PHY201: Classical Mechanics I

Problem Set III

1. A particle of mass m is constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$, where R is the radius of the cylinder. The particles is subjected to a a force directed toward the origin and proportional to the distance of the particle from the origin : $\vec{F} = -k\vec{r}$. (Take the origin at the center of the cylinder).

   (a) Find the Langrangian equations of motion.

   (b) Can you identify the motion along z direction?

   (c) Find the cyclic coordinates.

   (d) Which are the constants of motion?

2. (a) Find the equations of motion for the Lagrangian

   $$L = e^{\gamma t}\Big[\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\Big]$$

   (b) Are there any constants of motion?

   (c) Suppose a point transformation is made of the form

   $$s = e^{\gamma t} q$$

   what is the effective Lagrangian in terms of s?

   (d) Fins the equation of motion for s.

   (e) What do these results say about the conserved quantities for the system?

3. The Euler equation corresponding to a functional $F(y, \dot{y}, x)$ is

   $$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial \dot{y}} = 0$$

   Show that

   $$\frac{d}{dx}\big(F - \dot{y}\frac{\partial F}{\partial \dot{y}}\big) = \frac{\partial F}{\partial x}$$

   Hence in case of F does not depend explicitly on x, show that

   $$F - \dot{y}\frac{\partial F}{\partial \dot{y}} = const.$$

   If F is the Lagrangian, what does this mean?

4. Using spherical polar coordinates and the variational principle to show that the *geodesic* (curve that is the shortest distance between two points) on the surface of a sphere is a great circle. (A great circle of the sphere is the intersection of the sphere and the plane passing through the center of the sphere. Its equation is given by $cot\ \theta = Acos(\phi + \phi_0)$ where A and ϕ_0 are constants.)

5. The Fermat principle states that light always propagates along a path that takes the minimum amount of time. Consider a medium with an index of refraction given by $n(x, y) = n_0(1 + ky)$. Recall that the speed of light in a medium with index n is given by $v = c/n$. Find the function that describes the path of light in this medium. Determine a specific equation for the path of a laser beam that initially starts at the origin propagating in the x-direction, as shown in the figure.