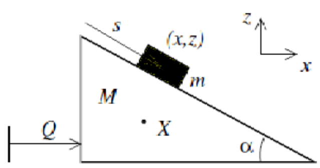
PHY201: Classical Mechanics I

Problem Set II

1. For each of the following systems determine the number of generalised coordinates.
   (a) A particle in a three dimensional potential, $V(\vec{r}) = ar^2$.
   (b) Two particles on a plane constrained such that $r_1^2 + r_2^2 = R^2$ where R is a constant.
   (c) Three particles in three dimensions such that $r_1 - r_2 = a$, $r_2 - r_3 = b$, $r_1 - r_3 = a + b$, where r_i is the spherical coordinate r of the i^{th} particle.

2. We use $T = 1/2m\dot{x}^2$ as the kinetic energy term. Here we consider two other possibilities
$$T = \frac{m\dot{x}^2}{2} + \frac{c_1}{\dot{x}^2}; \quad T = \frac{m\dot{x}^2}{2} + c_2\dot{x}^4$$
where c_1 and c_2 are constants.
   (a) What are the units of c1 and c2?
   (b) We consider a particle in one dimension in a potential $V(x)$. Using $L = T - V$, for each of the two possibilities above derive the Equation of motion of the particle.
   (c) Newtonian mechanics is defined in such a way that $c_1 = c_2 = 0$. Yet, Nature is not fully described by Newtonian mechanics. Can you think what c_2 corresponds to? Can you determine it?

3. A block of mass m is sliding down a wedge of mass M without friction by the gravitational force. The wedge also slides on a table frictionlessly. The angle of the wedge is given as α.



   (a) Introduce the generalized coordinates (s, Q) as shown in the figure. Express the block's center of mass coordinates (x, z) and the horizontal coordinate of the wedge X in terms of the generalized coordinates up to constants.
   (b) For a virtual displacement associated with $s(\delta s \neq 0$ and $\delta Q = 0)$, find the corresponding displacements for the block $(\delta x, \delta z)$ and the wedge δX.
   (c) Do the same for the virtual displacement for $Q(\delta s = 0$ and $\delta Q \neq 0)$.
   (d) What are the generalized forces Q_j for the coordinates (s, Q)?
   (e) From the forces above in (d), construct the Lagrangian-Euler equations
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q_j}}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

(f) Solve for $\ddot{s}$.

4. Find Lagrange's equations for a particle of mass m moving in two dimensions Under the influence of a potential $U(r, \theta)$, using polar coordinates.

5. Consider a planar double pendulum (in the $x - y$ plane) in the Earth's gravitational field in the y direction. The length of both strings is l and both masses are equal, and denoted by m.

   (a) How many generalised coordinates are required to describe the motion?

   (b) Write down the lagrangian of the system.

   (c) Write doen the equations of motion (do not solve them).

   (d) Solve the equations of motion in the small angle approximation.