PHY201: Classical Mechanics I
Problem set 5

1. Consider a two body central force problem with $V(r) = -k/r$ with $k > 0$. Let us assume that all the quantities have been made dimensionless through a suitable choice of units. Let $m_1 = 1$ and $m_2 = 2$. The initial position vectors and velocity vectors are $\mathbf{r}_1(0)$ = (2,2,-3), $\mathbf{r}_2(0)$ = (6,41), $\mathbf{v}_1(0)$ = (-12,11,0) and $\mathbf{v}_2(0)$ = (8,7,6) in the laboratory frame.
   (a) Calculate the total angular momentum vector in the center of mass frame.
   (b) Now choose two values for $k(k_1$ and $k_2)$ such that the total energy in the center of mass frame is positive and negative respectively (No change in the initial conditions).

2. A particle of mass m moves under the action of a central force whose potential is $V(r) = kr^3$ $(k > 0)$. For what energy and angular momentum will the orbit be a circle of radius R about the center of force?

3. Consider the motion of a particle moving in an attractive central force field described by $F = -k/r^3$. With l being the angular momentum and m being the reduced mass, obtain and discuss the orbits for 3 different situations; (a) $l^2 = mk$, (b) $l^2 > mk$ and (c) $l^2 < mk$.

4. Show that the areal velocity is constant for a particle moving under the influence of an attractive force given by $F(r) = -kr$, where $k > 0$. Calculate the time averages of the kinetic and potential energies and compare with the results of the virial theorem.

5. Consider a force law of the form
$$F(r) = -\frac{k}{r^2} - \frac{k'}{r^4}$$
with k and k' being positive defenite. Show that if $\rho^2 k > k'$, then a particle can move in a stable circular orbit at $r = \rho$.

6. Find the force law for a central force field that allows a particle to move in a spiral orbit given by $r = k\theta^2$, where k is a constant.

7. A particle of mass m moves in a circle of radius R under the influence of a central, attractive force
$$F = -\frac{K}{r^2}e^{-r/a}$$
where K and a are positive constants.
   (a) Determine the conditions on the constant a such that the circular orbit is stable.
   (b) Compute the frequency of small radial oscillations about this circular orbit, if the energy is slightly more than that of circular orbit.