# 2. ELECTROMAGNETIC DAMPING OF A COMPOUND PENDULUM

**Objective:** To study the Electromagnetic damping of a compound pendulum.

**Introduction:**

Damping plays an important role in controlling the motion of an object. It is an effect, which tends to reduce the velocity of a moving object. A number of damping techniques are used in various moving, oscillating and rotating systems. These techniques include, conventional friction damping, air friction damping, fluid friction damping and electromagnetic (eddy current) damping. Electromagnetic damping is one of the most interesting damping techniques, which uses electromagnetically induced currents to slow down the motion of a moving object without any physical contact with the moving object.

To understand the phenomenon of electromagnetic damping, we need to know about electromagnetic induction (discovered by Michael Faraday in 1831) and eddy currents (also known as Foucault currents - discovered by Leon Foucault in 1851). Electromagnetic induction is a phenomenon, in which an electromotive force (emf) is induced in a conductor, when it experiences a changing magnetic field. An emf is induced when either the conductor moves across a steady magnetic field or when the conductor is placed in a changing magnetic field. Due to this induced emf and the conducting path available, induced currents (flow of electrons) are set up in the body of the conductor. These induced currents are in the form of 'eddy currents' which are electrons swirling within the body of the conductor like water swirling in a whirlpool (eddy).

The eddy currents swirl in such a way as to create a magnetic field opposing the change in the magnetic field experienced by the conductor in accordance with Lenz's law. Thus the eddy currents swirl in a plane perpendicular to the magnetic field. These eddy currents interact with the magnetic field to produce a force, which opposes the motion of the moving conductor or object. The damping force increases as the distance of the conductor decreases from the magnet.

This damping force is also proportional to the strength of the magnetic field and the induced eddy currents and hence the velocity of the object. Thus faster the object moves the stronger is the damping force. This means that as the object slows down, the damping force is reduced, resulting in a smooth stopping motion.

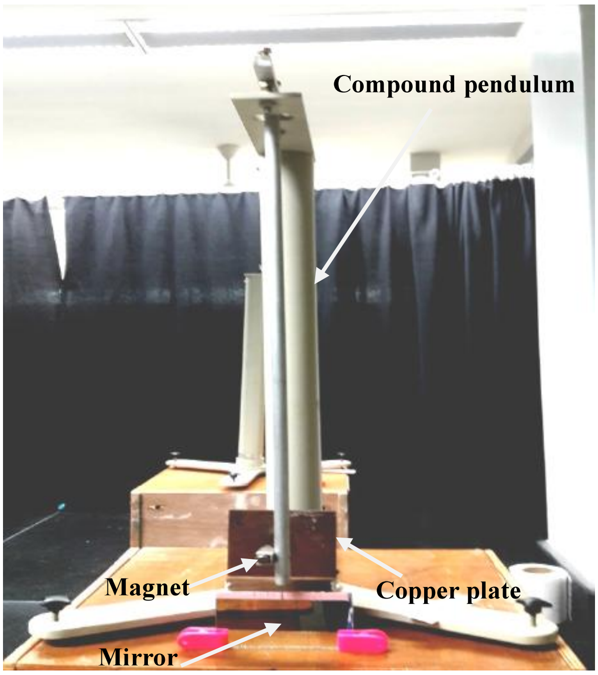

**Fig. 1: Experimental set up**

In this experiment, a magnet is attached to a compound pendulum and a metal sheet is placed at certain distance from the magnet. The metal sheet should be placed in such a way that it is parallel to the plane of oscillation and perpendicular to the length of the magnet as shown in Figure 1. While the pendulum oscillates, the magnetic flux passing through the metal will change and induce eddy current in the metal plate.

**Apparatus**

Compound pendulum with a pointer, tripod stand, set of magnets, copper plate with holder, mirror, graph paper to mark position of pointer, stop-watch.

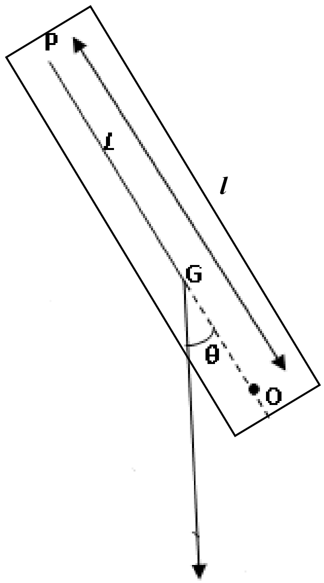

**Fig. 2: Compound Pendulum**

**Theory**

Consider a compound pendulum pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle θ (see Fig. 2). In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is L, the mass of the body is $m$ and $I$ is the moment of inertia of the body through the axis P. Then the equation of motion of for small amplitude oscillation is given by

$$I\ddot{\theta} = -mgL\theta \qquad \dots\dots (1)$$

Thus the solution of Eq. 1 becomes,

$$\theta = \theta_0 \sin(\omega_0 t) \quad \ldots \quad \ldots. \quad (2)$$

where $\theta_0$ is the maximum angular amplitude, $\omega_0 = 2\pi/\ T_0 = \sqrt{\frac{mgL}{I}}$. So time period for free oscillation of the pendulum is given by

$$T_0 = 2\pi\sqrt{\frac{I}{mgL}} \quad \ldots \quad \ldots \quad (3)$$

When the oscillation is damped due to any resistance in the path such as friction or eddy current etc, the damping force exerted at the free end of the rod is directly proportional to velocity, v, of the free end. Let $\gamma$ be the constant of proportionality called as damping coefficient, then this force can be written as

$$F = -\gamma v = -\gamma l \omega \quad \ldots \quad \ldots \quad (4)$$

where $l$= actual length of the rod, $\omega$= angular velocity and the negative sign indicates that the force is always directed opposite to the velocity. Then torque is given by

$$l F = -\gamma l^2 \omega = -\gamma l^2 \dot{\theta} \quad \ldots \quad \ldots \quad (5)$$

Thus the equation of motion for a damped oscillation is given by

$$I\ddot{\theta} = -mgL\theta - \gamma l^2 \dot{\theta} \quad \ldots \quad \ldots \quad (6)$$

The solution of this modified equation is

$$\theta = \theta_0\ e^{-\frac{t}{\tau}} cos(\omega_1\, t) \quad \ldots \quad \ldots \quad (7)$$

where $\tau = \frac{2I}{\gamma l^2}$, is called as decay constant and $\omega_1$ is the angular frequency of damped oscillation.

Thus, $\theta$ becomes maximum (but < $\theta_0$ due to exponential decay function in Eq. 7), for $\omega_1 t = 2n\pi$, where n= 0, 1, 2… If $T_1$is time period of this damped oscillator, then it attains maximum displacement at times, t=n$T_1$. Hence the maximum displacement of the pendulum decreases exponentially with time as given by the following equation:

$$\theta_{max} = \theta_0\ e^{-\frac{t}{\tau}} \quad \ldots \quad \ldots \quad (8)$$

Equation 8 can be rewritten in terms of an equation of a straight line as follows:

$$\ln\left(\frac{\theta_{max}}{\theta_0}\right) \approx \ln\frac{x_n}{x_0} = -\left(\frac{T_1}{\tau}\right) n \qquad \ldots \qquad \ldots \qquad (9)$$

where $x_0$ is the initial and $x_n$ is the final linear amplitude after n oscillations. Knowing $\tau$ from the slope of the straight line, the damping co-efficient can be calculated as

$$\gamma = \frac{2\ \mathrm{I}}{\tau l^2} \qquad \ldots \qquad \ldots \qquad (10)$$

**Procedure**

1. The compound pendulum is mounted on a tripod stand with two pin pivot arrangement. Make sure that the pendulum does not slip from the pivot.
2. Fix the magnet to the pendulum using adhesive tapes.
3. Place a mirror vertically very close to the pin attached to the pendulum as a pointer. Adjust the position so that the image of the tip of the pin can be seen in the mirror avoiding parallax error. A graph sheet is pasted on the mirror to mark and note the linear amplitudes of the pendulum. Mark the equilibrium position on the graph sheet.
4. Place the copper plate at a distance, say about 15 mm, so that the plane of copper plate is perpendicular to the axis of the magnets (see Fig.1).
5. Measure the time period for 10 oscillations by using a stop watch provided. Repeat it for 3 to 5 times to find the average time period of oscillation.
6. Displace the pendulum from the mean position to a position of initial amplitude, $x_0$ (say about 20mm), and then leave it to oscillate. Note the final amplitude after 2 oscillations. Repeat it for at least three times, and then find the average value of $x_2$.
7. Repeat the above step for 4, 6, 8, 10 oscillations keeping the initial position fixed.
8. Fill up the observation table and plot a graph between the number of oscillations, n vs $\ln\frac{x_n}{x_0}$. Determine the slope using straight line fitting and find decay time $\tau$.
9. Finally, calculate the damping co-efficient '$\gamma$' using the given values of I and *l*.

**Observations**

**Given**: Moment of inertia of the supplied rod, **I = 0.0235 kgm$^2$**

Length of the rod, ***l*** **= 0.61m**

**Table 1: Time period of damped oscillation**

d = ……

| Sl. No. | Time for 10 Oscillations (sec) | Time period $T_1$ (sec) | Average time period $T_1$ (sec) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

**Table 2: Measuring Amplitude with damping**

**$x_0$ = ……. mm**

| No. of Oscillations (n) | Sl. No. | $x_n$ (mm) | $x_n/x_0$ | $\ln(x_n/x_0)$ |
|---|---|---|---|---|
| 2 | 1 | | | |
| | 2 | | | |
| | 3 | | | |
| 4 | 4 | | | |
| | 5 | | | |
| | 6 | | | |
| .. | | | | |

**Graph:** Plot n vs ln $\frac{x_n}{x_0}$

Slope = …..

**Calculations:** τ = ……

**γ = …..**

**Estimation of error:**

**Precautions:**

1. Avoid parallax error while noting the amplitude.
2. Mark the amplitude carefully on the graph sheet pasted on the mirror without disturbing the set up.