48,\chi_{12};4}(n)+\frac{40}{3}a_{4,48,\chi_{12};5}(n)-\frac{56}{3}a_{4,48,\chi_{12};6}(n),$

$N(3^1,4^3,6^4;n) =$
$\frac{3}{92}\sigma_{3;\chi_{12},\mathbf{1},}(n) - \frac{1}{92}\sigma_{3;\chi_{-4},\chi_{-3}}(n) + \frac{1}{23}\sigma_{3;\mathbf{1},\chi_{12}}(n/4) - \frac{3}{23}\sigma_{3;\chi_{-3},\chi_{-4}}(n/4) - \frac{1}{46}a_{4,12,\chi_{12};1}(n) -$
$\frac{8}{23}a_{4,12,\chi_{12};2}(n) + \frac{173}{46}a_{4,12,\chi_{12};3}(n) + \frac{17}{46}a_{4,12,\chi_{12};4}(n) + 12a_{4,12,\chi_{12};4}(n/2) + \frac{140}{23}a_{4,12,\chi_{12};1}(n/4) +$
$\frac{272}{23}a_{4,12,\chi_{12};2}(n/4) - \frac{1312}{23}a_{4,12,\chi_{12};3}(n/4) - \frac{1072}{23}a_{4,12,\chi_{12};4}(n/4) + \frac{3}{4}a_{4,48,\chi_{12};1}(n) - \frac{1}{4}a_{4,48,\chi_{12};2}(n) -$
$\frac{3}{4}a_{4,48,\chi_{12};3}(n) - \frac{7}{4}a_{4,48,\chi_{12};4}(n) - 4a_{4,48,\chi_{12};5}(n) + 4a_{4,48,\chi_{12};6}(n).$

Formulas for the cases $(0,0,4,1,3),(1,1,1,3,2)$ **of Theorem 2.3(iv).**

$N(3^4,4^1,6^3;n) =$
$\frac{1}{261}\sigma_{3;\mathbf{1},\chi_{24}}(n/2) + \frac{16}{261}\sigma_{3;\chi_{-8},\chi_{-3}}(n) + \frac{16}{261}\sigma_{3;\chi_{24},\mathbf{1}}(n) - \frac{1}{261}\sigma_{3;\chi_{-3},\chi_{-8}}(n/2) +$
$\frac{1504}{261}a_{4,24,\chi_{24};1}(n) + \frac{512}{87}a_{4,24,\chi_{24};1}(n/2) + \frac{512}{87}a_{4,24,\chi_{24};2}(n) + \frac{1320}{29}a_{4,24,\chi_{24};2}(n/2) -$
$\frac{1432}{261}a_{4,24,\chi_{24};3}(n) - \frac{8432}{87}a_{4,24,\chi_{24};3}(n/2) - \frac{10096}{261}a_{4,24,\chi_{24};4}(n) - \frac{132608}{261}a_{4,24,\chi_{24};4}(n/2) -$
$\frac{1024}{87}a_{4,24,\chi_{24};5}(n) - 256a_{4,24,\chi_{24};5}(n/2) + \frac{6128}{87}a_{4,24,\chi_{24};6}(n) + \frac{227648}{261}a_{4,24,\chi_{24};6}(n/2) +$
$\frac{1640}{87}a_{4,24,\chi_{24};7}(n) + \frac{3760}{9}a_{4,24,\chi_{24};7}(n/2) - \frac{132608}{261}a_{4,24,\chi_{24};8}(n/2) - \frac{512}{87}a_{4,24,\chi_{24};9}(n) -$
$\frac{16756}{261}a_{4,24,\chi_{24};9}(n/2) + \frac{512}{87}a_{4,24,\chi_{24};10}(n) - \frac{15536}{261}a_{4,24,\chi_{24};10}(n/2)$

$N(1^1,2^1,3^1,4^3,6^2;n) =$
$\frac{1}{261}\sigma_{3;\mathbf{1},\chi_{24}}(n/2) - \frac{4}{261}\sigma_{3;\chi_{-8},\chi_{-3}}(n) + \frac{4}{29}\sigma_{3;\chi_{24},\mathbf{1}}(n) + \frac{1}{29}\sigma_{3;\chi_{-3},\chi_{-8}}(n/2) +$
$\frac{194}{261}a_{4,24,\chi_{24};1}(n) - \frac{13472}{261}a_{4,24,\chi_{24};1}(n/2) - \frac{202}{87}a_{4,24,\chi_{24};2}(n) - \frac{4664}{87}a_{4,24,\chi_{24};2}(n/2) +$
$\frac{1190}{261}a_{4,24,\chi_{24};3}(n) + \frac{776}{9}a_{4,24,\chi_{24};3}(n/2) + \frac{1172}{87}a_{4,24,\chi_{24};4}(n) + \frac{1352}{3}a_{4,24,\chi_{24};4}(n/2) -$
$\frac{304}{87}a_{4,24,\chi_{24};5}(n) + \frac{13144}{87}a_{4,24,\chi_{24};5}(n/2) + \frac{164}{261}a_{4,24,\chi_{24};6}(n) - \frac{218696}{261}a_{4,24,\chi_{24};6}(n/2) +$
$\frac{698}{261}a_{4,24,\chi_{24};7}(n) - \frac{68048}{261}a_{4,24,\chi_{24};7}(n/2) - \frac{14576}{261}a_{4,24,\chi_{24};8}(n) + \frac{96128}{261}a_{4,24,\chi_{24};8}(n/2) +$
$\frac{296}{261}a_{4,24,\chi_{24};9}(n) + \frac{13984}{261}a_{4,24,\chi_{24};9}(n/2) + \frac{286}{261}a_{4,24,\chi_{24};10}(n) + \frac{12956}{261}a_{4,24,\chi_{24};10}(n/2)$

Remark 4.5 . We note that formulas for the cases (i,j,k,l,m) (i.e., with coefficients $1,2,3,4,6$) in Table 2, with $i=k=0$ can be obtained from the work [4] (replacing n by $n/2$ in their formulas). There are 28 such cases (10 for χ_{24} character case and 6 each in the remaining 3 characters) in Table 2 with this condition. It is to be noted that different bases were used in [4] to get the formulas. So, replacing $n/2$ by n in our formulas in these 28 cases, we get different formulas for these cases (with coefficients 1,2,3) when compared with [4].

5. List of tables

In this section we list Tables 1 and 2, which give the list of exponents of the theta functions corresponding to the coefficients $1,2,3,4$ and $1,2,3,4,6$ respectively. We mention only the character

χ in these tables corresponding to the space $M_4(\Gamma_0(48), \chi)$. The rest of the tables 3 to 10 give the coefficients list for the formulas for the number of representations corresponding to Theorem 2.2 and Theorem 2.3. These tables (3 to 10) are kept at the end as an Appendix.

Table 1.

Octonary quadratic forms with coefficients $1, 2, 3, 4$ ($k, l \neq 0$).

(i, j, k, l)	Space
(0,0,6,2),(0,0,4,4),(0,0,2,6),(0,4,2,2),(0,2,4,2),(0,2,2,4),(5,0,2,1),(4,0,2,2),(3,0,4,1), (3,0,2,3),(2,0,4,2),(2,0,2,4),(1,0,6,1),(1,0,4,3),(1,0,2,5),(3,2,2,1),(2,2,2,2),(1,4,2,1),(1,2,4,1),(1,2,2,3)	triv.
(0,5,2,1),(0,3,4,1),(0,3,2,3),(0,1,6,1),(0,1,4,3),(0,1,2,5),(4,1,2,1),(3,1,2,2),(2,3,2,1),(2,1,4,1),(2,1,2,3),(1,3,2,2),(1,1,4,2),(1,1,2,4)	χ_8
(0,0,7,1),(0,0,5,3),(0,0,3,5),(0,0,1,7),(0,6,1,1),(0,4,3,1),(0,4,1,3),(0,2,5,1),(0,2,3,3),(0,2,1,5),(6,0,1,1),(5,0,1,2),(4,0,3,1),(4,0,1,3),(3,0,3,2),(3,0,1,4),(2,0,5,1),(2,0,3,3), (2,0,1,5),(1,0,5,2),(1,0,3,4),(1,0,1,6),(4,2,1,1),(3,2,1,2),(2,4,1,1),(2,2,3,1),(2,2,1,3),(1,4,1,2),(1,2,3,2), (1,2,1,4)	χ_{12}
(0,5,1,2),(0,3,3,2),(0,3,1,4),(0,1,5,2),(0,1,3,4),(0,1,1,6),(5,1,1,1),(4,1,1,2),(3,3,1,1).(3,1,3,1),(3,1,1,3),(2,3,1,2),(2,1,3,2),(2,1,1,4),(1,5,1,1),(1,3,3,1),(1,3,1,3),(1,1,5,1),(1,1,3,3),(1,1,1,5)	χ_{24}

Table 2.

Octonary quadratic forms with coefficients $1, 2, 3, 4, 6$ ($l, m \neq 0$).

(i, j, k, l, m)	Space
(0,0,4,2,2),(0,0,2,4,2),(0,0,2,2,4),(0,2,2,2,2),(0,4,0,2,2), (0,2,0,4,2),(0,2,0,2,4),(0,1,5,1,1),(0,3,3,1,1),(0,5,1,1,1),(0,1,3,1,3),(0,3,1,1,3),(0,1,1,3,3), (0,1,1,1,5) (0,1,3,3,1),(0,3,1,3,1),(0,1,1,5,1),(1,1,1,2,3),(1,1,1,4,1),(1,1,3,2,1),(1,3,1,2,1),(1,0,4,1,2),(1,2,2,1,2),(1,4,0,1,2),(1,0,0,3,4),(1,0,2,3,2), (1,2,0,3,2),(1,0,0,5,2),(1,0,2,1,4), (1,2,0,1,4),(1,0,0,1,6),(2,2,0,2,2),(2,1,3,1,1),(2,3,1,1,1),(2,1,1,1,3),(2,1,1,3,1),(2,0,2,2,2), (2,0,0,4,2),(2,0,0,2,4),(3,1,1,2,1),(3,0,2,1,2),(3,2,0,1,2),(3,0,0,3,2),(3,0,0,1,4),(4,1,1,1,1),(4,0,0,2,2),(5,0,0,1,2)	triv.
(0,0,3,2,3),(0,0,1,4,3),(0,0,3,4,1),(0,0,5,2,1),(0,0,1,2,5),(0,0,1,6,1),(0,1,4,1,2),(0,3,2,1,2) (0,5,0,1,2),(0,1,0,3,4),(0,1,2,3,2),(0,3,0,3,2),(0,1,0,5,2),(0,1,2,1,4),(0,3,0,1,4),(0,1,0,1,6) (0,2,1,2,3),(0,2,1,4,1),(0,2,3,2,1),(0,4,1,2,1),(1,1,2,2,2),(1,1,0,4,2),(1,1,0,2,4),(1,3,0,2,2) (1,0,3,1,3),(1,2,1,1,3),(1,0,1,1,5),(1,0,1,3,3),(1,0,5,1,1),(1,2,3,1,1),(1,4,1,1,1),(1,0,3,3,1) (1,2,1,3,1),(1,0,1,5,1),(2,2,1,2,1),(2,1,2,1,2),(2,3,0,1,2),(2,1,0,3,2),(2,1,0,1,4),(2,0,1,2,3) (2,0,1,4,1),(2,0,3,2,1),(3,1,0,2,2),(3,0,1,1,3),(3,0,3,1,1),(3,2,1,1,1),(3,0,1,3,1),(4,1,0,1,2) (4,0,1,2,1),(5,0,1,1,1)	χ_8
(0,0,5,1,2),(0,0,1,3,4),(0,0,3,3,2),(0,0,1,5,2),(0,0,3,1,4),(0,0,1,1,6),(0,2,3,1,2),(0,2,1,3,2) (0,4,1,1,2),(0,2,1,1,4),(0,3,0,2,3),(0,1,2,2,3),(0,1,0,4,3),(0,1,2,4,1),(0,3,0,4,1),(0,1,4,2,1) (0,3,2,2,1),(0,5,0,2,1),(0,1,0,2,5),(0,1,0,6,1),(1,1,2,1,3),(1,1,0,3,3),(1,1,4,1,1),(1,1,0,1,5) (1,1,2,3,1),(1,1,0,5,1),(1,0,3,2,2),(1,2,1,2,2)(1,0,1,4,2),(1,0,1,2,4),(1,3,0,1,3),(1,3,2,1,1) (1,5,0,1,1),(1,3,0,3,1),(2,2,1,1,2),(2,0,3,1,2),(2,0,1,3,2),(2,0,1,1,4),(2,1,0,2,3),(2,1,0,4,1) (2,1,2,2,1),(2,3,0,2,1),(3,3,0,1,1),(3,0,1,2,2),(3,1,0,1,3),(3,1,2,1,1),(3,1,0,3,1),(4,0,1,1,2) (4,1,0,2,1),(5,1,0,1,1)	χ_{12}
(0,0,4,1,3),(0,0,2,3,3),(0,0,6,1,1),(0,0,2,1,5), (0,0,4,3,1),(0,0,2,5,1),(0,2,2,1,3),(0,4,0,1,3), (0,2,0,3,3),(0,2,4,1,1),(0,4,2,1,1),(0,6,0,1,1), (0,2,0,1,5),(0,2,2,3,1),(0,4,0,3,1),(0,2,0,5,1), (0,1,3,2,2),(0,3,1,2,2),(0,1,1,4,2),(0,1,1,2,4), (1,1,1,3,2),(1,1,1,1,4),(1,1,3,1,2),(1,0,2,2,3), (1,2,0,2,3),(1,0,0,4,3),(1,0,2,4,1),(1,2,0,4,1), (1,0,4,2,1),(1,2,2,2,1),(1,4,0,2,1),(1,0,0,2,5), (1,0,0,6,1),(1,3,1,1,2),(2,2,2,1,1),(2,2,0,1,3), (2,2,0,3,1),(2,0,2,1,3),(2,0,0,3,3),(2,0,4,1,1), (2,4,0,1,1),(2,0,0,1,5),(2,0,2,3,1),(2,0,0,5,1),(2,1,1,2,2),(3,0,0,2,3),(3,0,0,4,1),(3,0,2,2,1), (3,2,0,2,1),(3,1,1,1,2),(4,0,0,1,3),(4,0,2,1,1),(4,2,0,1,1),(4,0,0,3,1),(5,0,0,2,1),(6,0,0,1,1)	χ_{24}

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Appendix

We list the tables 3 to 10 below.

Table 3. (Theorem 2.2 (i))

$ijkl$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
0062	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{-7}{400}$	0	$\frac{7}{400}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$	$\frac{1}{20}$	$\frac{21}{20}$	$\frac{2}{15}$	$\frac{-8}{3}$	$\frac{-112}{3}$
0044	$\frac{1}{2400}$	$\frac{1}{800}$	$\frac{3}{800}$	$\frac{-17}{600}$	$\frac{9}{800}$	$\frac{1}{50}$	$\frac{-51}{200}$	$\frac{8}{75}$	$\frac{9}{50}$	$\frac{24}{25}$	0	0	$\frac{2}{5}$	$\frac{28}{5}$	$\frac{112}{5}$
0026	$\frac{7}{19200}$	$\frac{-7}{19200}$	$\frac{-9}{6400}$	0	$\frac{9}{6400}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	$\frac{-7}{80}$	$\frac{-27}{80}$	$\frac{-19}{10}$	-7	-8
0422	$\frac{7}{4800}$	$\frac{-7}{4800}$	$\frac{-9}{1600}$	0	$\frac{9}{1600}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-8}{5}$	-4	0
0242	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{3}{400}$	0	$\frac{-3}{400}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$	0	0	$\frac{4}{5}$	8	16
0224	$\frac{7}{9600}$	$\frac{-7}{9600}$	$\frac{-9}{3200}$	0	$\frac{9}{3200}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-9}{5}$	-6	-8
5021	$\frac{7}{600}$	0	$\frac{-9}{200}$	$\frac{-119}{600}$	0	$\frac{21}{100}$	$\frac{153}{200}$	$\frac{-28}{75}$	$\frac{-81}{100}$	$\frac{36}{25}$	$\frac{7}{20}$	$\frac{27}{20}$	$\frac{-4}{5}$	$\frac{16}{5}$	$\frac{-176}{5}$
4022	$\frac{7}{1200}$	$\frac{-7}{1200}$	$\frac{-9}{400}$	0	$\frac{9}{400}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	$\frac{7}{20}$	$\frac{27}{20}$	$\frac{-2}{5}$	0	-48
3041	$\frac{1}{300}$	$\frac{-1}{150}$	$\frac{3}{100}$	$\frac{17}{300}$	$\frac{-3}{50}$	$\frac{-3}{50}$	$\frac{51}{100}$	$\frac{8}{75}$	$\frac{-27}{50}$	$\frac{24}{25}$	$\frac{-1}{10}$	$\frac{9}{10}$	$\frac{16}{5}$	$\frac{64}{5}$	$\frac{96}{5}$
3023	$\frac{7}{2400}$	$\frac{-7}{800}$	$\frac{-9}{800}$	$\frac{119}{1200}$	$\frac{27}{800}$	$\frac{-7}{100}$	$\frac{-153}{400}$	$\frac{-28}{75}$	$\frac{27}{100}$	$\frac{36}{25}$	$\frac{7}{40}$	$\frac{27}{40}$	$\frac{-1}{5}$	$\frac{-8}{5}$	$\frac{-212}{5}$
2042	$\frac{1}{600}$	$\frac{-1}{600}$	$\frac{3}{200}$	0	$\frac{-3}{200}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$	$\frac{-1}{10}$	$\frac{9}{10}$	$\frac{8}{5}$	8	32
2024	$\frac{7}{4800}$	$\frac{-7}{960}$	$\frac{-9}{1600}$	$\frac{119}{1200}$	$\frac{9}{320}$	$\frac{-7}{100}$	$\frac{-153}{400}$	$\frac{-28}{75}$	$\frac{27}{100}$	$\frac{36}{25}$	0	0	$\frac{-3}{5}$	$\frac{-18}{5}$	$\frac{-152}{5}$
1061	$\frac{1}{600}$	0	$\frac{-7}{200}$	$\frac{-17}{600}$	0	$\frac{3}{100}$	$\frac{119}{200}$	$\frac{-4}{75}$	$\frac{-63}{100}$	$\frac{28}{25}$	$\frac{1}{20}$	$\frac{21}{20}$	$\frac{4}{15}$	$\frac{-16}{15}$	$\frac{-304}{15}$
1043	$\frac{1}{1200}$	$\frac{1}{1200}$	$\frac{3}{400}$	$\frac{-17}{600}$	$\frac{3}{400}$	$\frac{1}{50}$	$\frac{-51}{200}$	$\frac{8}{75}$	$\frac{9}{50}$	$\frac{24}{25}$	$\frac{-1}{20}$	$\frac{9}{20}$	$\frac{4}{5}$	$\frac{28}{5}$	$\frac{152}{5}$
1025	$\frac{7}{9600}$	$\frac{-7}{1920}$	$\frac{-9}{3200}$	$\frac{119}{2400}$	$\frac{9}{640}$	$\frac{-7}{300}$	$\frac{-153}{800}$	$\frac{-28}{75}$	$\frac{9}{100}$	$\frac{36}{25}$	$\frac{-7}{80}$	$\frac{-27}{80}$	$\frac{-13}{10}$	$\frac{-29}{5}$	$\frac{-96}{5}$
3221	$\frac{7}{1200}$	$\frac{-7}{1200}$	$\frac{-9}{400}$	0	$\frac{9}{400}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-2}{5}$	0	-24
2222	$\frac{7}{2400}$	$\frac{-7}{2400}$	$\frac{-9}{800}$	0	$\frac{9}{800}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-6}{5}$	-4	-24
1421	$\frac{7}{2400}$	$\frac{-7}{2400}$	$\frac{-9}{800}$	0	$\frac{9}{800}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-6}{5}$	-4	-16
1241	$\frac{1}{600}$	$\frac{-1}{600}$	$\frac{3}{200}$	0	$\frac{-3}{200}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$	0	0	$\frac{8}{5}$	8	16
1223	$\frac{7}{4800}$	$\frac{-7}{4800}$	$\frac{-9}{1600}$	0	$\frac{9}{1600}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$	0	0	$\frac{-8}{5}$	-6	-20

Table 3. (Theorem 2.2 (i) (contd.))

$ijkl$	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
0062	$\frac{-2432}{15}$	$\frac{-3}{2}$	-9	$\frac{-9}{2}$	-27	$\frac{2}{3}$	$\frac{10}{3}$	$\frac{68}{3}$	$\frac{-9}{4}$	$\frac{27}{4}$	$\frac{1}{2}$	7	$\frac{38}{15}$	$\frac{17}{12}$	$\frac{-7}{4}$
0044	$\frac{128}{5}$	$\frac{3}{2}$	6	$\frac{27}{2}$	54	$\frac{-1}{2}$	-6	-8	1	9	$\frac{-3}{2}$	-6	-2	0	1
0026	$\frac{32}{5}$	$\frac{-9}{16}$	$\frac{-3}{2}$	$\frac{-243}{16}$	$\frac{-81}{2}$	$\frac{17}{16}$	$\frac{5}{2}$	8	$\frac{9}{16}$	$\frac{-243}{16}$	$\frac{21}{16}$	$\frac{3}{2}$	$\frac{19}{10}$	$\frac{-17}{16}$	$\frac{-21}{16}$
0422	$\frac{-128}{5}$	$\frac{-3}{4}$	-1	$\frac{-81}{4}$	-27	$\frac{5}{4}$	4	4	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{3}{4}$	3	2	-1	$\frac{-3}{2}$
0242	$\frac{128}{5}$	1	6	9	54	-1	-4	-8	1	9	-1	-6	-2	0	1
0224	$\frac{-128}{5}$	$\frac{-5}{8}$	-1	$\frac{-135}{8}$	-27	$\frac{9}{8}$	3	4	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{9}{8}$	3	2	-1	$\frac{-3}{2}$
5021	$\frac{-1408}{5}$	0	1	0	27	4	0	20	$\frac{1}{4}$	$\frac{-27}{4}$	0	9	$\frac{22}{5}$	$\frac{5}{4}$	$\frac{-9}{4}$
4022	$\frac{-1408}{5}$	$\frac{-1}{2}$	1	$\frac{-27}{2}$	27	2	2	20	$\frac{1}{4}$	$\frac{-27}{4}$	$\frac{3}{2}$	9	$\frac{22}{5}$	$\frac{5}{4}$	$\frac{-9}{4}$
3041	$\frac{768}{5}$	0	10	0	18	0	0	-8	$\frac{5}{2}$	$\frac{-9}{2}$	0	-10	$\frac{-12}{5}$	$\frac{-1}{2}$	$\frac{5}{2}$
3023	$\frac{-928}{5}$	$\frac{-3}{4}$	0	$\frac{-81}{4}$	0	1	3	20	$\frac{3}{8}$	$\frac{-81}{8}$	$\frac{9}{4}$	6	$\frac{37}{10}$	$\frac{5}{8}$	$\frac{-15}{8}$
2042	$\frac{768}{5}$	1	10	9	18	0	-4	-8	$\frac{5}{2}$	$\frac{-9}{2}$	-1	-10	$\frac{-12}{5}$	$\frac{-1}{2}$	$\frac{5}{2}$
2024	$\frac{-448}{5}$	$\frac{-3}{4}$	-1	$\frac{-81}{4}$	-27	$\frac{3}{4}$	3	20	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{9}{4}$	3	3	0	$\frac{-3}{2}$
1061	$\frac{-2432}{15}$	0	-9	0	-27	$\frac{4}{3}$	0	$\frac{68}{3}$	$\frac{-9}{4}$	$\frac{27}{4}$	0	7	$\frac{38}{15}$	$\frac{17}{12}$	$\frac{-7}{4}$
1043	$\frac{448}{5}$	$\frac{3}{2}$	8	$\frac{27}{2}$	36	0	-6	-8	$\frac{7}{4}$	$\frac{9}{4}$	$\frac{-3}{2}$	-8	$\frac{-11}{5}$	$\frac{-1}{4}$	$\frac{7}{4}$
1025	$\frac{-128}{5}$	$\frac{-5}{8}$	$\frac{-3}{2}$	$\frac{-135}{8}$	$\frac{-81}{2}$	$\frac{7}{8}$	$\frac{5}{2}$	16	$\frac{9}{16}$	$\frac{-243}{16}$	$\frac{15}{8}$	$\frac{3}{2}$	$\frac{12}{5}$	$\frac{-9}{16}$	$\frac{-21}{16}$
3221	$\frac{-448}{5}$	$\frac{-1}{2}$	-1	$\frac{-27}{2}$	-27	2	2	20	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{3}{2}$	3	3	0	$\frac{-3}{2}$
2222	$\frac{-448}{5}$	$\frac{-1}{2}$	-1	$\frac{-27}{2}$	-27	$\frac{3}{2}$	2	20	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{3}{2}$	3	3	0	$\frac{-3}{2}$
1421	$\frac{-128}{5}$	$\frac{-1}{2}$	-1	$\frac{-27}{2}$	-27	$\frac{3}{2}$	2	4	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{3}{2}$	3	2	-1	$\frac{-3}{2}$
1241	$\frac{128}{5}$	1	6	9	54	0	-4	-8	1	9	-1	-6	-2	0	1
1223	$\frac{-288}{5}$	$\frac{-1}{2}$	-1	$\frac{-27}{2}$	-27	$\frac{5}{4}$	2	12	$\frac{1}{2}$	$\frac{-27}{2}$	$\frac{3}{2}$	3	$\frac{5}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$

Table 4. (Theorem 2.2 (ii))

$ijkl$	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
0521	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{208}{451}$	0	$\frac{864}{451}$	0	$\frac{-3112}{451}$	$\frac{16768}{451}$	$\frac{324}{451}$	$\frac{98496}{451}$	$\frac{3504}{451}$	$\frac{832}{451}$
0341	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{160}{451}$	0	$\frac{-1152}{451}$	0	$\frac{14744}{1353}$	$\frac{-32392}{451}$	$\frac{-432}{451}$	$\frac{-64224}{451}$	$\frac{-7296}{451}$	$\frac{-320}{451}$
0323	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{104}{451}$	0	$\frac{432}{451}$	0	$\frac{-4713}{451}$	$\frac{16768}{451}$	$\frac{162}{451}$	$\frac{98496}{451}$	$\frac{5360}{451}$	$\frac{832}{451}$
0161	0	$\frac{-2}{451}$	0	$\frac{84}{451}$	$\frac{64}{451}$	0	$\frac{2688}{451}$	0	$\frac{-96320}{4059}$	$\frac{158704}{4059}$	$\frac{1008}{451}$	$\frac{33312}{451}$	$\frac{14400}{451}$	$\frac{-28800}{451}$
0143	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{80}{451}$	0	$\frac{-576}{451}$	0	$\frac{7268}{451}$	$\frac{628}{451}$	$\frac{-216}{451}$	$\frac{-64224}{451}$	$\frac{-10864}{451}$	$\frac{-14752}{451}$
0125	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{52}{451}$	0	$\frac{216}{451}$	0	$\frac{-3935}{451}$	$\frac{11356}{451}$	$\frac{81}{451}$	$\frac{98496}{451}$	$\frac{4484}{451}$	$\frac{8048}{451}$
4121	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{832}{451}$	0	$\frac{3456}{451}$	0	$\frac{-12448}{451}$	$\frac{38416}{451}$	$\frac{1296}{451}$	$\frac{98496}{451}$	$\frac{14016}{451}$	$\frac{-28032}{451}$
3122	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{416}{451}$	0	$\frac{1728}{451}$	0	$\frac{-12538}{451}$	$\frac{38416}{451}$	$\frac{648}{451}$	$\frac{98496}{451}$	$\frac{14224}{451)}$	$\frac{-28032}{451}$
2321	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{416}{451}$	0	$\frac{1728}{451}$	0	$\frac{-6224}{451}$	$\frac{16768}{451}$	$\frac{648}{451}$	$\frac{98496}{451}$	$\frac{7008}{451}$	$\frac{832}{451}$
2141	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{320}{451}$	0	$\frac{-2304}{451}$	0	$\frac{24076}{1353}$	$\frac{36160}{1353}$	$\frac{-864}{451}$	$\frac{-64224}{451}$	$\frac{-14592}{451}$	$\frac{-29184}{451}$
2123	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{208}{451}$	0	$\frac{864}{451}$	0	$\frac{-9426}{451}$	$\frac{27592}{451}$	$\frac{324}{451}$	$\frac{98496}{451}$	$\frac{10720}{451}$	$\frac{-13600}{451}$
1322	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{208}{451}$	0	$\frac{864}{451}$	0	$\frac{-6269}{451}$	$\frac{16768}{451}$	$\frac{324}{451}$	$\frac{98496}{451}$	$\frac{7112}{451}$	$\frac{832}{451}$
1142	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{160}{451}$	0	$\frac{-1152}{451}$	0	$\frac{26470}{1353}$	$\frac{36160}{1353}$	$\frac{-432}{451}$	$\frac{-64224}{451}$	$\frac{-14512}{451}$	$\frac{-29184}{451}$
1124	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{104}{451}$	0	$\frac{432}{451}$	0	$\frac{-12583}{902}$	$\frac{16768}{451}$	$\frac{162}{451}$	$\frac{98496}{451}$	$\frac{7164}{451}$	$\frac{832}{451}$

$ijkl$	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}	b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{27}	b_{28}
0521	$\frac{-432}{451}$	$\frac{-3456}{451}$	$\frac{208}{41}$	$\frac{-1712}{41}$	$\frac{810}{41}$	$\frac{-7272}{41}$	$\frac{-54}{41}$	$\frac{2856}{41}$	$\frac{-904}{41}$	$\frac{12800}{41}$	$\frac{-2808}{41}$	$\frac{23040}{41}$	$\frac{110}{41}$	$\frac{-2256}{41}$
0341	$\frac{576}{451}$	$\frac{-2304}{451}$	$\frac{-832}{123}$	$\frac{2480}{123}$	$\frac{-1080}{41}$	$\frac{4992}{41}$	$\frac{72}{41}$	$\frac{-1376}{41}$	$\frac{4928}{123}$	$\frac{-32128}{123}$	$\frac{3744}{41}$	$\frac{-16128}{41}$	$\frac{-256}{41}$	$\frac{1776}{41}$
0323	$\frac{-216}{451}$	$\frac{-3456}{451}$	$\frac{309}{41}$	$\frac{-1548}{41}$	$\frac{1266}{41}$	$\frac{-6780}{41}$	$\frac{-150}{41}$	$\frac{2364}{41}$	$\frac{1764}{41}$	$\frac{11488}{41}$	$\frac{-4356}{41}$	$\frac{23040}{41}$	$\frac{260}{41}$	$\frac{-2092}{41}$
0161	$\frac{-1344}{451}$	$\frac{-2688}{451}$	$\frac{4348}{369}$	$\frac{-10016}{369}$	$\frac{7232}{123}$	$\frac{-14344}{123}$	$\frac{-832}{123}$	$\frac{4040}{123}$	$\frac{-46304}{369}$	$\frac{92224}{369}$	$\frac{-8736}{41}$	$\frac{25792}{41}$	$\frac{2284}{123}$	$\frac{-5264}{123}$
0143	$\frac{288}{451}$	$\frac{-2304}{451}$	$\frac{-412}{41}$	$\frac{280}{41}$	$\frac{-1688}{41}$	$\frac{2040}{41}$	$\frac{200}{41}$	$\frac{-1048}{41}$	$\frac{3008}{41}$	$\frac{-3712}{41}$	$\frac{5808}{41}$	$\frac{-8256}{41}$	$\frac{-456}{41}$	$\frac{792}{41}$
0125	$\frac{-108}{451}$	$\frac{-3456}{451}$	$\frac{298}{41}$	$\frac{-1220}{41}$	$\frac{2127}{82}$	$\frac{-5304}{41}$	$\frac{-273}{82}$	$\frac{1872}{41}$	$\frac{-1210}{41}$	$\frac{8208}{41}$	$\frac{-3654}{41}$	$\frac{17136}{41}$	$\frac{383}{82}$	$\frac{-1600}{41}$
4121	$\frac{-1728}{451}$	$\frac{-3456}{451}$	$\frac{668}{41}$	$\frac{-2368}{41}$	$\frac{3240}{41}$	$\frac{-11208}{41}$	$\frac{-216}{41}$	$\frac{2856}{41}$	$\frac{-6240}{41}$	$\frac{20672}{41}$	$\frac{-11232}{41}$	$\frac{46656}{41}$	$\frac{932}{41}$	$\frac{-3568}{41}$
3122	$\frac{-864}{451}$	$\frac{-3456}{451}$	$\frac{744}{41}$	$\frac{-2368}{41}$	$\frac{3342}{41}$	$\frac{-11208}{41}$	$\frac{-354}{41}$	$\frac{2856}{41}$	$\frac{-6400}{41}$	$\frac{20672}{41}$	$\frac{-11520}{41}$	$\frac{46656}{41}$	$\frac{958}{41}$	$\frac{-3568}{41}$
2321	$\frac{-864}{451}$	$\frac{-3456}{451}$	$\frac{416}{41}$	$\frac{-1384}{41}$	$\frac{1620}{41}$	$\frac{-6288}{41}$	$\frac{-108}{41}$	$\frac{1872}{41}$	$\frac{-2464}{41}$	$\frac{10176}{41}$	$\frac{-5616}{41}$	$\frac{23040}{41}$	$\frac{384}{41}$	$\frac{-1928}{41}$
2141	$\frac{1152}{451}$	$\frac{-2304}{451}$	$\frac{-1664}{123}$	$\frac{800}{123}$	$\frac{-1832}{41}$	$\frac{-912}{41}$	$\frac{472}{41}$	$\frac{-720}{41}$	$\frac{9856}{123}$	$\frac{9856}{123}$	$\frac{7488}{41}$	$\frac{-384}{41}$	$\frac{-512}{41}$	$\frac{-192}{41}$
2123	$\frac{-432}{451}$	$\frac{-3456}{451}$	$\frac{618}{41}$	$\frac{-1876}{41}$	$\frac{2532}{41}$	$\frac{-8748}{41}$	$\frac{-300}{41}$	$\frac{2364}{41}$	$\frac{-4512}{41}$	$\frac{15424}{41}$	$\frac{-8712}{41}$	$\frac{34848}{41}$	$\frac{684}{41}$	$\frac{-2748}{41}$
1322	$\frac{-432}{451}$	$\frac{-3456}{451}$	$\frac{413}{41}$	$\frac{-1384}{41}$	$\frac{1671}{41}$	$\frac{-6288}{41}$	$\frac{-177}{41}$	$\frac{1872}{41}$	$\frac{-2544}{41}$	$\frac{10176}{41}$	$\frac{-5760}{41}$	$\frac{23040}{41}$	$\frac{397}{41}$	$\frac{-1928}{41}$
1142	$\frac{576}{451}$	$\frac{-2304}{451}$	$\frac{-1652}{123}$	$\frac{-800}{123}$	$\frac{-2064}{41}$	$\frac{-912}{41}$	$\frac{400}{41}$	$\frac{-720}{41}$	$\frac{11488}{123}$	$\frac{9856}{123}$	$\frac{7680}{41}$	$\frac{-384}{41}$	$\frac{-584}{41}$	$\frac{-192}{41}$
1124	$\frac{-216}{451}$	$\frac{-3456}{451}$	$\frac{905}{82}$	$\frac{-1384}{41}$	$\frac{3393}{82}$	$\frac{-6288}{41}$	$\frac{-423}{82}$	$\frac{1872}{41}$	$\frac{-2584}{41}$	$\frac{10176}{41)}$	$\frac{-5832}{41}$	$\frac{23040}{41}$	$\frac{807}{82}$	$\frac{-1928}{41}$

Table 5.(Theorem 2.2 (iii))

$ijkl$	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}
0071	$\frac{1}{46}$	$\frac{4}{23}$	$\frac{4}{23}$	$\frac{1}{46}$	$\frac{-1}{46}$	$\frac{-4}{23}$	$\frac{4}{23}$	$\frac{1}{46}$	$\frac{1}{23}$	$\frac{64}{23}$	$\frac{64}{23}$	$\frac{1}{23}$	$\frac{14}{23}$	0	$\frac{28}{23}$
0053	$\frac{-1}{92}$	$\frac{3}{23}$	$\frac{-1}{23}$	$\frac{3}{92}$	$\frac{1}{92}$	$\frac{6}{23}$	$\frac{2}{23}$	$\frac{3}{92}$	$\frac{1}{23}$	$\frac{-96}{23}$	$\frac{32}{23}$	$\frac{-3}{23}$	$\frac{-5}{46}$	$\frac{38}{23}$	$\frac{191}{23}$
0035	$\frac{-1}{184}$	$\frac{9}{92}$	$\frac{1}{92}$	$\frac{-9}{184}$	$\frac{1}{184}$	$\frac{9}{23}$	$\frac{-1}{23}$	$\frac{-9}{184}$	$\frac{1}{23}$	$\frac{-144}{23}$	$\frac{-16}{23}$	$\frac{9}{23}$	$\frac{-5}{92}$	$\frac{98}{23}$	$\frac{-677}{92}$
0017	$\frac{1}{368}$	$\frac{27}{368}$	$\frac{-1}{368}$	$\frac{-27}{368}$	$\frac{-1}{368}$	$\frac{-27}{46}$	$\frac{-1}{46}$	$\frac{-27}{368}$	$\frac{1}{23}$	$\frac{216}{23}$	$\frac{-8}{23}$	$\frac{-27}{23}$	0	$\frac{-273}{46}$	$\frac{819}{184}$
0611	0	$\frac{27}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{10}{23}$	$\frac{-112}{23}$	$\frac{94}{23}$
0431	0	$\frac{9}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{9}{23}$	$\frac{13}{23}$	$\frac{120}{23}$	$\frac{-149}{23}$
0413	0	$\frac{27}{92}$	$\frac{-1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{-13}{46}$	$\frac{-148}{23}$	$\frac{209}{46}$
0251	0	$\frac{6}{23}$	$\frac{-2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{3}{23}$	$\frac{19}{23}$	$\frac{28}{23}$	$\frac{94}{23}$
0233	0	$\frac{9}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{9}{23}$	$\frac{-5}{23}$	$\frac{106}{23}$	$\frac{-241}{46}$
0215	0	$\frac{27}{184}$	$\frac{-1}{184}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{-13}{92}$	$\frac{-143}{23}$	$\frac{104}{23}$
6011	$\frac{1}{46}$	$\frac{108}{23}$	$\frac{-4}{23}$	$\frac{-27}{46}$	$\frac{-1}{46}$	$\frac{108}{23}$	$\frac{4}{23}$	$\frac{-27}{46}$	$\frac{1}{23}$	$\frac{-1728}{23}$	$\frac{64}{23}$	$\frac{-27}{23}$	$\frac{70}{23}$	$\frac{56}{23}$	$\frac{28}{23}$
5012	0	$\frac{54}{23}$	$\frac{2}{23}$	0	0	$\frac{108}{23}$	$\frac{4}{23}$	0	$\frac{1}{23}$	$\frac{-1728}{23}$	$\frac{64}{23}$	$\frac{-27}{23}$	$\frac{63}{23}$	$\frac{12}{23}$	$\frac{54}{23}$
4031	$\frac{1}{46}$	$\frac{36}{23}$	$\frac{4}{23}$	$\frac{9}{46}$	$\frac{-1}{46}$	$\frac{-36}{23}$	$\frac{4}{23}$	$\frac{9}{46}$	$\frac{1}{23}$	$\frac{576}{23}$	$\frac{64}{23}$	$\frac{9}{23}$	$\frac{70}{23}$	$\frac{208}{23}$	$\frac{-68}{23}$
4013	$\frac{-1}{92}$	$\frac{27}{23}$	$\frac{-1}{23}$	$\frac{27}{92}$	$\frac{1}{92}$	$\frac{54}{23}$	$\frac{2}{23}$	$\frac{27}{92}$	$\frac{1}{23}$	$\frac{-864}{23}$	$\frac{32}{23}$	$\frac{-27}{23}$	$\frac{119}{46}$	$\frac{-10}{23}$	$\frac{67}{23}$
3032	0	$\frac{18}{23}$	$\frac{2}{23}$	0	0	$\frac{-36}{23}$	$\frac{4}{23}$	0	$\frac{1}{23}$	$\frac{576}{23}$	$\frac{64}{23}$	$\frac{9}{23}$	$\frac{49}{23}$	$\frac{148}{23}$	$\frac{-206}{23}$
3014	$\frac{-1}{92}$	$\frac{27}{46}$	$\frac{-1}{46}$	$\frac{27}{92}$	$\frac{1}{92}$	0	0	$\frac{27}{92}$	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{99}{46}$	$\frac{-36}{23}$	$\frac{153}{46}$
2051	$\frac{1}{46}$	$\frac{12}{23}$	$\frac{-4}{23}$	$\frac{-3}{46}$	$\frac{-1}{46}$	$\frac{12}{23}$	$\frac{4}{23}$	$\frac{-3}{46}$	$\frac{1}{23}$	$\frac{-192}{23}$	$\frac{64}{23}$	$\frac{-3}{23}$	$\frac{62}{23}$	$\frac{8}{23}$	$\frac{-100}{23}$
2033	$\frac{-1}{92}$	$\frac{9}{23}$	$\frac{1}{23}$	$\frac{-9}{92}$	$\frac{1}{92}$	$\frac{-18}{23}$	$\frac{2}{23}$	$\frac{-9}{92}$	$\frac{1}{23}$	$\frac{288}{23}$	$\frac{32}{23}$	$\frac{9}{23}$	$\frac{77}{46}$	$\frac{118}{23}$	$\frac{-275}{23}$
2015	$\frac{-1}{184}$	$\frac{27}{92}$	$\frac{-1}{92}$	$\frac{27}{184}$	$\frac{1}{184}$	$\frac{-27}{23}$	$\frac{-1}{23}$	$\frac{27}{184}$	$\frac{1}{23}$	$\frac{432}{23}$	$\frac{-16}{23}$	$\frac{-27}{23}$	$\frac{145}{92}$	$\frac{-64}{23}$	$\frac{337}{92}$
1052	0	$\frac{6}{23}$	$\frac{-2}{23}$	0	0	$\frac{12}{23}$	$\frac{4}{23}$	0	$\frac{1}{23}$	$\frac{-192}{23}$	$\frac{64}{23}$	$\frac{-3}{23}$	$\frac{19}{23}$	$\frac{28}{23}$	$\frac{94}{23}$
1034	$\frac{-1}{92}$	$\frac{9}{46}$	$\frac{1}{46}$	$\frac{-9}{92}$	$\frac{1}{92}$	0	0	$\frac{-9}{92}$	$\frac{1}{23}$	0	0	$\frac{9}{23}$	$\frac{41}{46}$	$\frac{104}{23}$	$\frac{-493}{46}$
1016	0	$\frac{27}{184}$	$\frac{-1}{184}$	0	0	$\frac{-27}{23}$	$\frac{-1}{23}$	0	$\frac{1}{23}$	$\frac{432}{23}$	$\frac{-16}{23}$	$\frac{-27}{23}$	$\frac{79}{92}$	$\frac{-97}{23}$	$\frac{185}{46}$
4211	0	$\frac{54}{23}$	$\frac{-2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{63}{23}$	$\frac{12}{23}$	$\frac{54}{23}$
3212	0	$\frac{27}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{43}{23}$	$\frac{-40}{23}$	$\frac{73}{23}$
2411	0	$\frac{27}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{43}{23}$	$\frac{-40}{23}$	$\frac{73}{23}$
2231	0	$\frac{18}{23}$	$\frac{2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{9}{23}$	$\frac{49}{23}$	$\frac{148}{23}$	$\frac{-206}{23}$
2213	0	$\frac{27}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{33}{23}$	$\frac{-66}{23}$	$\frac{165}{46}$
1412	0	$\frac{27}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{10}{23}$	$\frac{-112}{23}$	$\frac{94}{23}$
1232	0	$\frac{9}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{9}{23}$	$\frac{13}{23}$	$\frac{120}{23}$	$\frac{-149}{23}$
1214	0	$\frac{27}{92}$	$\frac{-1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0	0	$\frac{-27}{23}$	$\frac{33}{46}$	$\frac{-102}{23}$	$\frac{93}{23}$

Table 5.(Theorem 2.2 (iii)) (contd.)

$ijkl$	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
0071	$\frac{28}{23}$	0	$\frac{364}{23}$	$\frac{1092}{23}$	0	$\frac{-728}{23}$	0	$\frac{5824}{23}$	$\frac{5824}{23}$	-5	$\frac{7}{3}$	4	$\frac{28}{3}$	$\frac{80}{3}$	$\frac{-112}{3}$
0053	$\frac{-43}{23}$	$\frac{-78}{23}$	$\frac{-342}{23}$	$\frac{186}{23}$	$\frac{1224}{23}$	$\frac{204}{23}$	$\frac{1296}{23}$	$\frac{-5760}{23}$	$\frac{-4368}{23}$	$\frac{15}{4}$	$\frac{1}{4}$	$\frac{-15}{4}$	$\frac{-17}{4}$	-20	12
0035	$\frac{595}{92}$	$\frac{-122}{23}$	$\frac{350}{23}$	$\frac{-776}{23}$	$\frac{-1136}{23}$	$\frac{380}{23}$	$\frac{-2464}{23}$	$\frac{3992}{23}$	$\frac{1720}{23}$	$\frac{-5}{8}$	$\frac{-21}{8}$	$\frac{5}{8}$	$\frac{9}{8}$	-2	18
0017	$\frac{-1365}{184}$	$\frac{273}{46}$	$\frac{-1365}{46}$	$\frac{1911}{46}$	$\frac{273}{23}$	$\frac{140}{23}$	$\frac{112}{23}$	$\frac{56}{23}$	$\frac{-280}{23}$	0	$\frac{29}{8}$	0	$\frac{15}{8}$	0	-30
0611	$\frac{-170}{23}$	12	-60	84	24	$\frac{1360}{23}$	$\frac{3008}{23}$	$\frac{-896}{23}$	$\frac{-320}{23}$	0	4	-1	3	-16	-48
0431	$\frac{147}{23}$	0	12	-36	-48	$\frac{312}{23}$	$\frac{-2752}{23}$	$\frac{5472}{23}$	$\frac{1312}{23}$	$\frac{-3}{2}$	$\frac{-7}{2}$	$\frac{1}{2}$	$\frac{-3}{2}$	8	24
0413	$\frac{-331}{46}$	12	-48	48	12	$\frac{716}{23}$	$\frac{1168}{23}$	$\frac{-160}{23}$	$\frac{-688}{23}$	$\frac{3}{4}$	$\frac{15}{4}$	$\frac{-3}{4}$	$\frac{9}{4}$	-12	-36
0251	$\frac{-70}{23}$	0	0	0	40	$\frac{-320}{23}$	$\frac{2112}{23}$	$\frac{-4256}{23}$	$\frac{-2912}{23}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{-7}{2}$	$\frac{-1}{2}$	-8	-8
0233	$\frac{331}{46}$	-2	14	-26	-44	$\frac{220}{23}$	$\frac{-2384}{23}$	$\frac{5104}{23}$	$\frac{2048}{23}$	-1	$\frac{-5}{2}$	1	$\frac{3}{2}$	4	12
0215	$\frac{-169}{23}$	9	-39	45	12	$\frac{348}{23}$	$\frac{432}{23}$	$\frac{24}{23}$	$\frac{-504}{23}$	$\frac{3}{8}$	$\frac{29}{8}$	$\frac{-3}{8}$	$\frac{15}{8}$	-6	-30
6011	$\frac{-140}{23}$	$\frac{316}{23}$	$\frac{-316}{23}$	$\frac{-1580}{23}$	$\frac{-632}{23}$	$\frac{632}{23}$	$\frac{-2528}{23}$	$\frac{2528}{23}$	$\frac{-5056}{23}$	9	3	-4	0	-64	0
5012	$\frac{-174}{23}$	$\frac{348}{23}$	$\frac{-924}{23}$	$\frac{-12}{23}$	$\frac{-120}{23}$	$\frac{632}{23}$	$\frac{-2528}{23}$	$\frac{2528}{23}$	$\frac{-5056}{23}$	9	3	-4	0	-64	0
4031	$\frac{140}{23}$	$\frac{120}{23}$	$\frac{348}{23}$	$\frac{804}{23}$	$\frac{-240}{23}$	$\frac{-696}{23}$	$\frac{-960}{23}$	$\frac{8448}{23}$	$\frac{5568}{23}$	-1	1	4	12	16	-48
4013	$\frac{-191}{23}$	$\frac{282}{23}$	$\frac{-1134}{23}$	$\frac{1146}{23}$	$\frac{288}{23}$	$\frac{444}{23}$	$\frac{-1968}{23}$	$\frac{1920}{23}$	$\frac{-3792}{23}$	$\frac{27}{4}$	$\frac{13}{4}$	$\frac{-11}{4}$	$\frac{3}{4}$	-44	-12
3032	$\frac{110}{23}$	$\frac{32}{23}$	$\frac{780}{23}$	$\frac{-76}{23}$	$\frac{-912}{23}$	$\frac{-696}{23}$	$\frac{-960}{23}$	$\frac{8448}{23}$	$\frac{5568}{23}$	-1	1	4	12	16	48
3014	$\frac{-387}{46}$	$\frac{200}{23}$	$\frac{-1040}{23}$	$\frac{1520}{23}$	$\frac{440}{23}$	$\frac{256}{23}$	$\frac{-1408}{23}$	$\frac{1312}{23}$	$\frac{-2528}{23}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	-24	-24
2051	$\frac{-124}{23}$	$\frac{-60}{23}$	$\frac{-364}{23}$	$\frac{-972}{23}$	$\frac{120}{23}$	$\frac{728}{23}$	$\frac{480}{23}$	$\frac{-7264}{23}$	$\frac{-5824}{23}$	5	-1	-4	-8	-32	32
2033	$\frac{95}{23}$	$\frac{-74}{23}$	$\frac{790}{23}$	$\frac{-662}{23}$	$\frac{-1252}{23}$	$\frac{-284}{23}$	$\frac{-1488}{23}$	$\frac{6592}{23}$	$\frac{4176}{23}$	$\frac{-3}{4}$	$\frac{-1}{4}$	$\frac{11}{4}$	$\frac{33}{4}$	8	-24
2015	$\frac{-755}{92}$	$\frac{128}{23}$	$\frac{-818}{23}$	$\frac{1430}{23}$	$\frac{434}{23}$	$\frac{116}{23}$	$\frac{-944}{23}$	$\frac{824}{23}$	$\frac{-1528}{23}$	$\frac{21}{8}$	$\frac{29}{8}$	$\frac{-5}{8}$	$\frac{15}{8}$	-10	-30
1052	$\frac{-70}{23}$	$\frac{-108}{23}$	$\frac{-524}{23}$	$\frac{-300}{23}$	$\frac{824}{23}$	$\frac{728}{23}$	$\frac{480}{23}$	$\frac{-7264}{23}$	$\frac{-5824}{23}$	5	-1	-4	-8	-32	32
1034	$\frac{227}{46}$	$\frac{-136}{23}$	$\frac{584}{23}$	$\frac{-808}{23}$	$\frac{-1256}{23}$	$\frac{128}{23}$	$\frac{-2016}{23}$	$\frac{4736}{23}$	$\frac{2784}{23}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	0	0
1016	$\frac{-361}{46}$	$\frac{97}{23}$	$\frac{-643}{23}$	$\frac{1153}{23}$	$\frac{352}{23}$	$\frac{24}{23}$	$\frac{-576}{23}$	$\frac{456}{23}$	$\frac{-792}{23}$	$\frac{9}{8}$	$\frac{29}{8}$	$\frac{-1}{8}$	$\frac{15}{8}$	-2	-30
4211	$\frac{-174}{23}$	8	-32	32	8	$\frac{256}{23}$	$\frac{-1408}{23}$	$\frac{1312}{23}$	$\frac{-2528}{23}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	-24	-24
3212	$\frac{-179}{23}$	8	-32	32	8	$\frac{256}{23}$	$\frac{-1408}{23}$	$\frac{1312}{23}$	$\frac{-2528}{23}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	-24	24
2411	$\frac{-179}{23}$	4	-28	52	16	$\frac{72}{23}$	$\frac{-672}{23}$	$\frac{576}{23}$	$\frac{-1056}{23}$	$\frac{3}{2}$	$\frac{7}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	-8	-24
2231	$\frac{110}{23}$	-4	16	-16	-40	$\frac{128}{23}$	$\frac{-2016}{23}$	$\frac{4736}{23}$	$\frac{2784}{23}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	0	0
2213	$\frac{-363}{46}$	6	-30	42	12	$\frac{164}{23}$	$\frac{-1040}{23}$	$\frac{944}{23}$	$\frac{-1792}{23}$	3	$\frac{7}{2}$	-1	$\frac{3}{2}$	-16	-24
1412	$\frac{-170}{23}$	8	-32	32	8	$\frac{72}{23}$	$\frac{-672}{23}$	$\frac{576}{23}$	$\frac{-1056}{23}$	$\frac{3}{2}$	$\frac{7}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	-8	-24
1232	$\frac{147}{23}$	-4	16	-16	-40	$\frac{128}{23}$	$\frac{-2016}{23}$	$\frac{4736}{23}$	$\frac{2784}{23}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	0	0
1214	$\frac{-177}{23}$	6	-30	42	12	$\frac{72}{23}$	$\frac{-672}{23}$	$\frac{576}{23}$	$\frac{-1056}{23}$	$\frac{3}{2}$	$\frac{7}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	-8	-24

Table 6. (Theorem 2.2 (iv))

$ijkl$	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
0512	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{-3}{29}$	$\frac{3234}{261}$	$\frac{16192}{261}$
0332	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{1}{29}$	$\frac{-3788}{261}$	$\frac{-17648}{261}$
0314	0	$\frac{1}{261}$	$\frac{2}{261}$	0	$\frac{6}{29}$	0	0	$\frac{-3}{29}$	$\frac{3800}{261}$	$\frac{16192}{261}$
0152	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{87}$	0	0	$\frac{-1}{87}$	$\frac{1960}{261}$	$\frac{28432}{261}$
0134	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{1}{29}$	$\frac{-2416}{261}$	$\frac{-17648}{261}$
0116	0	$\frac{1}{261}$	$\frac{1}{261}$	0	$\frac{3}{29}$	0	0	$\frac{-3}{29}$	$\frac{3988}{261}$	$\frac{13060}{261}$
5111	0	$\frac{1}{261}$	$\frac{32}{261}$	0	$\frac{96}{29}$	0	0	$\frac{-3}{29}$	$\frac{9644}{261}$	$\frac{45424}{261}$
4112	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{48}{29}$	0	0	$\frac{-3}{29}$	$\frac{7432}{261}$	$\frac{45424}{261}$
3311	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{48}{29}$	0	0	$\frac{-3}{29}$	$\frac{5344}{261}$	$\frac{24544}{261}$
3131	0	$\frac{1}{261}$	$\frac{-32}{261}$	0	$\frac{32}{29}$	0	0	$\frac{1}{29}$	$\frac{-1580}{261}$	$\frac{-38528}{261}$
3113	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{-3}{29}$	$\frac{5282}{261}$	$\frac{34984}{261}$
2312	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{-3}{29}$	$\frac{5804}{261}$	$\frac{24544}{261}$
2132	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{1}{29}$	$\frac{-1312}{261}$	$\frac{-38528}{261}$
2114	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{-3}{29}$	$\frac{4468}{261}$	$\frac{24544}{261}$
1511	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{-3}{29}$	$\frac{3716}{261}$	$\frac{16192}{261}$
1331	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{1}{29}$	$\frac{-1312}{261}$	$\frac{-17648}{261}$
1313	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{-3}{29}$	$\frac{4990}{261}$	$\frac{20368}{261}$
1151	0	$\frac{1}{261}$	$\frac{32}{261}$	0	$\frac{32}{87}$	0	0	$\frac{-1}{87}$	$\frac{2876}{261}$	$\frac{28432}{261}$
1133	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{1}{29}$	$\frac{-1178}{261}$	$\frac{-28088}{261}$
1115	0	$\frac{1}{261}$	$\frac{2}{261}$	0	$\frac{6}{29}$	0	0	$\frac{-3}{29}$	$\frac{4322}{261}$	$\frac{17236}{261}$
$ijkl$	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}
0512	$\frac{1258}{87}$	$\frac{5224}{87}$	$\frac{-9470}{261}$	$\frac{-38912}{261}$	$\frac{-8264}{87}$	$\frac{-45728}{87}$	$\frac{-4844}{87}$	$\frac{-1184}{87}$	$\frac{57688}{261}$	$\frac{223456}{261}$
0332	$\frac{-1100}{87}$	$\frac{-6752}{87}$	$\frac{7078}{261}$	$\frac{1208}{261}$	$\frac{10696}{87}$	$\frac{1808}{3}$	$\frac{2524}{87}$	$\frac{6880}{87}$	$\frac{-45608}{261}$	$\frac{-295952}{261}$
0314	$\frac{1151}{87}$	$\frac{5224}{87}$	$\frac{-8650}{261}$	$\frac{-38912}{261}$	$\frac{-10048}{87}$	$\frac{-45728}{87}$	$\frac{-3988}{87}$	$\frac{-1184}{87}$	$\frac{54944}{261}$	$\frac{223456}{261}$
0152	$\frac{1114}{87}$	$\frac{9304}{87}$	$\frac{-2750}{261}$	$\frac{-45296}{261}$	$\frac{-2008}{29}$	$\frac{-83296}{87}$	$\frac{-428}{87}$	$\frac{-34784}{87}$	$\frac{24280}{261}$	$\frac{470176}{261}$
0134	$\frac{-811}{87}$	$\frac{-6752}{87}$	$\frac{5366}{261}$	$\frac{1208}{9}$	$\frac{7088}{87}$	$\frac{1808}{3}$	$\frac{740}{87}$	$\frac{6880}{87}$	$\frac{-28024}{261}$	$\frac{-295952}{261}$
0116	$\frac{1228}{87}$	$\frac{5224}{87}$	$\frac{-8240}{261}$	$\frac{-34736}{261}$	$\frac{-11288}{87}$	$\frac{-39464}{87}$	$\frac{-3560}{87}$	$\frac{-2992}{87}$	$\frac{56704}{261}$	$\frac{200488}{261}$
5111	$\frac{2234}{87}$	$\frac{13576}{87}$	$\frac{-12598}{261}$	$\frac{-80672}{261}$	$\frac{-23656}{87}$	$\frac{-123680}{87}$	$\frac{-8476}{87}$	$\frac{-42944}{87}$	$\frac{91928}{261}$	$\frac{624352}{261}$
4112	$\frac{1378}{87}$	$\frac{13576}{87}$	$\frac{-10214}{261}$	$\frac{-80672}{261}$	$\frac{-17048}{87}$	$\frac{-123680}{87}$	$\frac{-5804}{87}$	$\frac{-42944}{87}$	$\frac{57448}{261}$	$\frac{624352}{261}$
3311	$\frac{1378}{87}$	$\frac{9400}{87}$	$\frac{-8126}{261}$	$\frac{-51440}{261}$	$\frac{-13568}{87}$	$\frac{-72176}{87}$	$\frac{-3716}{87}$	$\frac{-13712}{87}$	$\frac{51184}{261}$	$\frac{377968}{261}$
3131	$\frac{-746}{87}$	$\frac{-10928}{87}$	$\frac{4822}{261}$	$\frac{2216}{9}$	$\frac{5896}{87}$	$\frac{3584}{3}$	$\frac{700}{87}$	$\frac{36112}{87}$	$\frac{-30008}{261}$	$\frac{-542336}{261}$
3113	$\frac{950}{87}$	$\frac{11488}{87}$	$\frac{-7978}{261}$	$\frac{-66056}{261}$	$\frac{-12004}{87}$	$\frac{-97928}{87}$	$\frac{-3424}{87}$	$\frac{-28328}{87}$	$\frac{37076}{261}$	$\frac{501160}{261}$
2312	$\frac{1472}{87}$	$\frac{9400}{87}$	$\frac{-9022}{261}$	$\frac{-51440}{261}$	$\frac{-15136}{87}$	$\frac{-72176}{87}$	$\frac{-4468}{87}$	$\frac{-13712}{87}$	$\frac{61088}{261}$	$\frac{377968}{261}$
2132	$\frac{-634}{87}$	$\frac{-10928}{87}$	$\frac{4238}{261}$	$\frac{2216}{9}$	$\frac{4688}{87}$	$\frac{3584}{3}$	$\frac{-172}{87}$	$\frac{36112}{87}$	$\frac{-20224}{261}$	$\frac{-542336}{261}$
2114	$\frac{997}{87}$	$\frac{9400}{87}$	$\frac{-7382}{261}$	$\frac{-51440}{261}$	$\frac{-11048}{87}$	$\frac{-72176}{87}$	$\frac{-2756}{87}$	$\frac{-13712}{87}$	$\frac{38896}{261}$	$\frac{377968}{261}$
1511	$\frac{950}{87}$	$\frac{5224}{87}$	$\frac{-6934}{261}$	$\frac{-38912}{261}$	$\frac{-10264}{87}$	$\frac{-45728}{87}$	$\frac{-2380}{87}$	$\frac{-1184}{87}$	$\frac{42296}{261}$	$\frac{223456}{261}$
1331	$\frac{-634}{87}$	$\frac{-6752}{87}$	$\frac{4238}{261}$	$\frac{1208}{9}$	$\frac{4688}{87}$	$\frac{1808}{3}$	$\frac{-172}{87}$	$\frac{6880}{87}$	$\frac{-20224}{261}$	$\frac{-295952}{261}$
1313	$\frac{1258}{87}$	$\frac{7312}{87}$	$\frac{-8426}{261}$	$\frac{-45176}{261}$	$\frac{-13484}{87}$	$\frac{-58952}{87}$	$\frac{-3800}{87}$	$\frac{-7448}{87}$	$\frac{56644}{261}$	$\frac{300712}{261}$
1151	$\frac{1010}{87}$	$\frac{9304}{87}$	$\frac{-3934}{261}$	$\frac{-45296}{261}$	$\frac{-2392}{29}$	$\frac{-83296}{87}$	$\frac{-1900}{87}$	$\frac{-34784}{87}$	$\frac{25592}{261}$	$\frac{470176}{261}$
1133	$\frac{-578}{87}$	$\frac{-8840}{87}$	$\frac{3946}{261}$	$\frac{1712}{9}$	$\frac{4084}{87}$	$\frac{2696}{3}$	$\frac{-608}{87}$	$\frac{21496}{87}$	$\frac{-15332}{261}$	$\frac{-419144}{261}$
1115	$\frac{1151}{87}$	$\frac{7312}{87}$	$\frac{-7606}{261}$	$\frac{-41000}{261}$	$\frac{-11788}{87}$	$\frac{-52688}{87}$	$\frac{-2944}{87}$	$\frac{-3272}{87}$	$\frac{48680}{261}$	$\frac{277744}{261}$

Table 6. (Theorem 2.2 (iv))(contd.)

$ijkl$	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}
0512	$\frac{26914}{261}$	$\frac{52480}{261}$	$\frac{14720}{261}$	$\frac{-52480}{261}$	$\frac{-3536}{261}$	$\frac{-14600}{261}$	$\frac{-3562}{261}$	$\frac{-14656}{261}$
0332	$\frac{-21050}{261}$	$\frac{-45080}{261}$	$\frac{-20800}{261}$	$\frac{129536}{261}$	$\frac{3724}{261}$	$\frac{18160}{261}$	$\frac{3704}{261}$	$\frac{18176}{261}$
0314	$\frac{26246}{261}$	$\frac{52480}{261}$	$\frac{15712}{261}$	$\frac{-52480}{261}$	$\frac{-3856}{261}$	$\frac{-14600}{261}$	$\frac{-3869}{261}$	$\frac{-14656}{261}$
0152	$\frac{-3038}{261}$	$\frac{171184}{261}$	$\frac{25088}{261}$	$\frac{-265216}{261}$	$\frac{-2024}{261}$	$\frac{-28952}{261}$	$\frac{-2554}{261}$	$\frac{-28960}{261}$
0134	$\frac{-14962}{261}$	$\frac{-45080}{261}$	$\frac{-27104}{261}$	$\frac{129536}{261}$	$\frac{2384}{261}$	$\frac{18160}{261}$	$\frac{2113}{261}$	$\frac{18176}{261}$
0116	$\frac{25912}{261}$	$\frac{35776}{261}$	$\frac{16208}{261}$	$\frac{-35776}{261}$	$\frac{-4984}{261}$	$\frac{-4016}{261}$	$\frac{-12512}{261}$	$\frac{-3892}{261}$
5111	$\frac{19562}{261}$	$\frac{252928}{261}$	$\frac{17536}{261}$	$\frac{-252928}{261}$	$\frac{-7930}{261}$	$\frac{-39656}{261}$	$\frac{-8138}{261}$	$\frac{-43888}{261}$
4112	$\frac{14218}{261}$	$\frac{252928}{261}$	$\frac{25472}{261}$	$\frac{-252928}{261}$	$\frac{-5792}{261}$	$\frac{-39656}{261}$	$\frac{-6418}{261}$	$\frac{-43888}{261}$
3311	$\frac{14218}{261}$	$\frac{119296}{261}$	$\frac{25472}{261}$	$\frac{-119296}{261}$	$\frac{-4226}{261}$	$\frac{-22952}{261}$	$\frac{-4330}{261}$	$\frac{-25096}{261}$
3131	$\frac{-24170}{261}$	$\frac{-178712}{261}$	$\frac{-16384}{261}$	$\frac{263168}{261}$	$\frac{2890}{261}$	$\frac{39040}{261}$	$\frac{2810}{261}$	$\frac{36968}{261}$
3113	$\frac{11546}{261}$	$\frac{186112}{261}$	$\frac{29440}{261}$	$\frac{-186112}{261}$	$\frac{-3940}{261}$	$\frac{-31304}{261}$	$\frac{-4514}{261}$	$\frac{-34492}{261}$
2312	$\frac{19898}{261}$	$\frac{119296}{261}$	$\frac{21088}{261}$	$\frac{-119296}{261}$	$\frac{-4984}{261}$	$\frac{-22952}{261}$	$\frac{-5036}{261}$	$\frac{-25096}{261}$
2132	$\frac{-16522}{261}$	$\frac{-178712}{261}$	$\frac{-24896}{261}$	$\frac{263168}{261}$	$\frac{2228}{261}$	$\frac{39040}{261}$	$\frac{1666}{261}$	$\frac{36968}{261}$
2114	$\frac{14386}{261}$	$\frac{119296}{261}$	$\frac{27248}{261}$	$\frac{-119296}{261}$	$\frac{-3536}{261}$	$\frac{-22952}{261}$	$\frac{-3823}{261}$	$\frac{-25096}{261}$
1511	$\frac{19898}{261}$	$\frac{52480}{261}$	$\frac{21088}{261}$	$\frac{-52480}{261}$	$\frac{-3418}{261}$	$\frac{-14600}{261}$	$\frac{-3470}{261}$	$\frac{-14656}{261}$
1331	$\frac{-16522}{261}$	$\frac{-45080}{261}$	$\frac{-24896}{261}$	$\frac{129536}{261}$	$\frac{1706}{261}$	$\frac{18160}{261}$	$\frac{1666}{261}$	$\frac{18176}{261}$
1313	$\frac{22738}{261}$	$\frac{85888}{261}$	$\frac{18896}{261}$	$\frac{-85888}{261}$	$\frac{-4580}{261}$	$\frac{-18776}{261}$	$\frac{-4606}{261}$	$\frac{-19876}{261}$
1151	$\frac{3842}{261}$	$\frac{171184}{261}$	$\frac{16768}{261}$	$\frac{-265216}{261}$	$\frac{-2482}{261}$	$\frac{-28952}{261}$	$\frac{-2498}{261}$	$\frac{-28960}{261}$
1133	$\frac{-12698}{261}$	$\frac{-111896}{261}$	$\frac{-29152}{261}$	$\frac{196352}{261}$	$\frac{1636}{261}$	$\frac{28600}{261}$	$\frac{1094}{261}$	$\frac{27572}{261}$
1115	$\frac{19982}{261}$	$\frac{69184}{261}$	$\frac{21976}{261}$	$\frac{-69184}{261}$	$\frac{-3856}{261}$	$\frac{-16688}{261}$	$\frac{-3869}{261}$	$\frac{-17788}{261}$

Table 7.(Theorem 2.3 (i))

$ijklm$	a'_1	a'_2	a'_3	a'_4	a'_5	a'_6	a'_7	a'_8	a'_9	a'_{10}
00026	0	$\frac{1}{600}$	0	$\frac{-1}{600}$	$\frac{-7}{200}$	$\frac{1}{300}$	$\frac{7}{200}$	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
00062	0	$\frac{7}{2400}$	0	$\frac{-7}{2400}$	$\frac{-9}{800}$	$\frac{7}{300}$	$\frac{9}{800}$	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
00044	0	$\frac{1}{600}$	0	$\frac{-1}{600}$	$\frac{3}{200}$	$\frac{-1}{150}$	$\frac{-3}{200}$	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
00422	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{-7}{800}$	0	$\frac{7}{800}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
00242	$\frac{1}{4800}$	$\frac{-1}{4800}$	$\frac{3}{1600}$	0	$\frac{-3}{1600}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
00224	$\frac{1}{4800}$	$\frac{-1}{4800}$	$\frac{-7}{1600}$	0	$\frac{7}{1600}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
02222	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{3}{800}$	0	$\frac{-3}{800}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
04022	0	$\frac{7}{600}$	0	$\frac{-7}{600}$	$\frac{-9}{200}$	$\frac{7}{300}$	$\frac{9}{200}$	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
02042	0	$\frac{7}{1200}$	0	$\frac{-7}{1200}$	$\frac{-9}{400}$	$\frac{7}{300}$	$\frac{9}{400}$	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
02024	0	$\frac{1}{300}$	0	$\frac{-1}{300}$	$\frac{3}{100}$	$\frac{-1}{150}$	$\frac{-3}{100}$	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
01511	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{13}{800}$	0	$\frac{-13}{800}$	$\frac{-1}{600}$	0	$\frac{2}{75}$	$\frac{-13}{200}$	$\frac{26}{25}$
03311	$\frac{1}{960}$	$\frac{-1}{960}$	$\frac{-3}{320}$	0	$\frac{3}{320}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
05111	$\frac{13}{9600}$	$\frac{-13}{9600}$	$\frac{9}{3200}$	0	$\frac{-9}{3200}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
01313	$\frac{1}{4800}$	$\frac{-1}{4800}$	$\frac{13}{1600}$	0	$\frac{-13}{1600}$	$\frac{-1}{600}$	0	$\frac{2}{75}$	$\frac{-13}{200}$	$\frac{26}{25}$
03113	$\frac{1}{1920}$	$\frac{-1}{1920}$	$\frac{-3}{640}$	0	$\frac{3}{640}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
01133	$\frac{1}{3840}$	$\frac{-1}{3840}$	$\frac{-3}{1280}$	0	$\frac{3}{1280}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
01115	$\frac{1}{9600}$	$\frac{-1}{9600}$	$\frac{13}{3200}$	0	$\frac{-13}{3200}$	$\frac{-1}{600}$	0	$\frac{2}{75}$	$\frac{-13}{200}$	$\frac{26}{25}$
01331	$\frac{1}{1920}$	$\frac{-1}{1920}$	$\frac{-3}{640}$	0	$\frac{3}{640}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
03131	$\frac{13}{19200}$	$\frac{-13}{19200}$	$\frac{9}{6400}$	0	$\frac{-9}{6400}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
01151	$\frac{13}{38400}$	$\frac{-13}{38400}$	$\frac{9}{12800}$	0	$\frac{-9}{12800}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
11123	$\frac{1}{1920}$	$\frac{-1}{1920}$	$\frac{-3}{640}$	0	$\frac{3}{640}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
11141	$\frac{13}{19200}$	$\frac{-13}{19200}$	$\frac{9}{6400}$	0	$\frac{-9}{6400}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
11321	$\frac{1}{960}$	$\frac{-1}{960}$	$\frac{-3}{320}$	0	$\frac{3}{320}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
13121	$\frac{13}{9600}$	$\frac{-13}{9600}$	$\frac{9}{3200}$	0	$\frac{-9}{3200}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
10412	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{-7}{400}$	0	$\frac{7}{400}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
12212	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{3}{400}$	0	$\frac{-3}{400}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
14012	$\frac{7}{4800}$	$\frac{-7}{4800}$	$\frac{-9}{1600}$	0	$\frac{9}{1600}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
10034	$\frac{1}{4800}$	$\frac{-1}{4800}$	$\frac{3}{1600}$	0	$\frac{-3}{1600}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
10232	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{3}{800}$	0	$\frac{-3}{800}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
12032	$\frac{7}{9600}$	$\frac{-7}{9600}$	$\frac{-9}{3200}$	0	$\frac{9}{3200}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
10052	$\frac{7}{19200}$	$\frac{-7}{19200}$	$\frac{-9}{6400}$	0	$\frac{9}{6400}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
10214	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{-7}{800}$	0	$\frac{7}{800}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
12014	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{3}{800}$	0	$\frac{-3}{800}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
10016	$\frac{1}{4800}$	$\frac{-1}{4800}$	$\frac{-7}{1600}$	0	$\frac{7}{1600}$	$\frac{1}{300}$	0	$\frac{-4}{75}$	$\frac{-7}{100}$	$\frac{28}{25}$
22022	$\frac{7}{4800}$	$\frac{-7}{4800}$	$\frac{-9}{1600}$	0	$\frac{9}{1600}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
21311	$\frac{1}{480}$	$\frac{-1}{480}$	$\frac{-3}{160}$	0	$\frac{3}{160}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
23111	$\frac{13}{4800}$	$\frac{-13}{4800}$	$\frac{9}{1600}$	0	$\frac{-9}{1600}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
21113	$\frac{1}{960}$	$\frac{-1}{960}$	$\frac{-3}{320}$	0	$\frac{3}{320}$	$\frac{1}{120}$	0	$\frac{-2}{15}$	$\frac{-3}{40}$	$\frac{6}{5}$
21131	$\frac{13}{9600}$	$\frac{-13}{9600}$	$\frac{9}{3200}$	0	$\frac{-9}{3200}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
20222	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{3}{400}$	0	$\frac{-3}{400}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$

Table 7 (contd.)

$ijklm$	a'_{11}	a'_{12}	a'_{13}	a'_{14}	a'_{15}	a'_{16}	a'_{17}	a'_{18}	a'_{19}	a'_{20}
00026	0	0	0	$\frac{4}{15}$	$\frac{-16}{3}$	$\frac{-512}{15}$	0	-3	0	-9
00062	0	0	0	$\frac{-16}{5}$	-8	$\frac{32}{5}$	0	$\frac{-3}{2}$	0	$\frac{-81}{2}$
00044	0	0	0	$\frac{8}{5}$	16	$\frac{128}{5}$	0	2	0	18
00422	0	0	$\frac{-4}{15}$	$\frac{-4}{3}$	-16	$\frac{-384}{5}$	$\frac{-1}{2}$	-5	$\frac{-3}{2}$	-15
00242	0	0	$\frac{1}{5}$	2	16	$\frac{128}{5}$	$\frac{3}{4}$	2	$\frac{27}{4}$	18
00224	0	0	$\frac{-2}{15}$	0	$\frac{-16}{3}$	$\frac{-512}{15}$	$\frac{-1}{4}$	-3	$\frac{-3}{4}$	-9
02222	0	0	$\frac{2}{5}$	4	16	$\frac{128}{5}$	$\frac{1}{2}$	2	$\frac{9}{2}$	8
04022	0	0	0	$\frac{-4}{5}$	0	$\frac{-128}{5}$	0	-1	0	-27
02042	0	0	0	$\frac{-12}{5}$	-8	$\frac{-128}{5}$	0	-1	0	-27
02024	0	0	0	$\frac{16}{5}$	16	$\frac{128}{5}$	0	2	0	18
01511	0	0	$\frac{2}{5}$	$\frac{8}{3}$	16	$\frac{896}{15}$	$\frac{3}{4}$	$\frac{9}{2}$	$\frac{21}{4}$	$\frac{63}{2}$
03311	0	0	$\frac{-1}{2}$	-3	-12	-32	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{-9}{2}$	$\frac{-63}{2}$
05111	0	0	$\frac{4}{5}$	4	16	$\frac{256}{5}$	$\frac{7}{8}$	$\frac{7}{2}$	$\frac{81}{8}$	$\frac{81}{2}$
01313	0	0	$\frac{11}{30}$	$\frac{5}{3}$	$\frac{28}{3}$	$\frac{416}{15}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{9}{2}$	$\frac{27}{2}$
03113	0	0	0	0	0	0	$\frac{-1}{8}$	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{-9}{2}$
01133	0	0	$\frac{-1}{4}$	$\frac{-3}{2}$	-6	-16	$\frac{-7}{16}$	-1	$\frac{-9}{16}$	-9
01115	0	0	$\frac{4}{15}$	$\frac{4}{3}$	$\frac{16}{3}$	$\frac{256}{15}$	$\frac{3}{8}$	$\frac{3}{2}$	$\frac{21}{8}$	$\frac{21}{2}$
01331	0	0	$\frac{-3}{4}$	$\frac{-9}{2}$	-18	-48	$\frac{-7}{8}$	$\frac{-5}{2}$	$\frac{-45}{8}$	$\frac{-63}{2}$
03131	0	0	$\frac{13}{20}$	$\frac{7}{2}$	10	$\frac{176}{5}$	$\frac{9}{16}$	3	$\frac{135}{16}$	27
01151	0	0	$\frac{7}{10}$	3	8	$\frac{96}{5}$	$\frac{21}{32}$	$\frac{7}{4}$	$\frac{243}{32}$	$\frac{81}{4}$
11123	0	0	0	0	-4	-32	$\frac{-1}{8}$	$\frac{-5}{2}$	$\frac{9}{8}$	$\frac{-27}{2}$
11141	0	0	$\frac{9}{10}$	3	12	$\frac{96}{5}$	$\frac{17}{16}$	$\frac{5}{2}$	$\frac{135}{16}$	$\frac{27}{2}$
11321	0	0	$\frac{-1}{2}$	-3	-16	-64	$\frac{-1}{2}$	$\frac{-9}{2}$	$\frac{-9}{2}$	$\frac{-63}{2}$
13121	0	0	$\frac{4}{5}$	4	12	$\frac{96}{5}$	$\frac{7}{8}$	$\frac{5}{2}$	$\frac{81}{8}$	$\frac{27}{2}$
10412	0	0	$\frac{2}{15}$	0	$\frac{-16}{3}$	$\frac{-384}{5}$	$\frac{1}{2}$	-5	$\frac{3}{2}$	-15
12212	0	0	$\frac{4}{5}$	4	16	$\frac{128}{5}$	1	2	9	18
14012	0	0	$\frac{2}{5}$	0	0	$\frac{-128}{5}$	$\frac{1}{4}$	-1	$\frac{27}{4}$	-27
10034	0	0	$\frac{1}{5}$	2	16	$\frac{128}{5}$	$\frac{1}{4}$	2	$\frac{9}{4}$	18
10232	0	0	$\frac{2}{5}$	2	20	$\frac{288}{5}$	1	3	9	9
12032	0	0	$\frac{1}{5}$	-2	-8	$\frac{-128}{5}$	$\frac{1}{8}$	-1	$\frac{27}{8}$	-27
10052	0	0	$\frac{3}{5}$	-2	-8	$\frac{32}{5}$	$\frac{-1}{16}$	$\frac{-3}{2}$	$\frac{-27}{16}$	$\frac{-81}{2}$
10214	0	0	$\frac{2}{5}$	$\frac{4}{3}$	0	$\frac{-512}{15}$	$\frac{1}{2}$	-3	$\frac{3}{2}$	-9
12014	0	0	$\frac{2}{5}$	4	16	$\frac{128}{5}$	$\frac{1}{2}$	2	$\frac{9}{2}$	18
10016	0	0	$\frac{8}{15}$	$\frac{4}{3}$	$\frac{-16}{3}$	$\frac{-512}{15}$	$\frac{3}{4}$	-3	$\frac{9}{4}$	-9
22022	0	0	$\frac{2}{5}$	0	0	$\frac{-128}{5}$	$\frac{1}{4}$	-1	$\frac{27}{4}$	-27
21311	0	0	0	0	-8	-64	$\frac{1}{4}$	$\frac{-9}{2}$	$\frac{-9}{4}$	$\frac{-63}{2}$
23111	0	0	$\frac{11}{10}$	5	20	$\frac{96}{5}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{27}{2}$	$\frac{27}{2}$
21113	0	0	$\frac{1}{2}$	3	4	-32	$\frac{1}{2}$	$\frac{-5}{2}$	$\frac{9}{2}$	$\frac{-27}{2}$
21131	0	0	$\frac{21}{20}$	$\frac{7}{2}$	18	$\frac{176}{5}$	$\frac{11}{8}$	$\frac{9}{2}$	$\frac{81}{8}$	$\frac{27}{2}$
20222	0	0	$\frac{4}{5}$	4	24	$\frac{448}{5}$	1	4	9	0

Table 7. (contd.)

$ijklm$	a'_{21}	a'_{22}	a'_{23}	a'_{24}	a'_{25}	a'_{26}	a'_{27}	a'_{28}	a'_{29}	a'_{30}
00026	0	$\frac{4}{3}$	$\frac{20}{3}$	0	0	0	1	0	0	0
00062	0	$\frac{5}{2}$	8	0	0	0	$\frac{3}{2}$	0	0	0
00044	0	-2	-8	0	0	0	-2	0	0	0
00422	$\frac{1}{6}$	2	12	-1	3	$\frac{1}{2}$	3	$\frac{4}{3}$	$\frac{2}{3}$	-1
00242	$\frac{-1}{4}$	-2	-8	$\frac{1}{2}$	$\frac{9}{2}$	$\frac{-3}{4}$	-2	-1	0	$\frac{1}{2}$
00224	$\frac{1}{12}$	$\frac{4}{3}$	$\frac{20}{3}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{1}{4}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{-1}{2}$
02222	$\frac{-1}{2}$	0	-8	$\frac{1}{2}$	$\frac{9}{2}$	$\frac{-1}{2}$	-2	-1	0	$\frac{1}{2}$
04022	0	4	4	0	0	0	3	0	0	0
02042	0	3	4	0	0	0	3	0	0	0
02024	0	0	-8	0	0	0	-2	0	0	0
01511	0	$\frac{-5}{3}$	$\frac{-50}{3}$	1	3	$\frac{-1}{4}$	$\frac{-7}{2}$	$\frac{-5}{3}$	$\frac{-5}{6}$	$\frac{1}{2}$
03311	$\frac{3}{4}$	3	6	$\frac{-3}{4}$	$\frac{-27}{4}$	1	$\frac{7}{2}$	1	$\frac{-1}{2}$	$\frac{-3}{4}$
05111	$\frac{-5}{8}$	0	10	0	0	$\frac{-3}{8}$	$\frac{3}{2}$	-2	1	0
01313	$\frac{1}{12}$	$\frac{-1}{3}$	$\frac{-26}{3}$	$\frac{1}{4}$	$\frac{9}{4}$	0	$\frac{-3}{2}$	-1	$\frac{-1}{2}$	$\frac{1}{4}$
03113	$\frac{3}{8}$	2	6	$\frac{-1}{2}$	$\frac{-9}{2}$	$\frac{5}{8}$	$\frac{5}{2}$	0	0	$\frac{-1}{2}$
01133	$\frac{3}{16}$	$\frac{3}{2}$	6	$\frac{-3}{8}$	$\frac{-27}{8}$	$\frac{7}{16}$	2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{-3}{8}$
01115	$\frac{5}{24}$	0	$\frac{-10}{3}$	0	0	$\frac{1}{8}$	$\frac{-1}{2}$	$\frac{-2}{3}$	$\frac{-1}{3}$	0
01331	$\frac{5}{8}$	3	10	$\frac{-5}{8}$	$\frac{-45}{8}$	$\frac{7}{8}$	$\frac{7}{2}$	$\frac{3}{2}$	0	$\frac{-7}{8}$
03131	$\frac{-13}{16}$	$\frac{-1}{2}$	2	$\frac{3}{8}$	$\frac{27}{8}$	$\frac{-9}{16}$	0	$\frac{-3}{2}$	$\frac{3}{4}$	$\frac{3}{8}$
01151	$\frac{-25}{32}$	$\frac{-5}{4}$	0	$\frac{1}{4}$	$\frac{27}{4}$	$\frac{-21}{32}$	$\frac{-3}{4}$	$\frac{-5}{4}$	$\frac{5}{8}$	$\frac{3}{8}$
11123	$\frac{3}{8}$	2	6	$\frac{-1}{4}$	$\frac{-9}{4}$	$\frac{5}{8}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{-1}{4}$
11141	$\frac{-9}{16}$	$\frac{-3}{2}$	-6	$\frac{3}{4}$	$\frac{27}{4}$	$\frac{-9}{16}$	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{3}{4}$
11321	$\frac{3}{4}$	3	14	$\frac{-1}{2}$	$\frac{-9}{2}$	1	$\frac{7}{2}$	2	$\frac{1}{2}$	-1
13121	$\frac{-5}{8}$	0	-6	$\frac{3}{4}$	$\frac{27}{4}$	$\frac{-3}{8}$	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{3}{4}$
10412	$\frac{2}{3}$	$\frac{2}{3}$	12	-1	3	$\frac{1}{2}$	3	$\frac{4}{3}$	$\frac{2}{3}$	-1
12212	0	0	-8	$\frac{1}{2}$	$\frac{9}{2}$	0	-2	-1	0	$\frac{1}{2}$
14012	$\frac{1}{4}$	4	4	0	0	$\frac{3}{4}$	3	0	0	0
10034	$\frac{1}{4}$	-2	-8	$\frac{1}{2}$	$\frac{-9}{2}$	$\frac{1}{4}$	-2	0	0	$\frac{1}{2}$
10232	0	-2	-8	1	0	$\frac{-1}{2}$	-3	-1	0	1
12032	$\frac{1}{8}$	3	4	0	0	$\frac{3}{8}$	3	$\frac{1}{2}$	$\frac{1}{2}$	0
10052	$\frac{-3}{16}$	$\frac{5}{2}$	8	0	0	$\frac{9}{16}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
10214	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{20}{3}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{-1}{2}$
12014	$\frac{1}{2}$	0	-8	0	0	$\frac{1}{2}$	-2	0	0	0
10016	$\frac{5}{12}$	$\frac{4}{3}$	$\frac{20}{3}$	0	0	$\frac{1}{4}$	1	0	0	0
22022	$\frac{1}{4}$	4	4	0	0	$\frac{3}{4}$	3	1	1	0
21311	1	3	14	$\frac{-1}{2}$	$\frac{-9}{2}$	$\frac{5}{4}$	$\frac{7}{2}$	2	$\frac{1}{2}$	-1
23111	$\frac{-1}{4}$	1	-6	$\frac{3}{4}$	$\frac{27}{4}$	0	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{3}{4}$
21113	$\frac{3}{4}$	3	6	$\frac{-1}{4}$	$\frac{-9}{4}$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{-1}{4}$
21131	$\frac{-3}{8}$	-1	-10	$\frac{11}{8}$	$\frac{27}{8}$	$\frac{-3}{8}$	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{9}{8}$
20222	0	0	-8	$\frac{3}{2}$	$\frac{-9}{2}$	0	-4	-1	0	$\frac{3}{2}$

Table 7.(contd.)

$ijklm$	a'_1	a'_2	a'_3	a'_4	a'_5	a'_6	a'_7	a'_8	a'_9	a'_{10}
20042	$\frac{7}{9600}$	$\frac{-7}{9600}$	$\frac{-9}{3200}$	0	$\frac{9}{3200}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
20024	$\frac{1}{2400}$	$\frac{-1}{2400}$	$\frac{3}{800}$	0	$\frac{-3}{800}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
31121	$\frac{13}{4800}$	$\frac{-13}{4800}$	$\frac{9}{1600}$	0	$\frac{-9}{1600}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
30212	$\frac{1}{600}$	$\frac{-1}{600}$	$\frac{3}{200}$	0	$\frac{-3}{200}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
32012	$\frac{7}{2400}$	$\frac{-7}{2400}$	$\frac{-9}{800}$	0	$\frac{9}{800}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
30032	$\frac{7}{4800}$	$\frac{-7}{4800}$	$\frac{-9}{1600}$	0	$\frac{9}{1600}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
30014	$\frac{1}{1200}$	$\frac{-1}{1200}$	$\frac{3}{400}$	0	$\frac{-3}{400}$	$\frac{-1}{150}$	0	$\frac{8}{75}$	$\frac{-3}{50}$	$\frac{24}{25}$
41111	$\frac{13}{2400}$	$\frac{-13}{2400}$	$\frac{9}{800}$	0	$\frac{-9}{800}$	$\frac{-13}{600}$	0	$\frac{26}{75}$	$\frac{-9}{200}$	$\frac{18}{25}$
40022	$\frac{7}{2400}$	$\frac{-7}{2400}$	$\frac{-9}{800}$	0	$\frac{9}{800}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$
50012	$\frac{7}{1200}$	$\frac{-7}{1200}$	$\frac{-9}{400}$	0	$\frac{9}{400}$	$\frac{7}{300}$	0	$\frac{-28}{75}$	$\frac{-9}{100}$	$\frac{36}{25}$

$ijklm$	a'_{11}	a'_{12}	a'_{13}	a'_{14}	a'_{15}	a'_{16}	a'_{17}	a'_{18}	a'_{19}	a'_{20}
20042	0	0	$\frac{6}{5}$	0	-8	$\frac{-128}{5}$	$\frac{-1}{8}$	-1	$\frac{-27}{8}$	-27
20024	0	0	$\frac{2}{5}$	4	16	$\frac{128}{5}$	$\frac{1}{2}$	2	$\frac{9}{2}$	18
31121	0	0	$\frac{11}{10}$	5	24	$\frac{256}{5}$	$\frac{3}{2}$	$\frac{13}{2}$	$\frac{27}{2}$	$\frac{27}{2}$
30212	0	0	$\frac{8}{5}$	8	24	$\frac{448}{5}$	1	4	9	0
32012	0	0	$\frac{4}{5}$	4	16	$\frac{-128}{5}$	$\frac{1}{2}$	-1	$\frac{27}{2}$	-27
30032	0	0	$\frac{7}{5}$	2	-8	$\frac{-448}{5}$	0	0	0	0
30014	0	0	$\frac{4}{5}$	8	16	$\frac{128}{5}$	1	2	9	18
41111	0	0	$\frac{6}{5}$	8	32	$\frac{256}{5}$	$\frac{7}{4}$	$\frac{13}{2}$	$\frac{81}{4}$	$\frac{27}{2}$
40022	0	0	$\frac{4}{5}$	4	0	$\frac{-768}{5}$	$\frac{1}{2}$	1	$\frac{27}{2}$	27
50012	0	0	$\frac{-2}{5}$	8	32	$\frac{-768}{5}$	$\frac{3}{2}$	1	$\frac{81}{2}$	27

Table 7.(contd.)

$ijklm$	a'_{21}	a'_{22}	a'_{23}	a'_{24}	a'_{25}	a'_{26}	a'_{27}	a'_{28}	a'_{29}	a'_{30}
20042	$\frac{-3}{8}$	3	4	0	0	$\frac{9}{8}$	3	1	1	0
20024	$\frac{1}{2}$	0	-8	1	-9	$\frac{1}{2}$	-2	0	0	1
31121	$\frac{-1}{4}$	1	-14	2	0	0	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{3}{2}$
30212	0	4	-8	$\frac{3}{2}$	$\frac{-9}{2}$	1	-4	-1	0,	$\frac{3}{2}$
32012	$\frac{1}{2}$	6	4	0	0	$\frac{3}{2}$	3	1	1	0
30032	$\frac{-1}{4}$	4	-4	0	0	$\frac{3}{2}$	6	$\frac{3}{2}$	$\frac{3}{2}$	0
30014	1	4	-8	1	-9	1	-2	0	0	1
41111	0	5	-14	2	0	$\frac{3}{4}$	$\frac{-3}{2}$	-1	$\frac{1}{2}$	$\frac{3}{2}$
40022	$\frac{1}{2}$	6	-12	0	0	$\frac{3}{2}$	9	2	2	0
50012	2	10	-12	0	0	$\frac{3}{2}$	9	2	2	0

Table 8.(Theorem 2.3(ii))

$ijklm$	b'_1	b'_2	b'_3	b'_4	b'_5	b'_6	b'_7	b'_8	b'_9	b'_{10}
00323	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{32}{451}$	0	$\frac{-624}{451}$	0	$\frac{44746}{4059}$	$\frac{-17944}{4059}$
00143	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{16}{451}$	0	$\frac{180}{451}$	0	$\frac{-8177}{1353}$	$\frac{3544}{1353}$
00341	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{32}{451}$	0	$\frac{360}{451}$	0	$\frac{-13648}{1353}$	$\frac{5992}{451}$
00521	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{64}{451}$	0	$\frac{-1248}{451}$	0	$\frac{26824}{1353}$	$\frac{-7184}{1353}$
00125	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{16}{451}$	0	$\frac{-312}{451}$	0	$\frac{23275}{4059}$	$\frac{3704}{4059}$
00161	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{28}{451}$	0	$\frac{-54}{451}$	0	$\frac{11889}{1804}$	$\frac{-9246}{451}$
01412	0	$\frac{-2}{451}$	0	$\frac{84}{451}$	$\frac{32}{451}$	0	$\frac{1344}{451}$	0	$\frac{-44552}{4059}$	$\frac{-3656}{4059}$
03212	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{80}{451}$	0	$\frac{-576}{451}$	0	$\frac{7372}{1353}$	$\frac{-17960}{1353}$
05012	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	0	$\frac{1664}{451}$	0	$\frac{6912}{451}$	0	$\frac{-912}{451}$
01034	0	$\frac{10}{451}$	0	$\frac{72}{451}$	0	$\frac{320}{451}$	0	$\frac{-2304}{451}$	0	$\frac{76}{41}$
01232	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{40}{451}$	0	$\frac{-288}{451}$	0	$\frac{3634}{451}$	$\frac{-12548}{1353}$
03032	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	0	$\frac{832}{451}$	0	$\frac{3456}{451}$	0	$\frac{-1092}{451}$
01052	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	0	$\frac{416}{451}$	0	$\frac{1728}{451}$	0	$\frac{-1182}{451}$
01214	0	$\frac{-2}{451}$	0	$\frac{84}{451}$	$\frac{16}{451}$	0	$\frac{672}{451}$	0	$\frac{-22276}{4059}$0	$\frac{-7264}{4059}$
03014	0	$\frac{10}{451}$	0	$\frac{72}{451}$	0	$\frac{640}{451}$	0	$\frac{-4608}{451}$	0	$\frac{-760}{451}$
01016	0	$\frac{-2}{451}$	0	$\frac{84}{451}$	0	$\frac{128}{451}$	0	$\frac{5376}{451}$	0	$\frac{-4096}{1353}$
02123	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{32}{451}$	0	$\frac{360}{451}$	0	$\frac{-7334}{1353}$	$\frac{3544}{1353}$
02141	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{56}{451}$	0	$\frac{-108}{451}$	0	$\frac{2111}{451}$	$\frac{-8344}{451}$
02321	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{64}{451}$	0	$\frac{720}{451}$	0	$\frac{-4739}{451}$	$\frac{5992}{451}$
04121	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{112}{451}$	0	$\frac{-216}{451}$	0	$\frac{1967}{451}$	$\frac{-8344}{451}$
11222	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{80}{451}$	0	$\frac{-576}{451}$	0	$\frac{4562}{451}$	$\frac{-7136}{1353}$
11042	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{52}{451}$	0	$\frac{216}{451}$	0	$\frac{-4713}{902}$	$\frac{-1272}{451}$
11024	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{40}{451}$	0	$\frac{-288}{451}$	0	$\frac{928}{451}$	$\frac{2432}{451}$
13022	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{104}{451}$	0	$\frac{432}{451}$	0	$\frac{-1556}{451}$	$\frac{-1272}{451}$
10313	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{64}{451}$	0	$\frac{-1248}{451}$	0	$\frac{39431}{4059}$	$\frac{-17944}{4059}$
12113	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{64}{451}$	0	$\frac{720}{451}$	0	$\frac{-8354}{1353}$	$\frac{3544}{1353}$
10115	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{32}{451}$	0	$\frac{-624}{451}$	0	$\frac{19490}{4059}$	$\frac{3704}{4059}$
10133	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{32}{451}$	0	$\frac{360}{451}$	0	$\frac{-8347}{902}$	$\frac{4188}{451}$
10511	0	$\frac{4}{451}$	0	$\frac{78}{451}$	$\frac{128}{451}$	0	$\frac{-2496}{451}$	0	$\frac{26588}{1353}$	$\frac{-7184}{1353}$
12311	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{128}{451}$	0	$\frac{1440}{451}$	0	$\frac{-5419}{451}$	$\frac{5992}{451}$
14111	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{224}{451}$	0	$\frac{-432}{451}$	0	$\frac{2130}{451}$	$\frac{-8344}{451}$
10331	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{64}{451}$	0	$\frac{720}{451}$	0	$\frac{-39709}{2706}$	$\frac{7796}{451}$
12131	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{112}{451}$	0	$\frac{-216}{451}$	0	$\frac{6189}{902}$	$\frac{-6540}{451}$
10151	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{56}{451}$	0	$\frac{-108}{451}$	0	$\frac{5575}{902}$	$\frac{-7442}{451}$
22121	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{224}{451}$	0	$\frac{-432}{451}$	0	$\frac{3934}{451}$	$\frac{-4736}{451}$
21212	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{160}{451}$	0	$\frac{-1152}{451}$	0	$\frac{12940}{1353}$	$\frac{-7136}{1353}$
23012	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{208}{451}$	0	$\frac{864}{451}$	0	$\frac{-3112}{451}$	$\frac{-1272}{451}$
21032	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{104}{451}$	0	$\frac{432}{451}$	0	$\frac{-4713}{451}$	$\frac{-1272}{451}$
21014	0	$\frac{10}{451}$	0	$\frac{72}{451}$	$\frac{80}{451}$	0	$\frac{-576}{451}$	0	$\frac{1856}{451}$	$\frac{2432}{451}$
20123	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{64}{451}$	0	$\frac{720}{451}$	0	$\frac{-16021}{1353}$	$\frac{21584}{1353}$

Table 8.(contd.)

$ijklm$	b'_{11}	b'_{12}	b'_{13}	b'_{14}	b'_{15}	b'_{16}	b'_{17}	b'_{18}	b'_{19}	b'_{20}
00323	$\frac{-234}{451}$	$\frac{-15456}{451}$	$\frac{-7232}{451}$	$\frac{-128}{451}$	$\frac{312}{451}$	$\frac{-2496}{451}$	$\frac{-2582}{369}$	$\frac{1664}{369}$	$\frac{-3682}{123}$	$\frac{2776}{123}$
00143	$\frac{-1962}{451}$	$\frac{17136}{451}$	$\frac{3600}{451}$	$\frac{256}{451}$	$\frac{8028}{451}$	$\frac{-2880}{451}$	$\frac{499}{123}$	$\frac{-344}{123}$	$\frac{589}{41}$	$\frac{-648}{41}$
00341	$\frac{4194}{451}$	$\frac{49608}{451}$	$\frac{7200}{451}$	$\frac{256}{451}$	$\frac{-180}{451}$	$\frac{-2880}{451}$	$\frac{752}{123}$	$\frac{-552}{41}$	$\frac{932}{41}$	$\frac{-2616}{41}$
00521	$\frac{-3174}{451}$	$\frac{-4632}{451}$	$\frac{-14464}{451}$	$\frac{-128}{451}$	$\frac{624}{451}$	$\frac{-89088}{451}$	$\frac{-1448}{123}$	$\frac{664}{123}$	$\frac{-2072}{41}$	$\frac{1800}{41}$
00125	$\frac{-1470}{451}$	$\frac{-4632}{451}$	$\frac{-3616}{451}$	$\frac{-128}{451}$	$\frac{5568}{451}$	$\frac{-2496}{451}$	$\frac{-1373}{369}$	$\frac{-304}{369}$	$\frac{-2005}{123}$	$\frac{-176}{123}$
00161	$\frac{3024}{451}$	$\frac{-96876}{451}$	$\frac{-2720}{451}$	$\frac{-896}{451}$	$\frac{12204}{451}$	$\frac{193104}{451}$	$\frac{-831}{164}$	$\frac{866}{41}$	$\frac{-3585}{164}$	$\frac{3006}{41}$
01412	$\frac{504}{451}$	$\frac{22488}{451}$	$\frac{7200}{451}$	$\frac{64}{451}$	$\frac{-672}{451}$	$\frac{-2688}{451}$	$\frac{2584}{369}$	$\frac{-1160}{369}$	$\frac{3452}{123}$	$\frac{-568}{123}$
03212	$\frac{-216}{451}$	$\frac{-31752}{451}$	$\frac{-3648}{451}$	$\frac{-320}{451}$	$\frac{288}{451}$	$\frac{-2304}{451}$	$\frac{-416}{123}$	$\frac{1168}{123}$	$\frac{-540}{41}$	$\frac{2368}{41}$
05012	0	$\frac{3672}{451}$	0	0	0	$\frac{-6912}{451}$	0	$\frac{-48}{41}$	0	$\frac{192}{41}$
01034	0	$\frac{-144}{451}$	0	$\frac{-480}{451}$	0	$\frac{-1152}{451}$	0	$\frac{-56}{41}$	0	$\frac{-448}{41}$
01232	$\frac{-108}{451}$	$\frac{-31752}{451}$	$\frac{-5432}{451}$	$\frac{-320}{451}$	$\frac{144}{451}$	$\frac{-2304}{451}$	$\frac{-206}{41}$	$\frac{1168}{123}$	$\frac{-844}{41}$	$\frac{1876}{41}$
03032	0	$\frac{216}{41}$	0	$\frac{416}{451}$	0	$\frac{-5184}{451}$	0	$\frac{104}{41}$	0	$\frac{396}{41}$
01052	0	$\frac{1728}{451}$	0	$\frac{624}{451}$	0	$\frac{-4320}{451}$	0	$\frac{98}{41}$	0	$\frac{498}{41}$
01214	$\frac{252}{451}$	$\frac{11664}{451}$	$\frac{3600}{451}$	$\frac{64}{451}$	$\frac{-336}{451}$	$\frac{-2688}{451}$	$\frac{1292}{369}$	$\frac{-832}{369}$	$\frac{1726}{123}$	$\frac{88}{123}$
03014	0	$\frac{-1008}{451}$	0	$\frac{-640}{451}$	0	0	0	$\frac{-64}{41}$	0	$\frac{16}{41}$
01016	0	$\frac{2856}{451}$	0	0	0	$\frac{-5376}{451}$	0	$\frac{-112}{123}$	0	$\frac{368}{41}$
02123	$\frac{-3924}{451}$	$\frac{17136}{451}$	$\frac{3592}{451}$	$\frac{256}{451}$	$\frac{16056}{451}$	$\frac{-2880}{451}$	$\frac{424}{123}$	$\frac{-344}{123}$	$\frac{481}{41}$	$\frac{-648}{41}$
02141	$\frac{-6129}{451}$	$\frac{-48168}{451}$	$\frac{-3636}{451}$	$\frac{-896}{451}$	$\frac{24408}{451}$	$\frac{-1728}{451}$	$\frac{-108}{41}$	$\frac{784}{41}$	$\frac{-1125}{82}$	$\frac{3744}{41}$
02321	$\frac{270}{451}$	$\frac{49608}{451}$	$\frac{7184}{451}$	$\frac{256}{451}$	$\frac{-360}{451}$	$\frac{-2880}{451}$	$\frac{269}{41}$	$\frac{-552}{41}$	$\frac{1085}{41}$	$\frac{-2616}{41}$
04121	$\frac{-12258}{451}$	$\frac{-48168}{451}$	$\frac{-3664}{451}$	$\frac{-896}{451}$	$\frac{48816}{451}$	$\frac{-1728}{451}$	$\frac{-93}{41}$	$\frac{784}{41}$	$\frac{-633}{41}$	$\frac{3744}{41}$
11222	$\frac{-216}{451}$	$\frac{-31752}{451}$	$\frac{-7256}{451}$	$\frac{-320}{451}$	$\frac{288}{451}$	$\frac{-2304}{451}$	$\frac{-248}{41}$	$\frac{1168}{123}$	$\frac{-1032}{41}$	$\frac{1384}{41}$
11042	$\frac{81}{451}$	$\frac{1080}{451}$	$\frac{2680}{451}$	$\frac{832}{451}$	$\frac{-108}{451}$	$\frac{-3456}{451}$	$\frac{309}{82}$	$\frac{256}{41}$	$\frac{633}{41}$	$\frac{600}{41}$
11024	$\frac{-108}{451}$	$\frac{720}{451}$	$\frac{-1824}{451}$	$\frac{-320}{451}$	$\frac{144}{451}$	$\frac{-2304}{451}$	$\frac{-42}{41}$	$\frac{-48}{41}$	$\frac{-188}{41}$	$\frac{-912}{41}$
13022	$\frac{162}{451}$	$\frac{1080}{451}$	$\frac{1752}{451}$	$\frac{832}{451}$	$\frac{-216}{451}$	$\frac{-3456}{451}$	$\frac{104}{41}$	$\frac{256}{41}$	$\frac{405}{41}$	$\frac{600}{41}$
10313	$\frac{-468}{451}$	$\frac{-15456}{451}$	$\frac{-7248}{451}$	$\frac{-128}{451}$	$\frac{624}{451}$	$\frac{-2496}{451}$	$\frac{-2089}{369}$	$\frac{1664}{369}$	$\frac{-3059}{123}$	$\frac{2776}{123}$
12113	$\frac{270}{451}$	$\frac{17136}{451}$	$\frac{3576}{451}$	$\frac{256}{451}$	$\frac{-360}{451}$	$\frac{-2880}{451}$	$\frac{520}{123}$	$\frac{-344}{123}$	$\frac{675}{41}$	$\frac{-648}{41}$
10115	$\frac{-234}{451}$	$\frac{-4632}{451}$	$\frac{-3624}{451}$	$\frac{-128}{451}$	$\frac{312}{451}$	$\frac{-2496}{451}$	$\frac{-1024}{369}$	$\frac{-304}{369}$	$\frac{-1427}{123}$	$\frac{-176}{123}$
10133	$\frac{135}{451}$	$\frac{17136}{451}$	$\frac{5396}{451}$	$\frac{256}{451}$	$\frac{-180}{451}$	$\frac{-2880}{451}$	$\frac{515}{82}$	$\frac{-224}{41}$	$\frac{1055}{41}$	$\frac{-1632}{41}$
10511	$\frac{-936}{451}$	$\frac{-4632}{451}$	$\frac{-14496}{451}$	$\frac{-128}{451}$	$\frac{1248}{451}$	$\frac{-89088}{451}$	$\frac{-1420}{123}$	$\frac{664}{123}$	$\frac{-2094}{41}$	$\frac{1800}{41}$
12311	$\frac{540}{451}$	$\frac{49608}{451}$	$\frac{7152}{451}$	$\frac{256}{451}$	$\frac{-720}{451}$	$\frac{-2880}{451}$	$\frac{333}{41}$	$\frac{-552}{41}$	$\frac{1309}{41}$	$\frac{-2616}{41}$
14111	$\frac{-162}{451}$	$\frac{-48168}{451}$	$\frac{-3720}{451}$	$\frac{-896}{451}$	$\frac{216}{451}$	$\frac{-1728}{451}$	$\frac{-104}{41}$	$\frac{784}{41}$	$\frac{-405}{41}$	$\frac{3744}{41}$
10331	$\frac{8388}{451}$	$\frac{17136}{451}$	$\frac{10792}{451}$	$\frac{256}{451}$	$\frac{-360}{451}$	$\frac{127008}{451}$	$\frac{2147}{246}$	$\frac{-552}{41}$	$\frac{2867}{82}$	$\frac{-3764}{41}$
12131	$\frac{-81}{451}$	$\frac{-48168}{451}$	$\frac{-5468}{451}$	$\frac{-896}{451}$	$\frac{108}{451}$	$\frac{-1728}{451}$	$\frac{-309}{82}$	$\frac{784}{41}$	$\frac{-633}{41}$	$\frac{3252}{41}$
10151	$\frac{-81}{902}$	$\frac{-96876}{451}$	$\frac{-4538}{451}$	$\frac{-896}{451}$	$\frac{54}{451}$	$\frac{193104}{451}$	$\frac{-149}{41}$	$\frac{866}{41}$	$\frac{-2127}{164}$	$\frac{2514}{41}$
22121	$\frac{-162}{451}$	$\frac{-48168}{451}$	$\frac{-7328}{451}$	$\frac{-896}{451}$	$\frac{216}{451}$	$\frac{-1728}{451}$	$\frac{-186}{41}$	$\frac{784}{41}$	$\frac{-774}{41}$	$\frac{2760}{41}$
21212	$\frac{-432}{451}$	$\frac{-31752}{451}$	$\frac{-7296}{451}$	$\frac{-320}{451}$	$\frac{576}{451}$	$\frac{-2304}{451}$	$\frac{-668}{123}$	$\frac{1168}{123}$	$\frac{-916}{41}$	$\frac{1384}{41}$
23012	$\frac{324}{451}$	$\frac{1080}{451}$	$\frac{3504}{451}$	$\frac{832}{451}$	$\frac{-432}{451}$	$\frac{-3456}{451}$	$\frac{208}{41}$	$\frac{256}{41}$	$\frac{810}{41}$	$\frac{600}{41}$
21032	$\frac{162}{451}$	$\frac{1080}{451}$	$\frac{5360}{451}$	$\frac{832}{451}$	$\frac{-216}{451}$	$\frac{-3456}{451}$	$\frac{309}{41}$	$\frac{420}{41}$	$\frac{1266}{41}$	$\frac{600}{41}$
21014	$\frac{-216}{451}$	$\frac{720}{451}$	$\frac{-3648}{451}$	$\frac{-320}{451}$	$\frac{288}{451}$	$\frac{-2304}{451}$	$\frac{-84}{41}$	$\frac{-48}{41}$	$\frac{-376}{41}$	$\frac{-912}{41}$
20123	$\frac{270}{451}$	$\frac{17136}{451}$	$\frac{7184}{451}$	$\frac{256}{451}$	$\frac{-360}{451}$	$\frac{-2880}{451}$	$\frac{971}{123}$	$\frac{-1000}{123}$	$\frac{1413}{41}$	$\frac{-2616}{41}$

Table 8.(contd.)

$ijklm$	b'_{21}	b'_{22}	b'_{23}	b'_{24}	b'_{25}	b'_{26}	b'_{27}	b'_{28}
00323	$\frac{650}{123}$	$\frac{-1520}{123}$	$\frac{23752}{369}$	$\frac{-13888}{369}$	$\frac{4488}{41}$	$\frac{-6976}{41}$	$\frac{-1154}{123}$	$\frac{1016}{123}$
00143	$\frac{-83}{41}$	$\frac{248}{41}$	$\frac{-2984}{123}$	$\frac{3136}{123}$	$\frac{-2184}{41}$	$\frac{3456}{41}$	$\frac{163}{41}$	$\frac{-240}{41}$
00341	$\frac{-84}{41}$	$\frac{904}{41}$	$\frac{-4984}{123}$	$\frac{4544}{41}$	$\frac{-3384}{41}$	$\frac{11328}{41}$	$\frac{244}{41}$	$\frac{-896}{41}$
00521	$\frac{324}{41}$	$\frac{-944}{41}$	$\frac{13648}{123}$	$\frac{-13376}{123}$	$\frac{7664}{41}$	$\frac{-12224}{41}$	$\frac{-660}{41}$	$\frac{776}{41}$
00125	$\frac{407}{123}$	$\frac{-536}{123}$	$\frac{14008}{369}$	$\frac{1856}{369}$	$\frac{2408}{41}$	$\frac{-1728}{41}$	$\frac{-659}{123}$	$\frac{32}{123}$
00161	$\frac{567}{164}$	$\frac{-786}{41}$	$\frac{1266}{41}$	$\frac{-2784}{41}$	$\frac{2574}{41}$	$\frac{-12096}{41}$	$\frac{-827}{164}$	$\frac{758}{41}$
01412	$\frac{-580}{123}$	$\frac{1088}{123}$	$\frac{-20528}{369}$	$\frac{-2240}{369}$	$\frac{-4368}{41}$	$\frac{4800}{41}$	$\frac{1060}{123}$	$\frac{-344}{123}$
03212	$\frac{36}{41}$	$\frac{-392}{41}$	$\frac{2464}{123}$	$\frac{-13760}{123}$	$\frac{1872}{41}$	$\frac{-8256}{41}$	$\frac{-128}{41}$	$\frac{792}{41}$
05012	0	$\frac{456}{41}$	0	$\frac{320}{41}$	0	$\frac{576}{41}$	0	$\frac{-64}{41}$
01034	0	$\frac{80}{41}$	0	$\frac{448}{41}$	0	$\frac{-768}{41}$	0	$\frac{-48}{41}$
01232	$\frac{100}{41}$	$\frac{-556}{41}$	$\frac{1504}{41}$	$\frac{-9824}{123}$	$\frac{2904}{41}$	$\frac{-8256}{41}$	$\frac{-228}{41}$	$\frac{628}{41}$
03032	0	$\frac{180}{41}$	0	0	0	0	0	$\frac{-12}{41}$
01052	0	$\frac{42}{41}$	0	$\frac{-160}{41}$	0	$\frac{-288}{41}$	0	$\frac{14}{41}$
01214	$\frac{-290}{123}$	$\frac{760}{123}$	$\frac{-10264}{369}$	$\frac{-4864}{369}$	$\frac{-2184}{41}$	$\frac{2176}{41}$	$\frac{530}{123}$	$\frac{-16}{123}$
03014	0	$\frac{224}{41}$	0	$\frac{-640}{41}$	0	$\frac{-1152}{41}$	0	$\frac{96}{41}$
01016	0	$\frac{136}{41}$	0	$\frac{-2752}{123}$	0	$\frac{448}{41}$	0	$\frac{96}{41}$
02123	$\frac{-43}{41}$	$\frac{248}{41}$	$\frac{-1868}{123}$	$\frac{3136}{123}$	$\frac{-1908}{41}$	$\frac{3456}{41}$	$\frac{121}{41}$	$\frac{-240}{41}$
02141	$\frac{75}{82}$	$\frac{-1032}{41}$	$\frac{974}{41}$	$\frac{-6720}{41}$	$\frac{1458}{41}$	$\frac{-12096}{41}$	$\frac{-253}{82}$	$\frac{1168}{41}$
02321	$\frac{-127}{41}$	$\frac{904}{41}$	$\frac{-1792}{41}$	$\frac{4544}{41}$	$\frac{-3816}{41}$	$\frac{11328}{41}$	$\frac{283}{41}$	$\frac{-896}{41}$
04121	$\frac{75}{41}$	$\frac{-1032}{41}$	$\frac{1456}{41}$	$\frac{-6720}{41}$	$\frac{1440}{41}$	$\frac{-12096}{41}$	$\frac{-171}{41}$	$\frac{1168}{41}$
11222	$\frac{200}{41}$	$\frac{-720}{41}$	$\frac{2024}{41}$	$\frac{-5888}{123}$	$\frac{3840}{41}$	$\frac{-8256}{41}$	$\frac{-292}{41}$	$\frac{464}{41}$
11042	$\frac{-75}{41}$	$\frac{-96}{41}$	$\frac{-1374}{41}$	$\frac{-320}{41}$	$\frac{-2178}{41}$	$\frac{-576}{41}$	$\frac{212}{41}$	$\frac{40}{41}$
11024	$\frac{100}{41}$	$\frac{-64}{41}$	$\frac{520}{41}$	$\frac{1536}{41}$	$\frac{936}{41}$	$\frac{-384}{41}$	$\frac{-64}{41}$	$\frac{-192}{41}$
13022	$\frac{-27}{41}$	$\frac{-96}{41}$	$\frac{-780}{41}$	$\frac{-320}{41}$	$\frac{-1404}{41}$	$\frac{-576}{41}$	$\frac{137}{41}$	$\frac{40}{41}$
10313	$\frac{685}{123}$	$\frac{-1520}{123}$	$\frac{19952}{369}$	$\frac{-13888}{369}$	$\frac{4056}{41}$	$\frac{-6976}{41}$	$\frac{-955}{123}$	$\frac{1016}{123}$
12113	$\frac{-45}{41}$	$\frac{248}{41}$	$\frac{-3572}{123}$	$\frac{3136}{123}$	$\frac{-2340}{41}$	$\frac{3456}{41}$	$\frac{201}{41}$	$\frac{-240}{41}$
10115	$\frac{445}{123}$	$\frac{-536}{123}$	$\frac{9812}{369}$	$\frac{1856}{369}$	$\frac{2028}{41}$	$\frac{-1728}{41}$	$\frac{-457}{123}$	$\frac{32}{123}$
10133	$\frac{-125}{41}$	$\frac{412}{41}$	$\frac{-2126}{41}$	$\frac{3232}{41}$	$\frac{-3630}{41}$	$\frac{7392}{41}$	$\frac{326}{41}$	$\frac{-568}{41}$
10511	$\frac{402}{41}$	$\frac{-944}{41}$	$\frac{13520}{123}$	$\frac{-13376}{123}$	$\frac{8112}{41}$	$\frac{-12224}{41}$	$\frac{-664}{41}$	$\frac{776}{41}$
12311	$\frac{-131}{41}$	$\frac{904}{41}$	$\frac{-2272}{41}$	$\frac{4544}{41}$	$\frac{-4680}{41}$	$\frac{11328}{41}$	$\frac{361}{41}$	$\frac{-896}{41}$
14111	$\frac{27}{41}$	$\frac{-1032}{41}$	$\frac{452}{41}$	$\frac{-6720}{41}$	$\frac{1404}{41}$	$\frac{-12096}{41}$	$\frac{-55}{41}$	$\frac{1168}{41}$
10331	$\frac{-213}{82}$	$\frac{1232}{41}$	$\frac{-8984}{123}$	$\frac{8480}{41}$	$\frac{-5292}{41}$	$\frac{15264}{41}$	$\frac{853}{82}$	$\frac{-1388}{41}$
12131	$\frac{75}{41}$	$\frac{-1032}{41}$	$\frac{882}{41}$	$\frac{-5408}{41}$	$\frac{2178}{41}$	$\frac{-12096}{41}$	$\frac{-130}{41}$	$\frac{1004}{41}$
10151	$\frac{273}{164}$	$\frac{-786}{41}$	$\frac{605}{41}$	$\frac{-1472}{41}$	$\frac{1827}{41}$	$\frac{-12096}{41}$	$\frac{-383}{164}$	$\frac{594}{41}$
22121	$\frac{150}{41}$	$\frac{-1032}{41}$	$\frac{1272}{41}$	$\frac{-4096}{41}$	$\frac{2880}{41}$	$\frac{-12096}{41}$	$\frac{-178}{41}$	$\frac{840}{41}$
21212	$\frac{236}{41}$	$\frac{-720}{41}$	$\frac{5584}{123}$	$\frac{-5888}{123}$	$\frac{3744}{41}$	$\frac{-8256}{41}$	$\frac{-256}{41}$	$\frac{464}{41}$
23012	$\frac{-54}{41}$	$\frac{-96}{41}$	$\frac{-1560}{41}$	$\frac{-320}{41}$	$\frac{-2808}{41}$	$\frac{-576}{41}$	$\frac{274}{41}$	$\frac{40}{41}$
21032	$\frac{-150}{41}$	$\frac{-96}{41}$	$\frac{-2748}{41}$	$\frac{-320}{41}$	$\frac{-4356}{41}$	$\frac{-576}{41}$	$\frac{424}{41}$	$\frac{40}{41}$
21014	$\frac{200}{41}$	$\frac{-64}{41}$	$\frac{1040}{41}$	$\frac{1536}{41}$	$\frac{1872}{41}$	$\frac{-384}{41}$	$\frac{-128}{41}$	$\frac{-192}{41}$
20123	$\frac{-127}{41}$	$\frac{576}{41}$	$\frac{-8656}{123}$	$\frac{16256}{123}$	$\frac{-4800}{41}$	$\frac{11328}{41}$	$\frac{447}{41}$	$\frac{-896}{41}$

Table 8.(contd.)

$ijklm$	b'_1	b'_2	b'_3	b'_4	b'_5	b'_6	b'_7	b'_8	b'_9	b'_{10}
20141	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{112}{451}$	0	$\frac{-216}{451}$	0	$\frac{3483}{902}$	$\frac{-4736}{451}$
20321	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{128}{451}$	0	$\frac{1440}{451}$	0	$\frac{-26630}{1353}$	$\frac{9600}{451}$
31022	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{208}{451}$	0	$\frac{864}{451}$	0	$\frac{-6269}{451}$	$\frac{-1272}{451}$
30113	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{128}{451}$	0	$\frac{1440}{451}$	0	$\frac{-15355}{1353}$	$\frac{21584}{1353}$
30311	0	$\frac{-8}{451}$	0	$\frac{90}{451}$	$\frac{256}{451}$	0	$\frac{2880}{451}$	0	$\frac{-9936}{451}$	$\frac{9600}{451}$
32111	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{448}{451}$	0	$\frac{-864}{451}$	0	$\frac{3809}{451}$	$\frac{-4736}{451}$
30131	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{224}{451}$	0	$\frac{-432}{451}$	0	$\frac{2005}{902}$	$\frac{-1128}{451}$
41012	0	$\frac{-26}{451}$	0	$\frac{108}{451}$	$\frac{416}{451}$	0	$\frac{1728}{451}$	0	$\frac{-6224}{451}$	$\frac{-1272}{451}$
40121	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{448}{451}$	0	$\frac{-864}{451}$	0	$\frac{2456}{451}$	$\frac{2480}{451}$
50111	0	$\frac{28}{451}$	0	$\frac{54}{451}$	$\frac{896}{451}$	0	$\frac{-1728}{451}$	0	$\frac{6716}{451}$	$\frac{2480}{451}$

$ijklm$	b'_{11}	b'_{12}	b'_{13}	b'_{14}	b'_{15}	b'_{16}	b'_{17}	b'_{18}	b'_{19}	b'_{20}
20141	$\frac{-12258}{451}$	$\frac{-48168}{451}$	$\frac{-7272}{451}$	$\frac{-896}{451}$	$\frac{108}{451}$	$\frac{-1728}{451}$	$\frac{19}{82}$	$\frac{784}{41}$	$\frac{333}{82}$	$\frac{2760}{41}$
20321	$\frac{8658}{451}$	$\frac{-15336}{451}$	$\frac{14368}{451}$	$\frac{256}{451}$	$\frac{-720}{451}$	$\frac{256896}{451}$	$\frac{1450}{123}$	$\frac{-552}{41}$	$\frac{2088}{41}$	$\frac{-4912}{41}$
31022	$\frac{324}{451}$	$\frac{1080}{451}$	$\frac{7112}{451}$	$\frac{832}{451}$	$\frac{-432}{451}$	$\frac{-3456}{451}$	$\frac{413}{41}$	$\frac{584}{41}$	$\frac{1671}{41}$	$\frac{600}{41}$
30113	$\frac{540}{451}$	$\frac{17136}{451}$	$\frac{7152}{451}$	$\frac{256}{451}$	$\frac{-720}{451}$	$\frac{-2880}{451}$	$\frac{917}{123}$	$\frac{-1000}{123}$	$\frac{1391}{41}$	$\frac{-2616}{41}$
30311	$\frac{1080}{451}$	$\frac{-15336}{451}$	$\frac{14304}{451}$	$\frac{256}{451}$	$\frac{-1440}{451}$	$\frac{256896}{451}$	$\frac{584}{41}$	$\frac{-552}{41}$	$\frac{2618}{41}$	$\frac{-4912}{41}$
32111	$\frac{-324}{451}$	$\frac{-48168}{451}$	$\frac{-7440}{451}$	$\frac{-896}{451}$	$\frac{432}{451}$	$\frac{-1728}{451}$	$\frac{-167}{41}$	$\frac{784}{41}$	$\frac{-687}{41}$	$\frac{2760}{41}$
30131	$\frac{-24516}{451}$	$\frac{49248}{451}$	$\frac{-10936}{451}$	$\frac{-896}{451}$	$\frac{216}{451}$	$\frac{-391392}{451}$	$\frac{325}{82}$	$\frac{620}{41}$	$\frac{1527}{82}$	$\frac{3744}{41}$
41012	$\frac{648}{451}$	$\frac{1080}{451}$	$\frac{7008}{451}$	$\frac{832}{451}$	$\frac{-864}{451}$	$\frac{-3456}{451}$	$\frac{416}{41}$	$\frac{584}{41}$	$\frac{1620}{41}$	$\frac{600}{41}$
40121	$\frac{-24678}{451}$	$\frac{146664}{451}$	$\frac{-14656}{451}$	$\frac{-896}{451}$	$\frac{432}{451}$	$\frac{-781056}{451}$	$\frac{120}{41}$	$\frac{456}{41}$	$\frac{420}{41}$	$\frac{4728}{41}$
50111	$\frac{-648}{451}$	$\frac{146664}{451}$	$\frac{-14880}{451}$	$\frac{-896}{451}$	$\frac{864}{451}$	$\frac{-781056}{451}$	$\frac{-252}{41}$	$\frac{456}{41}$	$\frac{-1374}{41}$	$\frac{4728}{41}$

Table 8.(contd.)

$ijklm$	b'_{21}	b'_{22}	b'_{23}	b'_{24}	b'_{25}	b'_{26}	b'_{27}	b'_{28}
20141	$\frac{-219}{82}$	$\frac{-1032}{41}$	$\frac{-348}{41}$	$\frac{-4096}{41}$	$\frac{-36}{41}$	$\frac{-12096}{41}$	$\frac{191}{82}$	$\frac{840}{41}$
20321	$\frac{-172}{41}$	$\frac{1560}{41}$	$\frac{-13376}{123}$	$\frac{12416}{41}$	$\frac{-7632}{41}$	$\frac{19200}{41}$	$\frac{648}{41}$	$\frac{-1880}{41}$
31022	$\frac{-177}{41}$	$\frac{-96}{41}$	$\frac{-3528}{41}$	$\frac{-320}{41}$	$\frac{-5760}{41}$	$\frac{-576}{41}$	$\frac{561}{41}$	$\frac{40}{41}$
30113	$\frac{-49}{41}$	$\frac{576}{41}$	$\frac{-8128}{123}$	$\frac{16256}{123}$	$\frac{-4680}{41}$	$\frac{11328}{41}$	$\frac{443}{41}$	$\frac{-896}{41}$
30311	$\frac{-262}{41}$	$\frac{1560}{41}$	$\frac{-5200}{41}$	$\frac{12416}{41}$	$\frac{-9360}{41}$	$\frac{19200}{41}$	$\frac{804}{41}$	$\frac{-1880}{41}$
32111	$\frac{177}{41}$	$\frac{-1032}{41}$	$\frac{1232}{41}$	$\frac{-4096}{41}$	$\frac{2808}{41}$	$\frac{-12096}{41}$	$\frac{-151}{41}$	$\frac{840}{41}$
30131	$\frac{-561}{82}$	$\frac{-1524}{41}$	$\frac{-1024}{41}$	$\frac{-10656}{41}$	$\frac{-1548}{41}$	$\frac{-12096}{41}$	$\frac{505}{82}$	$\frac{1496}{41}$
41012	$\frac{-108}{41}$	$\frac{-96}{41}$	$\frac{-3120}{41}$	$\frac{-320}{41}$	$\frac{-5616}{41}$	$\frac{-576}{41}$	$\frac{548}{41}$	$\frac{40}{41}$
40121	$\frac{-192}{41}$	$\frac{-2016}{41}$	$\frac{-80}{41}$	$\frac{-17216}{41}$	$\frac{-144}{41}$	$\frac{-12096}{41}$	$\frac{136}{41}$	$\frac{2152}{41}$
50111	$\frac{354}{41}$	$\frac{-2016}{41}$	$\frac{3120}{41}$	$\frac{-17216}{41}$	$\frac{5616}{41}$	$\frac{-12096}{41}$	$\frac{-384}{41}$	$\frac{2152}{41}$

Table 9.(Theorem 2.3(iii))

$ijklm$	c_1'	c_2'	c_3'	c_4'	c_5'	c_6'	c_7'	c_8'	c_9'	c_{10}'
00512	0	$\frac{2}{23}$	$\frac{2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
00134	0	$\frac{3}{92}$	$\frac{-1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
00332	0	$\frac{3}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
00152	0	$\frac{9}{184}$	$\frac{1}{184}$	0	0	0	0	0	$\frac{1}{23}$	0
00314	0	$\frac{1}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
00116	0	$\frac{1}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
02312	0	$\frac{3}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
02132	0	$\frac{9}{92}$	$\frac{1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
04112	0	$\frac{9}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
02114	0	$\frac{3}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
03023	0	0	0	0	0	$\frac{36}{23}$	$\frac{4}{23}$	0	$\frac{1}{23}$	0
01223	0	$\frac{3}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
01043	0	0	0	0	0	$\frac{18}{23}$	$\frac{2}{23}$	0	$\frac{1}{23}$	0
01241	0	$\frac{9}{92}$	$\frac{-1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
03041	0	0	0	0	0	$\frac{54}{23}$	$\frac{-2}{23}$	0	$\frac{1}{23}$	0
01421	0	$\frac{3}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
03221	0	$\frac{9}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
05021	0	0	0	0	0	$\frac{108}{23}$	$\frac{-4}{23}$	0	$\frac{1}{23}$	0
01025	0	0	0	0	0	$\frac{12}{23}$	$\frac{-4}{23}$	0	$\frac{1}{23}$	0
01061	0	0	0	0	0	$\frac{27}{23}$	$\frac{-1}{23}$	0	$\frac{1}{23}$	0
11213	0	$\frac{3}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
11033	0	$\frac{9}{92}$	$\frac{-1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
11411	0	$\frac{6}{23}$	$\frac{2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
11015	0	$\frac{3}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
11231	0	$\frac{9}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
11051	0	$\frac{27}{184}$	$\frac{1}{184}$	0	0	0	0	0	$\frac{1}{23}$	0
10322	0	$\frac{3}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
12122	0	$\frac{9}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
10142	0	$\frac{9}{92}$	$\frac{1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
10124	0	$\frac{3}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
13013	0	$\frac{9}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
13211	0	$\frac{9}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
15011	0	$\frac{27}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
13031	0	$\frac{27}{92}$	$\frac{1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0
22112	0	$\frac{9}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
20312	0	$\frac{6}{23}$	$\frac{-2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
20132	0	$\frac{9}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
20114	0	$\frac{3}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
21023	0	$\frac{9}{46}$	$\frac{-1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
21041	0	$\frac{27}{92}$	$\frac{1}{92}$	0	0	0	0	0	$\frac{1}{23}$	0

Table 9.(contd.)

$ijklm$	c'_{11}	c'_{12}	c'_{13}	c'_{14}	c'_{15}	c'_{16}	c'_{17}	c'_{18}	c'_{19}	c'_{20}
00512	0	$\frac{1}{23}$	$\frac{19}{23}$	0	$\frac{22}{23}$	$\frac{22}{23}$	0	16	24	0
00134	0	$\frac{-3}{23}$	$\frac{-1}{46}$	$\frac{-8}{23}$	$\frac{173}{46}$	$\frac{17}{46}$	0	0	0	12
00332	0	$\frac{-3}{23}$	$\frac{-1}{23}$	$\frac{-16}{23}$	$\frac{277}{46}$	$\frac{-35}{46}$	0	-6	6	24
00152	0	$\frac{9}{23}$	$\frac{-5}{92}$	$\frac{-8}{23}$	$\frac{-56}{23}$	$\frac{50}{23}$	0	5	-15	-20
00314	0	$\frac{1}{23}$	$\frac{21}{23}$	0	$\frac{11}{23}$	$\frac{11}{23}$	0	12	12	0
00116	0	$\frac{1}{23}$	$\frac{22}{23}$	0	$\frac{-6}{23}$	$\frac{-6}{23}$	0	4	12	0
02312	0	$\frac{-3}{23}$	$\frac{21}{23}$	$\frac{-32}{23}$	$\frac{93}{23}$	$\frac{-35}{23}$	4	4	4	16
02132	0	$\frac{9}{23}$	$\frac{-5}{46}$	$\frac{-16}{23}$	$\frac{-17}{46}$	$\frac{131}{46}$	4	4	-8	-16
04112	0	$\frac{9}{23}$	$\frac{18}{23}$	$\frac{-32}{23}$	$\frac{6}{23}$	$\frac{62}{23}$	8	4	-4	-16
02114	0	$\frac{-3}{23}$	$\frac{22}{23}$	$\frac{-16}{23}$	$\frac{58}{23}$	$\frac{-6}{23}$	4	4	4	8
03023	0	$\frac{-9}{23}$	0	0	0	0	$\frac{98}{23}$	$\frac{296}{23}$	$\frac{-412}{23}$	$\frac{220}{23}$
01223	0	$\frac{3}{23}$	$\frac{-2}{23}$	$\frac{-8}{23}$	$\frac{-26}{23}$	$\frac{34}{23}$	2	8	-4	-12
01043	0	$\frac{-9}{23}$	0	0	0	0	$\frac{26}{23}$	$\frac{240}{23}$	$\frac{-298}{23}$	$\frac{294}{23}$
01241	0	$\frac{-9}{23}$	$\frac{-2}{23}$	$\frac{-43}{23}$	$\frac{91}{23}$	$\frac{-104}{23}$	3	-8	18	26
03041	0	$\frac{27}{23}$	0	0	0	0	$\frac{86}{23}$	$\frac{-80}{23}$	$\frac{146}{23}$	$\frac{-358}{23}$
01421	0	$\frac{3}{23}$	$\frac{-4}{23}$	$\frac{-16}{23}$	$\frac{-52}{23}$	$\frac{68}{23}$	2	8	-4	-28
03221	0	$\frac{-9}{23}$	$\frac{-4}{23}$	$\frac{-40}{23}$	$\frac{44}{23}$	$\frac{-116}{23}$	6	-8	12	20
05021	0	$\frac{27}{23}$	0	0	0	0	$\frac{126}{23}$	$\frac{24}{23}$	$\frac{108}{23}$	$\frac{-348}{23}$
01025	0	$\frac{3}{23}$	0	0	0	0	$\frac{38}{23}$	$\frac{56}{23}$	$\frac{188}{23}$	$\frac{-140}{23}$
01061	0	$\frac{27}{23}$	0	0	0	0	$\frac{20}{23}$	$\frac{-224}{23}$	$\frac{188}{23}$	$\frac{-340}{23}$
11213	0	$\frac{3}{23}$	$\frac{42}{23}$	$\frac{-16}{23}$	$\frac{-6}{23}$	$\frac{22}{23}$	2	16	8	-8
11033	0	$\frac{-9}{23}$	$\frac{21}{23}$	$\frac{-20}{23}$	$\frac{45}{23}$	$\frac{11}{23}$	2	16	-10	14
11411	0	$\frac{3}{23}$	$\frac{38}{23}$	$\frac{14}{23}$	$\frac{-58}{23}$	$\frac{44}{23}$	0	16	12	-20
11015	0	$\frac{3}{23}$	$\frac{44}{23}$	$\frac{-8}{23}$	$\frac{20}{23}$	$\frac{-12}{23}$	2	8	20	-4
11231	0	$\frac{-9}{23}$	$\frac{19}{23}$	$\frac{-63}{23}$	$\frac{136}{23}$	$\frac{-93}{23}$	3	-8	18	26
11051	0	$\frac{27}{23}$	$\frac{39}{46}$	$\frac{-75}{46}$	$\frac{-87}{46}$	$\frac{-21}{23}$	$\frac{5}{2}$	-10	10	-20
10322	0	$\frac{-3}{23}$	$\frac{21}{23}$	$\frac{-32}{23}$	$\frac{93}{23}$	$\frac{-35}{23}$	0	-8	0	16
12122	0	$\frac{9}{23}$	$\frac{18}{23}$	$\frac{-32}{23}$	$\frac{6}{23}$	$\frac{62}{23}$	4	16	-8	-16
10142	0	$\frac{9}{23}$	$\frac{41}{46}$	$\frac{-16}{23}$	$\frac{-89}{23}$	$\frac{31}{23}$	0	18	-18	-24
10124	0	$\frac{-3}{23}$	$\frac{22}{23}$	$\frac{-16}{23}$	$\frac{58}{23}$	$\frac{-6}{23}$	0	0	0	8
13013	0	$\frac{-9}{23}$	$\frac{42}{23}$	$\frac{-40}{23}$	$\frac{90}{23}$	$\frac{22}{23}$	6	24	-12	12
13211	0	$\frac{-9}{23}$	$\frac{38}{23}$	$\frac{-80}{23}$	$\frac{134}{23}$	$\frac{-94}{23}$	6	0	24	24
15011	0	$\frac{27}{23}$	$\frac{32}{23}$	$\frac{-104}{23}$	$\frac{56}{23}$	$\frac{8}{23}$	10	8	4	-20
13031	0	$\frac{27}{23}$	$\frac{16}{23}$	$\frac{-52}{23}$	$\frac{28}{23}$	$\frac{4}{23}$	6	0	6	-18
22112	0	$\frac{9}{23}$	$\frac{59}{23}$	$\frac{-64}{23}$	$\frac{35}{23}$	$\frac{55}{23}$	8	28	-4	-16
20312	0	$\frac{-3}{23}$	$\frac{65}{23}$	$\frac{-64}{23}$	$\frac{2}{23}$	$\frac{-70}{23}$	4	0	-8	0
20132	0	$\frac{9}{23}$	$\frac{41}{23}$	$\frac{-32}{23}$	$\frac{-149}{46}$	$\frac{55}{46}$	4	30	-14	-24
20114	0	$\frac{-3}{23}$	$\frac{67}{23}$	$\frac{-32}{23}$	$\frac{1}{23}$	$\frac{-35}{23}$	4	4	4	0
21023	0	$\frac{-9}{23}$	$\frac{42}{23}$	$\frac{-40}{23}$	$\frac{90}{23}$	$\frac{22}{23}$	6	24	-12	12
21041	0	$\frac{27}{23}$	$\frac{39}{23}$	$\frac{-75}{23}$	$\frac{-87}{23}$	$\frac{-42}{23}$	7	-4	10	-26

Table 9.(contd.)

$ijklm$	c'_{21}	c'_{22}	c'_{23}	c'_{24}	c'_{25}	c'_{26}	c'_{27}	c'_{28}	c'_{29}	c'_{30}
00512	$\frac{-416}{23}$	0	$\frac{2912}{23}$	$\frac{2912}{23}$	$\frac{-5}{2}$	$\frac{7}{6}$	$\frac{3}{2}$	$\frac{31}{6}$	$\frac{40}{3}$	$\frac{-56}{3}$
00134	$\frac{140}{23}$	$\frac{272}{23}$	$\frac{-1312}{23}$	$\frac{-1072}{23}$	$\frac{3}{4}$	$\frac{-1}{4}$	$\frac{-3}{4}$	$\frac{-7}{4}$	-4	4
00332	$\frac{140}{23}$	$\frac{272}{23}$	$\frac{-2416}{23}$	$\frac{-2176}{23}$	$\frac{3}{2}$	0	$\frac{-3}{2}$	-3	-8	8
00152	$\frac{220}{23}$	$\frac{-544}{23}$	$\frac{1240}{23}$	$\frac{760}{23}$	$\frac{-5}{8}$	$\frac{-7}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	-2	6
00314	$\frac{-232}{23}$	0	$\frac{1440}{23}$	$\frac{1440}{23}$	$\frac{-3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	8	-8
00116	$\frac{-48}{23}$	0	$\frac{704}{23}$	$\frac{704}{23}$	-1	$\frac{1}{3}$	0	$\frac{4}{3}$	$\frac{16}{3}$	$\frac{-16}{3}$
02312	$\frac{-136}{23}$	$\frac{640}{23}$	$\frac{-1312}{23}$	$\frac{-1440}{23}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{-3}{2}$	$\frac{-3}{2}$	0	0
02132	$\frac{220}{23}$	$\frac{-176}{23}$	$\frac{1792}{23}$	$\frac{944}{23}$	$\frac{-3}{4}$	$\frac{-3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	0	0
04112	$\frac{496}{23}$	$\frac{192}{23}$	$\frac{1792}{23}$	$\frac{576}{23}$	-1	-1	0	0	0	0
02114	$\frac{48}{23}$	$\frac{640}{23}$	$\frac{-576}{23}$	$\frac{-704}{23}$	0	0	-1	-1	0	0
03023	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	0	0	0	0	0	0
01223	$\frac{-96}{23}$	$\frac{-320}{23}$	$\frac{1344}{23}$	$\frac{1408}{23}$	-1	0	1	2	4	-4
01043	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	0	0	0	0	0	0
01241	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	1	2	-1	0	-4	-12
03041	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	0	0	0	0	0
01421	$\frac{-96}{23}$	$\frac{-1056}{23}$	$\frac{3552}{23}$	$\frac{2880}{23}$	-2	0	2	4	8	-8
03221	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	1	2	-1	0	-4	-12
05021	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	0	0	0	0	0
01025	$\frac{-96}{23}$	$\frac{-320}{23}$	$\frac{1344}{23}$	$\frac{1408}{23}$	0	0	0	0	0	0
01061	$\frac{248}{23}$	$\frac{-1400}{23}$	$\frac{2216}{23}$	$\frac{656}{23}$	0	0	0	0	0	0
11213	$\frac{-96}{23}$	$\frac{-320}{23}$	$\frac{1344}{23}$	$\frac{1408}{23}$	-1	0	1	2	4	-4
11033	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	1	$\frac{-1}{2}$	0	$\frac{-3}{2}$	-6	6
11411	$\frac{-96}{23}$	$\frac{-1056}{23}$	$\frac{3552}{23}$	$\frac{2880}{23}$	-2	0	2	4	8	-8
11015	$\frac{-96}{23}$	$\frac{-320}{23}$	$\frac{1344}{23}$	$\frac{1408}{23}$	0	0	0	0	0	0
11231	$\frac{8}{23}$	$\frac{496}{23}$	$\frac{-2128}{23}$	$\frac{-2080}{23}$	2	$\frac{3}{2}$	-1	$\frac{-3}{2}$	-10	-6
11051	$\frac{248}{23}$	$\frac{-1400}{23}$	$\frac{2216}{23}$	$\frac{656}{23}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	-2	-6
10322	$\frac{416}{23}$	$\frac{-96}{23}$	$\frac{-3520}{23}$	$\frac{-2912}{23}$	$\frac{5}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-9}{2}$	-16	16
12122	$\frac{-56}{23}$	$\frac{-544}{23}$	$\frac{1792}{23}$	$\frac{1312}{23}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	0
10142	$\frac{-56}{23}$	$\frac{-544}{23}$	$\frac{1792}{23}$	$\frac{1312}{23}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	0
10124	$\frac{232}{23}$	$\frac{-96}{23}$	$\frac{-2048}{23}$	$\frac{-1440}{23}$	$\frac{3}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{-5}{2}$	-8	8
13013	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	0	0	0	0	0	0
13211	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	1	2	-1	0	-4	-12
15011	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	0	0	0	0	0
13031	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	-2	-6
22112	$\frac{-56}{23}$	$\frac{-544}{23}$	$\frac{1792}{23}$	$\frac{1312}{23}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	0
20312	$\frac{416}{23}$	$\frac{-96}{23}$	$\frac{-3520}{23}$	$\frac{-2912}{23}$	$\frac{5}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	$\frac{-9}{2}$	-16	16
20132	$\frac{-332}{23}$	$\frac{-176}{23}$	$\frac{2896}{23}$	$\frac{2048}{23}$	$\frac{-1}{2}$	0	$\frac{5}{2}$	3	4	-12
20114	$\frac{232}{23}$	$\frac{-96}{23}$	$\frac{-2048}{23}$	$\frac{-1440}{23}$	$\frac{3}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{-5}{2}$	-8	8
21023	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	2	-1	0	-3	-12	12
21041	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	1	2	3	-4	-12

$ijklm$	c'_1	c'_2	c'_3	c'_4	c'_5	c'_6	c'_7	c'_8	c'_9	c'_{10}
21221	0	$\frac{9}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
23021	0	$\frac{27}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
33011	0	$\frac{27}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
30122	0	$\frac{9}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
31013	0	$\frac{9}{23}$	$\frac{-1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
31211	0	$\frac{18}{23}$	$\frac{-2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
31031	0	$\frac{27}{46}$	$\frac{1}{46}$	0	0	0	0	0	$\frac{1}{23}$	0
40112	0	$\frac{18}{23}$	$\frac{2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
41021	0	$\frac{27}{23}$	$\frac{1}{23}$	0	0	0	0	0	$\frac{1}{23}$	0
51011	0	$\frac{54}{23}$	$\frac{2}{23}$	0	0	0	0	0	$\frac{1}{23}$	0

Table 9.(contd.)

$ijklm$	c'_{11}	c'_{12}	c'_{13}	c'_{14}	c'_{15}	c'_{16}	c'_{17}	c'_{18}	c'_{19}	c'_{20}
21221	0	$\frac{-9}{23}$	$\frac{38}{23}$	$\frac{-80}{23}$	$\frac{134}{23}$	$\frac{-94}{23}$	6	-8	12	20
23021	0	$\frac{27}{23}$	$\frac{32}{23}$	$\frac{-104}{23}$	$\frac{56}{23}$	$\frac{8}{23}$	10	8	4	-20
33011	0	$\frac{27}{23}$	$\frac{64}{23}$	$\frac{-208}{23}$	$\frac{112}{23}$	$\frac{16}{23}$	18	24	0	-24
30122	0	$\frac{9}{23}$	$\frac{59}{23}$	$\frac{-64}{23}$	$\frac{35}{23}$	$\frac{55}{23}$	12	40	0	-16
31013	0	$\frac{-9}{23}$	$\frac{84}{23}$	$\frac{-80}{23}$	$\frac{180}{23}$	$\frac{44}{23}$	14	40	-16	8
31211	0	$\frac{-9}{23}$	$\frac{76}{23}$	$\frac{-114}{23}$	$\frac{130}{23}$	$\frac{-96}{23}$	12	0	12	12
31031	0	$\frac{27}{23}$	$\frac{55}{23}$	$\frac{-127}{23}$	$\frac{-59}{23}$	$\frac{-38}{23}$	15	12	6	-30
40112	0	$\frac{9}{23}$	$\frac{95}{23}$	$\frac{-128}{23}$	$\frac{254}{23}$	$\frac{110}{23}$	24	48	24	0
41021	0	$\frac{27}{23}$	$\frac{64}{23}$	$\frac{-208}{23}$	$\frac{112}{23}$	$\frac{16}{23}$	26	40	-4	-28
51011	0	$\frac{27}{23}$	$\frac{82}{23}$	$\frac{-370}{23}$	$\frac{454}{23}$	$\frac{124}{23}$	40	80	-20	-20

Table 9.(contd.)

$ijklm$	c'_{21}	c'_{22}	c'_{23}	c'_{24}	c'_{25}	c'_{26}	c'_{27}	c'_{28}	c'_{29}	c'_{30}
21221	$\frac{192}{23}$	$\frac{128}{23}$	$\frac{-2496}{23}$	$\frac{-2816}{23}$	3	1	-1	-3	-16	0
23021	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	1	2	3	-4	-12
33011	$\frac{64}{23}$	$\frac{-1216}{23}$	$\frac{3136}{23}$	$\frac{1024}{23}$	0	1	2	3	-4	-12
30122	$\frac{-608}{23}$	$\frac{192}{23}$	$\frac{4000}{23}$	$\frac{2784}{23}$	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	8	-24
31013	$\frac{-176}{23}$	$\frac{864}{23}$	$\frac{-1760}{23}$	$\frac{-1344}{23}$	2	-1	0	-3	-12	12
31211	$\frac{192}{23}$	$\frac{128}{23}$	$\frac{-2496}{23}$	$\frac{-2816}{23}$	3	1	-1	-3	-16	0
31031	$\frac{-304}{23}$	$\frac{-848}{23}$	$\frac{4976}{23}$	$\frac{1760}{23}$	0	$\frac{3}{2}$	3	$\frac{9}{2}$	-6	-18
40112	$\frac{-608}{23}$	$\frac{192}{23}$	$\frac{4000}{23}$	$\frac{2784}{23}$	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	8	-24
41021	$\frac{-672}{23}$	$\frac{-480}{23}$	$\frac{6816}{23}$	$\frac{2496}{23}$	0	2	4	6	-8	-24
51011	$\frac{-672}{23}$	$\frac{-480}{23}$	$\frac{6816}{23}$	$\frac{2496}{23}$	0	2	4	6	-8	-24

Table 10.(Theorem 2.3(iv))

$ijklm$	d'_1	d'_2	d'_3	d'_4	d'_5	d'_6	d'_7	d'_8	d'_9	d'_{10}
00035	0	$\frac{1}{261}$	0	$\frac{32}{261}$	0	$\frac{32}{87}$	0	$\frac{1}{87}$	0	$\frac{-376}{87}$
00071	0	$\frac{1}{261}$	0	$\frac{8}{261}$	0	$\frac{24}{29}$	0	$\frac{3}{29}$	0	$\frac{168}{29}$
00053	0	$\frac{1}{261}$	0	$\frac{-16}{261}$	0	$\frac{16}{29}$	0	$\frac{-1}{29}$	0	$\frac{-1376}{87}$
00017	0	$\frac{1}{261}$	0	$\frac{-64}{261}$	0	$\frac{64}{261}$	0	$\frac{-1}{261}$	0	$\frac{448}{29}$
00413	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{261}$	0	0	$\frac{-1}{261}$	$\frac{1504}{261}$	$\frac{512}{87}$
00233	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{4}{87}$	0	0	$\frac{1}{87}$	$\frac{374}{261}$	$\frac{-13400}{261}$
00611	0	$\frac{1}{261}$	$\frac{32}{261}$	0	$\frac{32}{261}$	0	0	$\frac{-1}{261}$	$\frac{3008}{261}$	$\frac{33280}{261}$
00215	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{261}$	0		$\frac{-1}{261}$	$\frac{752}{261}$	$\frac{8224}{261}$
00431	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{261}$	0	0	$\frac{-1}{261}$	$\frac{1504}{261}$	$\frac{16576}{261}$
00251	0	$\frac{1}{261}$	$\frac{2}{261}$	0	$\frac{2}{29}$	0	0	$\frac{-1}{29}$	$\frac{938}{261}$	$\frac{11800}{261}$
02213	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{87}$	0	0	$\frac{1}{87}$	$\frac{-296}{261}$	$\frac{-7136}{261}$
04013	0	$\frac{1}{261}$	0	$\frac{-64}{261}$	0	$\frac{64}{29}$	0	$\frac{-1}{29}$	0	$\frac{32}{29}$
02033	0	$\frac{1}{261}$	0	$\frac{-32}{261}$	0	$\frac{32}{29}$	0	$\frac{-1}{29}$	0	$\frac{824}{261}$
02411	0	$\frac{1}{261}$ 0	$\frac{-16}{261}$	0	$\frac{16}{87}$	0	0	$\frac{1}{87}$	$\frac{-592}{261}$	$\frac{-15488}{261}$
04211	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{-1}{29}$	$\frac{1664}{261}$	$\frac{13888}{261}$
06011	0	$\frac{1}{261}$	0	$\frac{64}{261}$	0	$\frac{192}{29}$	0	$\frac{3}{29}$	0	$\frac{5600}{261}$
02015	0	$\frac{1}{261}$	0	$\frac{64}{261}$	0	$\frac{64}{87}$	0	$\frac{1}{87}$	0	$\frac{704}{261}$
02231	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{-1}{29}$	$\frac{2398}{261}$	$\frac{15976}{261}$
04031	0	$\frac{1}{261}$	0	$\frac{32}{261}$	0	$\frac{96}{29}$	0	$\frac{3}{29}$	0	$\frac{392}{87}$
02051	0	$\frac{1}{261}$	0	$\frac{16}{261}$	0	$\frac{48}{29}$	0	$\frac{3}{29}$	0	$\frac{2096}{261}$
01322	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{87}$	0	0	$\frac{-1}{87}$	$\frac{980}{261}$	$\frac{15904}{261}$
03122	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{1}{29}$	$\frac{-1372}{261}$	$\frac{-9296}{261}$
01142	0	$\frac{1}{261}$	$\frac{-2}{261}$	0	$\frac{2}{29}$	0	0	$\frac{1}{29}$	$\frac{-686}{261}$	$\frac{-9296}{261}$
01124	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{4}{87}$	0	0	$\frac{-1}{87}$	$\frac{-32}{261}$	$\frac{7552}{261}$
11132	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{1}{29}$	$\frac{194}{261}$	$\frac{-13472}{261}$
11114	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{87}$	0	0	$\frac{-1}{87}$	$\frac{980}{261}$	$\frac{7552}{261}$
11312	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{87}$	0	0	$\frac{-1}{87}$	$\frac{1960}{261}$	$\frac{15904}{261}$
10223	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{87}$	0	0	$\frac{1}{87}$	$\frac{226}{261}$	$\frac{-19664}{261}$
12023	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{-1}{29}$ 0	$\frac{832}{261}$ 0	$\frac{1360}{261}$
10043	0	$\frac{1}{261}$	$\frac{2}{261}$	0	$\frac{2}{29}$	0	0	$\frac{-1}{29}$	$\frac{-367}{261}$	$\frac{1360}{261}$
10241	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{-1}{29}$	$\frac{571}{261}$	$\frac{18064}{261}$
12041	0	$\frac{1}{261}$	$\frac{-2}{261}$	0	$\frac{6}{29}$	0	0	$\frac{3}{29}$	$\frac{1393}{261}$	$\frac{-112}{9}$
10421	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{16}{87}$	0	0	$\frac{1}{87}$	$\frac{-592}{261}$	$\frac{-36368}{261}$
12221	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{-1}{29}$	$\frac{3230}{261}$	$\frac{18064}{261}$
14021	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{3}{29}$	$\frac{1220}{261}$	$\frac{-112}{9}$
10025	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{4}{87}$	0	0	$\frac{1}{87}$	$\frac{-148}{261}$	$\frac{-2960}{261}$
10061	0	$\frac{1}{261}$	$\frac{-1}{261}$	0	$\frac{3}{29}$	0	0	$\frac{3}{29}$	$\frac{2915}{261}$	$\frac{-184}{9}$
13112	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{1}{29}$	$\frac{388}{261}$	$\frac{-9296}{261}$
22211	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{-1}{29}$	$\frac{3328}{261}$	$\frac{18064}{261}$
22013	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{-1}{29}$	$\frac{1664}{261}$	$\frac{1360}{261}$

Table 10.(contd.)

$ijklm$	d'_{11}	d'_{12}	d'_{13}	d'_{14}	d'_{15}	d'_{16}	d'_{17}	d'_{18}	d'_{19}	d'_{20}
00035	0	$\frac{8}{29}$	0	$\frac{632}{87}$	0	$\frac{352}{29}$	0	$\frac{1328}{29}$	0	$\frac{-4192}{87}$
00071	0	$\frac{420}{29}$	0	$\frac{-1064}{29}$	0	$\frac{-672}{29}$	0	$\frac{-1680}{29}$	0	$\frac{5600}{29}$
00053	0	$\frac{-292}{29}$	0	$\frac{1984}{87}$	0	$\frac{3840}{29}$	0	$\frac{544}{29}$	0	$\frac{-16736}{87}$
00017	0	$\frac{280}{29}$	0	$\frac{-672}{29}$	0	$\frac{-3584}{29}$	0	$\frac{-2016}{29}$	0	$\frac{5824}{29}$
00413	$\frac{512}{87}$	$\frac{1320}{29}$	$\frac{-1432}{261}$	$\frac{-8432}{87}$	$\frac{-10096}{261}$	$\frac{-132608}{261}$	$\frac{-1024}{87}$	-256	$\frac{6128}{87}$	$\frac{227648}{261}$
00233	$\frac{86}{87}$	$\frac{-136}{3}$	$\frac{-40}{261}$	$\frac{20968}{261}$	$\frac{-1184}{87}$	$\frac{12256}{29}$	$\frac{-1012}{87}$	$\frac{15280}{87}$	$\frac{10880}{261}$	$\frac{-198944}{261}$
00611	$\frac{1024}{87}$	$\frac{2944}{29}$	$\frac{-3908}{261}$	$\frac{-16784}{87}$	$\frac{-22280}{261}$	$\frac{-266240}{261}$	$\frac{-2048}{87}$	-544	$\frac{11560}{87}$	$\frac{478208}{261}$
00215	$\frac{256}{87}$	$\frac{624}{29}$	$\frac{-716}{261}$	$\frac{-4256}{87}$	$\frac{-5048}{261}$	$\frac{-65792}{261}$	$\frac{-512}{87}$	-128	$\frac{3064}{87}$	$\frac{110720}{261}$
00431	$\frac{512}{87}$	$\frac{1320}{29}$	$\frac{-1432}{261}$	$\frac{-8432}{87}$	$\frac{-10096}{261}$	$\frac{-132608}{261}$	$\frac{-1024}{87}$	-256	$\frac{6128}{87}$	$\frac{227648}{261}$
00251	$\frac{278}{87}$	$\frac{4804}{87}$	$\frac{-1360}{261}$	$\frac{-21776}{261}$	$\frac{-2968}{87}$	$\frac{-39104}{87}$	$\frac{-1396}{87}$	$\frac{-1328}{87}$	$\frac{7856}{261}$	$\frac{187168}{261}$
02213	$\frac{-176}{87}$	$\frac{-112}{3}$	$\frac{964}{261}$	$\frac{12616}{261}$	$\frac{1112}{87}$	$\frac{7616}{29}$	$\frac{64}{87}$	$\frac{6928}{87}$	$\frac{-1208}{261}$	$\frac{-140480}{261}$
04013	0	$\frac{-56}{29}$	0	$\frac{160}{29}$	0	$\frac{640}{29}$	0	$\frac{-672}{29}$	0	$\frac{-2112}{29}$
02033	0	$\frac{56}{87}$	0	$\frac{272}{261}$	0	$\frac{-496}{87}$	0	$\frac{-3760}{87}$	0	$\frac{560}{261}$
02411	$\frac{-352}{87}$	$\frac{-232}{3}$	$\frac{1928}{261}$	$\frac{25144}{261}$	$\frac{2224}{87}$	$\frac{15968}{29}$	$\frac{128}{87}$	$\frac{15280}{87}$	$\frac{-2416}{261}$	$\frac{-282464}{261}$
04211	$\frac{416}{87}$	$\frac{5152}{87}$	$\frac{-4396}{261}$	$\frac{-25952}{261}$	$\frac{-5608}{87}$	$\frac{-39104}{87}$	$\frac{-1408}{87}$	$\frac{-5504}{87}$	$\frac{16808}{261}$	$\frac{195520}{261}$
06011	0	$\frac{2168}{87}$	0	$\frac{-7480}{261}$	0	$\frac{-16096}{87}$	0	$\frac{-3952}{87}$	0	$\frac{68768}{261}$
02015	0	$\frac{-184}{87}$	0	$\frac{-472}{261}$	0	$\frac{-416}{29}$	0	$\frac{1040}{87}$	0	$\frac{-9952}{261}$
02231	$\frac{730}{87}$	$\frac{5848}{87}$	$\frac{-4808}{261}$	$\frac{-30128}{261}$	$\frac{-7328}{87}$	$\frac{-46064}{87}$	$\frac{-1748}{87}$	$\frac{-9680}{87}$	$\frac{28240}{261}$	$\frac{241456}{261}$
04031	0	$\frac{152}{29}$	0	$\frac{-904}{87}$	0	$\frac{-960}{29}$	0	16	0	$\frac{-64}{87}$
02051	0	$\frac{644}{87}$	0	$\frac{-4504}{261}$	0	$\frac{-6016}{87}$	0	$\frac{-112}{87}$	0	$\frac{19616}{261}$
01322	$\frac{644}{87}$	$\frac{5128}{87}$	$\frac{-1114}{261}$	$\frac{-26504}{261}$	$\frac{-888}{29}$	$\frac{-45712}{87}$	$\frac{308}{87}$	$\frac{-18080}{87}$	$\frac{15272}{261}$	$\frac{248848}{261}$
03122	$\frac{-202}{87}$	$\frac{-3968}{87}$	$\frac{2234}{261}$	$\frac{560}{9}$	$\frac{5000}{87}$	$\frac{992}{3}$	$\frac{740}{87}$	$\frac{6880}{87}$	$\frac{-7144}{261}$	$\frac{-166496}{261}$
01142	$\frac{-275}{87}$	$\frac{-3968}{87}$	$\frac{1378}{261}$	$\frac{560}{9}$	$\frac{2848}{87}$	$\frac{992}{3}$	$\frac{-152}{87}$	$\frac{6880}{87}$	$\frac{-4616}{261}$	$\frac{-166496}{261}$
01124	$\frac{322}{87}$	$\frac{2344}{87}$	$\frac{226}{261}$	$\frac{-11888}{261}$	$\frac{136}{29}$	$\frac{-22048}{87}$	$\frac{676}{87}$	$\frac{-9728}{87}$	$\frac{4504}{261}$	$\frac{119392}{261}$
11132	$\frac{-202}{87}$	$\frac{-4664}{87}$	$\frac{1190}{261}$	$\frac{776}{9}$	$\frac{1172}{87}$	$\frac{1352}{3}$	$\frac{-304}{87}$	$\frac{13144}{87}$	$\frac{164}{261}$	$\frac{-218696}{261}$
11114	$\frac{470}{87}$	$\frac{2344}{87}$	$\frac{-70}{261}$	$\frac{-11888}{261}$	$\frac{-424}{29}$	$\frac{-22048}{87}$	$\frac{308}{87}$	$\frac{-9728}{87}$	$\frac{11096}{261}$	$\frac{119392}{261}$
11312	$\frac{766}{87}$	$\frac{5128}{87}$	$\frac{-1706}{261}$	$\frac{-26504}{261}$	$\frac{-1312}{29}$	$\frac{-45712}{87}$	$\frac{-428}{87}$	$\frac{-18080}{87}$	$\frac{22192}{261}$	$\frac{248848}{261}$
10223	$\frac{172}{87}$	$\frac{-160}{3}$	$\frac{964}{261}$	$\frac{29320}{261}$	$\frac{68}{87}$	$\frac{16896}{29}$	$\frac{64}{87}$	$\frac{23632}{87}$	$\frac{8188}{261}$	$\frac{-257408}{261}$
12023	$\frac{382}{87}$	$\frac{280}{87}$	$\frac{412}{261}$	$\frac{-896}{261}$	$\frac{-716}{87}$	$\frac{-2912}{87}$	$\frac{340}{87}$	$\frac{-5504}{87}$	$\frac{10492}{261}$	$\frac{20128}{261}$
10043	$\frac{-331}{87}$	$\frac{280}{87}$	$\frac{1250}{261}$	$\frac{-896}{261}$	$\frac{2948}{87}$	$\frac{-2912}{87}$	$\frac{1214}{87}$	$\frac{-5504}{87}$	$\frac{-16156}{261}$	$\frac{20128}{261}$
10241	$\frac{-53}{87}$	$\frac{6544}{87}$	$\frac{-110}{261}$	$\frac{-34304}{261}$	$\frac{-20}{87}$	$\frac{-53024}{87}$	$\frac{-182}{87}$	$\frac{-13856}{87}$	$\frac{-8300}{261}$	$\frac{287392}{261}$
12041	$\frac{163}{87}$	$\frac{-1256}{87}$	$\frac{-1214}{261}$	$\frac{2056}{261}$	$\frac{-2492}{87}$	$\frac{10336}{87}$	$\frac{-1178}{87}$	$\frac{6736}{87}$	$\frac{14284}{261}$	$\frac{-69152}{261}$
10421	$\frac{-178}{87}$	$\frac{-328}{3}$	$\frac{2972}{261}$	$\frac{58552}{261}$	$\frac{1876}{87}$	$\frac{30816}{29}$	$\frac{-916}{87}$	$\frac{48688}{87}$	$\frac{716}{261}$	$\frac{-516320}{261}$
12221	$\frac{1112}{87}$	$\frac{6544}{87}$	$\frac{-4396}{261}$	$\frac{-34304}{261}$	$\frac{-8044}{87}$	$\frac{-53024}{87}$	$\frac{-1408}{87}$	$\frac{-13856}{87}$	$\frac{38732}{261}$	$\frac{287392}{261}$
10025	$\frac{326}{87}$	$\frac{-1256}{87}$	$\frac{-340}{261}$	$\frac{2056}{261}$	$\frac{-1852}{87}$	$\frac{10336}{87}$	$\frac{-268}{87}$	$\frac{6736}{87}$	$\frac{17084}{261}$	$\frac{-69152}{261}$
10025	$\frac{86}{87}$	$\frac{8}{3}$	$\frac{1004}{261}$	$\frac{4264}{261}$	$\frac{1252}{87}$	$\frac{1120}{29}$	$\frac{1076}{87}$	$\frac{6928}{87}$	$\frac{-2692}{261}$	$\frac{-15200}{261}$
10061	$\frac{734}{87}$	$\frac{-212}{87}$	$\frac{-5566}{261}$	$\frac{-6296}{261}$	$\frac{-7336}{87}$	$\frac{21472}{87}$	$\frac{-850}{87}$	$\frac{-1616}{87}$	$\frac{36896}{261}$	$\frac{-27392}{261}$
13112	$\frac{118}{87}$	$\frac{-3968}{87}$	$\frac{1858}{261}$	$\frac{560}{9}$	$\frac{952}{87}$	$\frac{992}{3}$	$\frac{436}{87}$	$\frac{6880}{87}$	$\frac{8680}{261}$	$\frac{-166496}{261}$
22211	$\frac{1180}{87}$	$\frac{6544}{87}$	$\frac{-3572}{261}$	$\frac{-34304}{261}$	$\frac{-7040}{87}$	$\frac{-53024}{87}$	$\frac{-728}{87}$	$\frac{-13856}{87}$	$\frac{37792}{261}$	$\frac{287392}{261}$
22013	$\frac{764}{87}$	$\frac{280}{87}$	$\frac{824}{261}$	$\frac{-896}{261}$	$\frac{-1432}{87}$	$\frac{-2912}{87}$	$\frac{680}{87}$	$\frac{-5504}{87}$	$\frac{20984}{261}$	$\frac{20128}{261}$

Table 10.(contd.)

$ijklm$	d'_{21}	d'_{22}	d'_{23}	d'_{24}	d'_{25}	d'_{26}	d'_{27}	d'_{28}
00035	0	$\frac{-9544}{87}$	0	$\frac{16384}{87}$	0	$\frac{332}{87}$	0	$\frac{328}{87}$
00071	0	$\frac{2744}{29}$	0	$\frac{1792}{29}$	0	$\frac{-196}{29}$	0	$\frac{-196}{29}$
00053	0	$\frac{-1664}{87}$	0	$\frac{-12160}{87}$	0	$\frac{1336}{87}$	0	$\frac{1316}{87}$
00017	0	$\frac{2912}{29}$	0	$\frac{-3584}{29}$	0	$\frac{-448}{29}$	0	$\frac{-336}{29}$
00413	$\frac{1640}{87}$	$\frac{3760}{9}$	0	$\frac{-132608}{261}$	$\frac{-512}{87}$	$\frac{-16576}{261}$	$\frac{512}{87}$	$\frac{-15536}{261}$
00233	$\frac{3404}{261}$	$\frac{-75800}{261}$	$\frac{-12512}{261}$	$\frac{99200}{261}$	$\frac{-382}{261}$	$\frac{13396}{261}$	$\frac{-386}{261}$	$\frac{13400}{261}$
00611	$\frac{2236}{87}$	$\frac{7504}{9}$	0	$\frac{-266240}{261}$	$\frac{-1024}{87}$	$\frac{-33280}{261}$	$\frac{-1024}{87}$	$\frac{-32240}{261}$
00215	$\frac{820}{87}$	$\frac{1888}{9}$	0	$\frac{-65792}{261}$	$\frac{-256}{87}$	$\frac{-8224}{261}$	$\frac{-256}{87}$	$\frac{-7184}{261}$
00431	$\frac{1640}{87}$	$\frac{3760}{9}$	0	$\frac{-132608}{261}$	$\frac{-512}{87}$	$\frac{-16576}{261}$	$\frac{-512}{87}$	$\frac{-15536}{261}$
00251	$\frac{244}{9}$	$\frac{36064}{261}$	$\frac{20896}{261}$	$\frac{-78400}{261}$	$\frac{-958}{261}$	$\frac{-11792}{261}$	$\frac{-962}{261}$	$\frac{-11812}{261}$
02213	$\frac{-500}{261}$	$\frac{-42392}{261}$	$\frac{-8320}{261}$	$\frac{65792}{261}$	$\frac{280}{261}$	$\frac{8176}{261}$	$\frac{272}{261}$	$\frac{9224}{261}$
04013	0	$\frac{-1248}{29}$	0	$\frac{-3072}{29}$	0	$\frac{144}{29}$	0	$\frac{240}{29}$
02033	0	$\frac{4064}{261}$	0	$\frac{-44672}{261}$	0	$\frac{-28}{261}$	0	$\frac{-128}{261}$
02411	$\frac{-1000}{261}$	$\frac{-71624}{261}$	$\frac{-16640}{261}$	$\frac{132608}{261}$	$\frac{560}{261}$	$\frac{16528}{261}$	$\frac{544}{261}$	$\frac{17576}{261}$
04211	$\frac{364}{9}$	$\frac{19360}{261}$	$\frac{16768}{261}$	$\frac{-61696}{261}$	$\frac{-1744}{261}$	$\frac{-11792}{261}$	$\frac{-1760}{261}$	$\frac{-10768}{261}$
06011	0	$\frac{9992}{261}$	0	$\frac{21760}{261}$	0	$\frac{-4288}{261}$	0	$\frac{-3608}{261}$
02015	0	$\frac{-14872}{261}$	0	$\frac{32512}{261}$	0	$\frac{80}{261}$	0	$\frac{1096}{261}$
02231	$\frac{380}{9}$	$\frac{52768}{261}$	$\frac{16736}{261}$	$\frac{-95104}{261}$	$\frac{-2438}{261}$	$\frac{-14924}{261}$	$\frac{-2446}{261}$	$\frac{-14944}{261}$
04031	0	$\frac{-8}{3}$	0	$\frac{12544}{87}$	0	$\frac{-4}{87}$	0	$\frac{-56}{87}$
02051	0	$\frac{10664}{261}$	0	$\frac{28864}{261}$	0	$\frac{-1528}{261}$	0	$\frac{-1580}{261}$
01322	$\frac{830}{261}$	$\frac{93928}{261}$	$\frac{12544}{261}$	$\frac{-131584}{261}$	$\frac{-1012}{261}$	$\frac{-16424}{261}$	$\frac{-1016}{261}$	$\frac{-16432}{261}$
03122	$\frac{-3478}{261}$	$\frac{-34640}{261}$	$\frac{-10400}{261}$	$\frac{62720}{261}$	$\frac{1340}{261}$	$\frac{9808}{261}$	$\frac{1330}{261}$	$\frac{9824}{261}$
01142	$\frac{-434}{261}$	$\frac{-34640}{261}$	$\frac{-13552}{261}$	$\frac{62720}{261}$	$\frac{670}{261}$	$\frac{9808}{261}$	$\frac{665}{261}$	$\frac{9824}{261}$
01124	$\frac{-1934}{261}$	$\frac{45904}{261}$	$\frac{6272}{261}$	$\frac{-64768}{261}$	$\frac{16}{261}$	$\frac{-8072}{261}$	$\frac{14}{261}$	$\frac{-8080}{261}$
11132	$\frac{698}{261}$	$\frac{-68048}{261}$	$\frac{-14576}{261}$	$\frac{96128}{261}$	$\frac{296}{261}$	$\frac{13984}{261}$	$\frac{286}{261}$	$\frac{12956}{261}$
11114	$\frac{-214}{261}$	$\frac{45904}{261}$	$\frac{4192}{261}$	$\frac{-64768}{261}$	$\frac{-490}{261}$	$\frac{-8072}{261}$	$\frac{-494}{261}$	$\frac{-8080}{261}$
11312	$\frac{4270}{261}$	$\frac{93928}{261}$	$\frac{8384}{261}$	$\frac{-131584}{261}$	$\frac{-1502}{261}$	$\frac{-16424}{261}$	$\frac{-1510}{261}$	$\frac{-16432}{261}$
10223	$\frac{-500}{261}$	$\frac{-109208}{261}$	$\frac{-8320}{261}$	$\frac{132608}{261}$	$\frac{280}{261}$	$\frac{18616}{261}$	$\frac{272}{261}$	$\frac{17576}{261}$
12023	$\frac{-16}{9}$	$\frac{19360}{261}$	$\frac{32}{261}$	$\frac{-61696}{261}$	$\frac{-350}{261}$	$\frac{-1352}{261}$	$\frac{-358}{261}$	$\frac{-2416}{261}$
10043	$\frac{-152}{9}$	$\frac{19360}{261}$	$\frac{4192}{261}$	$\frac{-61696}{261}$	$\frac{869}{261}$	$\frac{-1352}{261}$	$\frac{865}{261}$	$\frac{-2416}{261}$
10241	$\frac{92}{9}$	$\frac{86176}{261}$	$\frac{25088}{261}$	$\frac{-128512}{261}$	$\frac{-89}{261}$	$\frac{-18056}{261}$	$\frac{-97}{261}$	$\frac{-19120}{261}$
12041	$\frac{6208}{261}$	$\frac{-11384}{261}$	$\frac{-6208}{261}$	$\frac{53504}{261}$	$\frac{-923}{261}$	$\frac{4264}{261}$	$\frac{-937}{261}$	$\frac{3272}{261}$
10421	$\frac{-1000}{261}$	$\frac{-205256}{261}$	$\frac{-16640}{261}$	$\frac{266240}{261}$	$\frac{1082}{261}$	$\frac{35320}{261}$	$\frac{1066}{261}$	$\frac{34280}{261}$
12221	$\frac{364}{9}$	$\frac{86176}{261}$	$\frac{16768}{261}$	$\frac{-128512}{261}$	$\frac{-2788}{261}$	$\frac{-18056}{261}$	$\frac{-2804}{261}$	$\frac{-19120}{261}$
14021	$\frac{4064}{261}$	$\frac{-11384}{261}$	$\frac{-4064}{261}$	$\frac{53504}{261}$	$\frac{-802}{261}$	$\frac{4264}{261}$	$\frac{-830}{261}$	$\frac{3272}{261}$
10025	$\frac{-3904}{261}$	$\frac{-42392}{261}$	$\frac{4192}{261}$	$\frac{65792}{261}$	$\frac{662}{261}$	$\frac{1912}{261}$	$\frac{658}{261}$	$\frac{872}{261}$
10061	$\frac{9368}{261}$	$\frac{5320}{261}$	$\frac{-9368}{261}$	$\frac{36800}{261}$	$\frac{-2419}{261}$	$\frac{5308}{261}$	$\frac{-2426}{261}$	$\frac{4316}{261}$
13112	$\frac{-1214}{261}$	$\frac{-34640}{261}$	$\frac{-12448}{261}$	$\frac{62720}{261}$	$\frac{70}{261}$	$\frac{9808}{261}$	$\frac{50}{261}$	$\frac{9824}{261}$
22211	$\frac{332}{9}$	$\frac{86176}{261}$	$\frac{16832}{261}$	$\frac{-128512}{261}$	$\frac{-2444}{261}$	$\frac{-18056}{261}$	$\frac{-2476}{261}$	$\frac{-19120}{261}$
22013	$\frac{-32}{9}$	$\frac{19360}{261}$	$\frac{64}{261}$	$\frac{-61696}{261}$	$\frac{-700}{261}$	$\frac{-1352}{261}$	$\frac{-716}{261}$	$\frac{-2416}{261}$

$ijklm$	d'_1	d'_2	d'_3	d'_4	d'_5	d'_6	d'_7	d'_8	d'_9	d'_{10}
22031	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{3}{29}$	$\frac{2786}{261}$	$\frac{-328}{9}$
20213	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{16}{87}$	0	0	$\frac{1}{87}$	$\frac{-592}{261}$	$\frac{-19664}{261}$
20033	0	$\frac{1}{261}$	$\frac{4}{261}$	0	$\frac{4}{29}$	0	0	$\frac{-1}{29}$	$\frac{-734}{261}$	$\frac{11800}{261}$
20411	0	$\frac{1}{261}$	$\frac{-32}{261}$	0	$\frac{32}{87}$	0	0	$\frac{1}{87}$	$\frac{-3272}{261}$	$\frac{-36368}{261}$
24011	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{3}{29}$	$\frac{2440}{261}$	$\frac{-112}{9}$
20015	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{87}$	0	0	$\frac{1}{87}$	$\frac{-296}{261}$	$\frac{-2960}{261}$
20231	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{-1}{29}$	$\frac{1664}{261}$	$\frac{28504}{261}$
20051	0	$\frac{1}{261}$	$\frac{-2}{261}$	0	$\frac{6}{29}$	0	0	$\frac{3}{29}$	$\frac{5830}{261}$	$\frac{-400}{9}$
21122	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{1}{29}$	$\frac{388}{261}$	$\frac{-17648}{261}$
30023	0	$\frac{1}{261}$	$\frac{8}{261}$	0	$\frac{8}{29}$	0	0	$\frac{-1}{29}$	$\frac{98}{261}$	$\frac{22240}{261}$
30041	0	$\frac{1}{261}$	$\frac{-4}{261}$	0	$\frac{12}{29}$	0	0	$\frac{3}{29}$	$\frac{7223}{261}$	$\frac{-544}{9}$
30221	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{-1}{29}$	$\frac{5416}{261}$	$\frac{38944}{261}$
32021	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{3}{29}$	$\frac{4006}{261}$	$\frac{-544}{9}$
31112	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{1}{29}$	$\frac{776}{261}$	$\frac{-17648}{261}$
40013	0	$\frac{1}{261}$	$\frac{16}{261}$	0	$\frac{16}{29}$	0	0	$\frac{-1}{29}$	$\frac{3328}{261}$	$\frac{22240}{261}$
40211	0	$\frac{1}{261}$	$\frac{32}{261}$	0	$\frac{32}{29}$	0	0	$\frac{-1}{29}$	$\frac{10832}{261}$	$\frac{38944}{261}$
42011	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{48}{29}$	0	0	$\frac{3}{29}$	$\frac{4880}{261}$	$\frac{-544}{9}$
40031	0	$\frac{1}{261}$	$\frac{-8}{261}$	0	$\frac{24}{29}$	0	0	$\frac{3}{29}$	$\frac{5572}{261}$	$\frac{-616}{9}$
50021	0	$\frac{1}{261}$	$\frac{-16}{261}$	0	$\frac{48}{29}$	0	0	$\frac{3}{29}$	$\frac{704}{261}$	$\frac{-688}{9}$
60011	0	$\frac{1}{261}$	$\frac{-32}{261}$	0	$\frac{96}{29}$	0	0	$\frac{3}{29}$	$\frac{-4856}{261}$	$\frac{-688}{9}$

Table 10.(contd.)

$ijklm$	d'_{11}	d'_{12}	d'_{13}	d'_{14}	d'_{15}	d'_{16}	d'_{17}	d'_{18}	d'_{19}	d'_{20}
22031	$\frac{326}{87}$	$\frac{-3344}{87}$	$\frac{-2428}{261}$	$\frac{10408}{261}$	$\frac{-4984}{87}$	$\frac{29824}{87}$	$\frac{-2356}{87}$	$\frac{15088}{87}$	$\frac{28568}{261}$	$\frac{-177728}{261}$
20213	$\frac{-4}{87}$	$\frac{-160}{3}$	$\frac{2972}{261}$	$\frac{29320}{261}$	$\frac{3616}{87}$	$\frac{16896}{29}$	$\frac{2216}{87}$	$\frac{23632}{87}$	$\frac{-6592}{261}$	$\frac{-257408}{261}$
20033	$\frac{-662}{87}$	$\frac{2368}{87}$	$\frac{2500}{261}$	$\frac{-13424}{261}$	$\frac{5896}{87}$	$\frac{-29360}{87}$	$\frac{2428}{87}$	$\frac{-18032}{87}$	$\frac{-32312}{261}$	$\frac{141232}{261}$
20411	$\frac{-1052}{87}$	$\frac{-328}{3}$	$\frac{8032}{261}$	$\frac{58552}{261}$	$\frac{10712}{87}$	$\frac{30816}{29}$	$\frac{2344}{87}$	$\frac{48688}{87}$	$\frac{-44504}{261}$	$\frac{-516320}{261}$
24011	$\frac{652}{87}$	$\frac{-1256}{87}$	$\frac{-680}{261}$	$\frac{2056}{261}$	$\frac{-3704}{87}$	$\frac{10336}{87}$	$\frac{-536}{87}$	$\frac{6736}{87}$	$\frac{34168}{261}$	$\frac{-69152}{261}$
20015	$\frac{172}{87}$	$\frac{8}{3}$	$\frac{2008}{261}$	$\frac{4264}{261}$	$\frac{2504}{87}$	$\frac{1120}{29}$	$\frac{2152}{87}$	$\frac{6928}{87}$	$\frac{-5384}{261}$	$\frac{-15200}{261}$
20231	$\frac{68}{87}$	$\frac{9328}{87}$	$\frac{-2308}{261}$	$\frac{-55184}{261}$	$\frac{-1432}{87}$	$\frac{-73904}{87}$	$\frac{680}{87}$	$\frac{-34736}{87}$	$\frac{-4072}{261}$	$\frac{441904}{261}$
20051	$\frac{1468}{87}$	$\frac{-2300}{87}$	$\frac{-11132}{261}$	$\frac{2056}{261}$	$\frac{-14672}{87}$	$\frac{40960}{87}$	$\frac{-1700}{87}$	$\frac{6736}{87}$	$\frac{73792}{261}$	$\frac{-135968}{261}$
21122	$\frac{-56}{87}$	$\frac{-5360}{87}$	$\frac{1858}{261}$	$\frac{992}{9}$	$\frac{1648}{87}$	$\frac{1712}{3}$	$\frac{436}{87}$	$\frac{19408}{87}$	$\frac{2416}{261}$	$\frac{-270896}{261}$
30023	$\frac{-280}{87}$	$\frac{4456}{87}$	$\frac{2912}{261}$	$\frac{-25952}{261}$	$\frac{5180}{87}$	$\frac{-55808}{87}$	$\frac{2768}{87}$	$\frac{-30560}{87}$	$\frac{-21820}{261}$	$\frac{262336}{261}$
30041	$\frac{1631}{87}$	$\frac{-5432}{87}$	$\frac{-12346}{261}$	$\frac{18760}{261}$	$\frac{-17164}{87}$	$\frac{49312}{87}$	$\frac{-2878}{87}$	$\frac{23440}{87}$	$\frac{88076}{261}$	$\frac{-286304}{261}$
30221	$\frac{1354}{87}$	$\frac{12112}{87}$	$\frac{-8792}{261}$	$\frac{-76064}{261}$	$\frac{-10868}{87}$	$\frac{-94784}{87}$	$\frac{316}{87}$	$\frac{-55616}{87}$	$\frac{47188}{261}$	$\frac{596416}{261}$
32021	$\frac{652}{87}$	$\frac{-5432}{87}$	$\frac{-2768}{261}$	$\frac{18760}{261}$	$\frac{-6836}{87}$	$\frac{49312}{87}$	$\frac{-2624}{87}$	$\frac{23440}{87}$	$\frac{45652}{261}$	$\frac{-286304}{261}$
31112	$\frac{410}{87}$	$\frac{-5360}{87}$	$\frac{3194}{261}$	$\frac{992}{9}$	$\frac{1904}{87}$	$\frac{1712}{3}$	$\frac{1916}{87}$	$\frac{19408}{87}$	$\frac{13184}{261}$	$\frac{-270896}{261}$
40013	$\frac{1528}{87}$	$\frac{4456}{87}$	$\frac{1648}{261}$	$\frac{-25952}{261}$	$\frac{-2864}{87}$	$\frac{-55808}{87}$	$\frac{1360}{87}$	$\frac{-30560}{87}$	$\frac{41968}{261}$	$\frac{262336}{261}$
40211	$\frac{3752}{87}$	$\frac{12112}{87}$	$\frac{-16540}{261}$	$\frac{-76064}{261}$	$\frac{-25912}{87}$	$\frac{-94784}{87}$	$\frac{-1456}{87}$	$\frac{-55616}{87}$	$\frac{140312}{261}$	$\frac{596416}{261}$
42011	$\frac{1304}{87}$	$\frac{-5432}{87}$	$\frac{-1360}{261}$	$\frac{18760}{261}$	$\frac{-7408}{87}$	$\frac{49312}{87}$	$\frac{-1072}{87}$	$\frac{23440}{87}$	$\frac{68336}{261}$	$\frac{-286304}{261}$
40031	$\frac{652}{87}$	$\frac{-9608}{87}$	$\frac{-4856}{261}$	$\frac{43816}{261}$	$\frac{-9968}{87}$	$\frac{46528}{87}$	$\frac{-4712}{87}$	$\frac{48496}{87}$	$\frac{57136}{261}$	$\frac{-478400}{261}$
50021	$\frac{-1306}{87}$	$\frac{-13784}{87}$	$\frac{12212}{261}$	$\frac{68872}{261}$	$\frac{7556}{87}$	$\frac{43744}{87}$	$\frac{-6292}{87}$	$\frac{73552}{87}$	$\frac{-16228}{261}$	$\frac{-670496}{261}$
60011	$\frac{-2612}{87}$	$\frac{-13784}{87}$	$\frac{32776}{261}$	$\frac{68872}{261}$	$\frac{27640}{87}$	$\frac{43744}{87}$	$\frac{-4232}{87}$	$\frac{73552}{87}$	$\frac{-78392}{261}$	$\frac{-670496}{261}$

Table 10.(contd.)

$ijklm$	d'_{21}	d'_{22}	d'_{23}	d'_{24}	d'_{25}	d'_{26}	d'_{27}	d'_{28}
22031	$\frac{12416}{261}$	$\frac{-44792}{261}$	$\frac{-12416}{261}$	$\frac{86912}{261}$	$\frac{-1846}{261}$	$\frac{11572}{261}$	$\frac{-1874}{261}$	$\frac{9536}{261}$
20213	$\frac{-8308}{261}$	$\frac{-109208}{261}$	$\frac{64}{261}$	$\frac{132608}{261}$	$\frac{1604}{261}$	$\frac{18616}{261}$	$\frac{1588}{261}$	$\frac{17576}{261}$
20033	$\frac{-304}{9}$	$\frac{52768}{261}$	$\frac{8384}{261}$	$\frac{-95104}{261}$	$\frac{1738}{261}$	$\frac{-10748}{261}$	$\frac{1730}{261}$	$\frac{-12856}{261}$
20411	$\frac{-16616}{261}$	$\frac{-205256}{261}$	$\frac{128}{261}$	$\frac{266240}{261}$	$\frac{4252}{261}$	$\frac{35320}{261}$	$\frac{4220}{261}$	$\frac{34280}{261}$
24011	$\frac{8128}{261}$	$\frac{-11384}{261}$	$\frac{-8128}{261}$	$\frac{53504}{261}$	$\frac{-1604}{261}$	$\frac{4264}{261}$	$\frac{-1660}{261}$	$\frac{3272}{261}$
20015	$\frac{-7808}{261}$	$\frac{-42392}{261}$	$\frac{8384}{261}$	$\frac{65792}{261}$	$\frac{1324}{261}$	$\frac{1912}{261}$	$\frac{1316}{261}$	$\frac{872}{261}$
20231	$\frac{76}{9}$	$\frac{152992}{261}$	$\frac{25120}{261}$	$\frac{-195328}{261}$	$\frac{-700}{261}$	$\frac{-27452}{261}$	$\frac{-716}{261}$	$\frac{-29560}{261}$
20051	$\frac{18736}{261}$	$\frac{-28088}{261}$	$\frac{-18736}{261}$	$\frac{70208}{261}$	$\frac{-4838}{261}$	$\frac{12616}{261}$	$\frac{-4852}{261}$	$\frac{10580}{261}$
21122	$\frac{-1214}{261}$	$\frac{-101456}{261}$	$\frac{-12448}{261}$	$\frac{129536}{261}$	$\frac{592}{261}$	$\frac{18160}{261}$	$\frac{572}{261}$	$\frac{16088}{261}$
30023	$\frac{-320}{9}$	$\frac{86176}{261}$	$\frac{8416}{261}$	$\frac{-128512}{261}$	$\frac{1388}{261}$	$\frac{-20144}{261}$	$\frac{1372}{261}$	$\frac{-23296}{261}$
30041	$\frac{24944}{261}$	$\frac{-78200}{261}$	$\frac{-24944}{261}$	$\frac{120320}{261}$	$\frac{-5761}{261}$	$\frac{18880}{261}$	$\frac{-5789}{261}$	$\frac{15800}{261}$
30221	$\frac{332}{9}$	$\frac{219808}{261}$	$\frac{16832}{261}$	$\frac{-262144}{261}$	$\frac{-4010}{261}$	$\frac{-36848}{261}$	$\frac{-4042}{261}$	$\frac{-40000}{261}$
32021	$\frac{16480}{261}$	$\frac{-78200}{261}$	$\frac{-16480}{261}$	$\frac{120320}{261}$	$\frac{-2648}{261}$	$\frac{18880}{261}$	$\frac{-2704}{261}$	$\frac{15800}{261}$
31112	$\frac{-5038}{261}$	$\frac{-101456}{261}$	$\frac{-8192}{261}$	$\frac{129536}{261}$	$\frac{662}{261}$	$\frac{18160}{261}$	$\frac{622}{261}$	$\frac{16088}{261}$
40013	$\frac{-64}{9}$	$\frac{86176}{261}$	$\frac{128}{261}$	$\frac{-128512}{261}$	$\frac{-1400}{261}$	$\frac{-20144}{261}$	$\frac{-1432}{261}$	$\frac{-23296}{261}$
40211	$\frac{844}{9}$	$\frac{219808}{261}$	$\frac{256}{261}$	$\frac{-262144}{261}$	$\frac{-9064}{261}$	$\frac{-36848}{261}$	$\frac{-9128}{261}$	$\frac{-40000}{261}$
42011	$\frac{16256}{261}$	$\frac{-78200}{261}$	$\frac{-16256}{261}$	$\frac{120320}{261}$	$\frac{-3208}{261}$	$\frac{18880}{261}$	$\frac{-3320}{261}$	$\frac{15800}{261}$
40031	$\frac{24832}{261}$	$\frac{-145016}{261}$	$\frac{-24832}{261}$	$\frac{187136}{261}$	$\frac{-3692}{261}$	$\frac{24100}{261}$	$\frac{-3748}{261}$	$\frac{19976}{261}$
50021	$\frac{16256}{261}$	$\frac{-211832}{261}$	$\frac{-16256}{261}$	$\frac{253952}{261}$	$\frac{1490}{261}$	$\frac{29320}{261}$	$\frac{1378}{261}$	$\frac{24152}{261}$
60011	$\frac{-896}{261}$	$\frac{-211832}{261}$	$\frac{896}{261}$	$\frac{253952}{261}$	$\frac{7156}{261}$	$\frac{29320}{261}$	$\frac{6932}{261}$	$\frac{24152}{261}$

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