Emission spectra of Hydrogen (Balmer series) and determination of Rydberg’s constant

Objective

i) To measure the wavelengths of visible spectral lines in Balmer series of atomic hydrogen

ii) To determine the value of Rydberg’s constant

Introduction

In the 19th Century, much before the advent of Quantum Mechanics, physicists devoted a considerable effort to spectroscopy trying to deduce some physical properties of atoms and molecules by investigating the electromagnetic radiation emitted or absorbed by them. Many spectra have been studied in detail and amongst them, the hydrogen emission spectrum which is relatively simple and shows regularity, was most intensely studied.

Theoretical Background

According to Bohr's model of hydrogen atom, the wavelengths of Balmer series spectral lines are given by

$$\frac{1}{\lambda} = R_y \left[ \frac{1}{n^2} - \frac{1}{m^2} \right]$$

(1)

where, \(n = 2\) and \(m = 3, 4, 5, 6\ldots\) and Rydberg’s constant \(R_y\) is given by

$$R_y = \frac{e^2 m_e}{8 \varepsilon_0^2 h c} = 1.097 \times 10^7 \text{ m}^{-1}$$

(2)

where ‘\(e\)’ is the charge of 1 electron, ‘\(m_e\)’ is mass of one electron, ‘\(\varepsilon_0\)’ is permittivity of air = \(8.85 \times 10^{-12}\), ‘\(h\)’ is Planck’s constant and ‘\(c\)’ is velocity of light.

The wavelengths of the hydrogen spectral emission lines are spectrally resolved with the help of a diffraction grating. The principle is that if a monochromatic light of wavelength \(\lambda\) falls normally on an amplitude diffraction grating with periodicity of lines given by ‘\(g\)’
\( (= 1/N, \text{ where } N \text{ is the number of grating lines per unit length}), \) the intensity peaks due to principal maxima occur under the condition:

\[ g \sin \alpha = p\lambda \quad \ldots(2) \]

where ‘\( \alpha \)’ is the diffraction angle and \( p = 1, 2, 3, \ldots \) is the order of diffraction.

In the first part of the experiment, the grating constant ‘\( g \)’ is determined by measuring the diffraction angles for the known spectral lines of a mercury (Hg) spectral tube (Mercury-vapour lamp). The Hg spectral tube is then replaced by a hydrogen (atomic) spectral tube, in which, \( H_2 \) is converted into atomic hydrogen due to collision ionization. The electrons in \( H \) atoms are excited to higher energy levels through collisions with other electrons. When these electrons return to lower energy levels, the atom emits electromagnetic radiation of discrete frequencies in various spectral ranges. Balmer's series spectral lines fall in ultraviolet and visible ranges and the latter wavelengths are determined in this experiment by measuring the corresponding diffraction angles. Usually, only first order diffraction is studied, for which the value of \( 'p' \) in Eq. 2 is taken to be unity.

**Experimental Set-up**

The experimental set-up is shown in Fig. 1. Hydrogen or mercury spectral tubes are used as the source of light. The spectral tube is fixed between two high voltage electrodes. A grating and a spectrometer will be used to analyse different spectra. First, a mercury source is used to determine the grating element (\( g \)). Then using this value of \( g \) and a hydrogen spectral tube the unknown lines of Balmer series of hydrogen spectra are determined.

**Procedure**

A. Adjusting the spectrometer:

Follow the support manual for spectrometer provided in the appendix for basic adjustment of spectrometer. Determine the vernier constant of the spectrometer. Fix the grating on the prism table. Do not disturb the spectrometer henceforth throughout the experiment.
B. Determination of $g$

1. Bring in the Hg source close to the collimator of the previously leveled spectrometer.
2. Then switch ON the power supply and let the Hg lamp warm up.
3. Look through the telescope to notice the three first order spectral lines of Hg (yellow, green and blue) on both sides of the direct image of the slit at the center. Make the spectral lines vertical by turning the grating slightly in its plane.
4. Note down the positions of the cross wire for each line on one side using the two verniers on the spectrometer. Use a torch, if needed, to read the verniers.
5. Repeat the above step by turning the telescope to the other side too. Determine the diffraction angle, $\alpha$, for all the three spectral lines of Hg spectrum. Using the spectral data of Hg given below, calculate $g$.
6. Switch off the power supply and remove the Hg spectral tube.

<table>
<thead>
<tr>
<th>Colour</th>
<th>$\lambda$ /nm</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>yellow</td>
<td>581 ± 1</td>
<td>6 D1 → 6 P1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 D1 → 6 P1</td>
</tr>
<tr>
<td>green</td>
<td>550 ± 1</td>
<td>7 sS1 → 6 P1</td>
</tr>
<tr>
<td>green</td>
<td>494 ± 2</td>
<td>8 sS1 → 6 P1</td>
</tr>
<tr>
<td>blue</td>
<td>437 ± 2</td>
<td>7 sS → 6 P1</td>
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</tbody>
</table>

C. Studying hydrogen spectrum

1. In presence of the instructor/lab operator bring in the hydrogen source to the front of collimator. Then switch ON the power supply.
2. Repeat steps B.3 to B.4 (mentioned above) and note down the positions of the cursors for 3 spectral lines (red, green and violet) of the hydrogen spectrum.
3. Using the value of $g$ determined earlier, calculate the wavelength of each of the spectral lines.
Observation

Table 1: Determination of $g$

<table>
<thead>
<tr>
<th>Colour/ Line/ Literature Value (nm)</th>
<th>Left side</th>
<th>Right side</th>
<th>Vernier 1(deg)</th>
<th>Vernier 2(deg)</th>
<th>Vernier 1(deg)</th>
<th>Vernier2(deg)</th>
<th>Vernier 1 2θ(deg)</th>
<th>Vernier 2 2θ(deg)</th>
<th>Avg. 0(θ(deg))</th>
<th>$g$</th>
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<tbody>
<tr>
<td>MSR VSR TOTAL MSR VSR TOTAL MSR VSR TOTAL</td>
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</table>

Mean value of $g = \ldots$

Table 2: Determination of spectral lines of hydrogen

<table>
<thead>
<tr>
<th>Colour/ Line/ Literature Value (nm)</th>
<th>Left side</th>
<th>Right side</th>
<th>Vernier 1(deg)</th>
<th>Vernier 2(deg)</th>
<th>Vernier 1(deg)</th>
<th>Vernier2(deg)</th>
<th>Vernier 1 2θ(deg)</th>
<th>Vernier 2 2θ(deg)</th>
<th>Avg. 0(θ(deg))</th>
<th>$\lambda$ (nm)</th>
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Red/H$_\alpha$/656.28
Green/H$_\beta$/486.13
Violet/H$_\gamma$/434.05

Results and analysis

1. Assign appropriate values of $n$ and $m$ ($n = 2$ and $m = 3, 4 \ldots$) to each $\lambda$ (average value). Tabulate the values of $n$ and $m$, the corresponding values of $\left(1/n^2 - 1/m^2\right)$ and $1/\lambda$.

2. Plot $1/\lambda$ vs. $\left(1/n^2 - 1/m^2\right)$ on a graph sheet. Indicate the error bars of $1/\lambda$ by calculating $1/(\lambda \pm \Delta\lambda)$ in each case. Indicate $n$ and $m$ for the three data points on the graph.

3. Draw the linear graph. From the slope of the graph and the error in the slope, determine the value of Rydberg's constant and the corresponding random error. Write the final result with its uncertainty.
Precautions

1. Handle the spectral tube with utmost care.
2. Never touch the surface of the grating by hand. Always hold it from the sides.
3. Do not change the positions of the spectrometer and the spectral tube throughout the experiment.
4. Switch OFF power supply before making any changes in the spectral tube arrangement.