

Moment of inertia of different bodies

Aim: 1) Study moment of inertia of different bodies

Objectives of the experiment

1. Measuring the period of oscillation of a thin transverse rod with weights on a torsion axle as a function of the distance of the weights from the torsion axle.
2. Confirming the proportionality between the moment of inertia of the weights and the square of the distance.
3. Determining the restoring torque of the torsion axle.

Apparatus

Torsion axle, spring, rod with weights, different shape objects
Stand base
Stop clock

Principles

The moment of inertia is a measure of the inertia that a body exhibits when a torque acts on it causing a change of its rotational motion. It corresponds to the inertial mass in the case of translational motions. In rotational oscillations, for example, the period of oscillation T is the greater, the greater the moment of inertia J of the oscillating system is. More specifically:

$$T = 2\pi \sqrt{\frac{J}{D}} \dots \dots \dots (I)$$

D : restoring torque

The moment of inertia of a point like mass m moving on a circular path with radius r is

$$J_1 = m.r^2 \dots \dots \dots (II).$$

The moment of inertia of two equal masses m that is rigidly connected and has the same distance r from the axis of rotation is

$$J_2 = 2 . m . r^2 \dots \dots \dots (III).$$

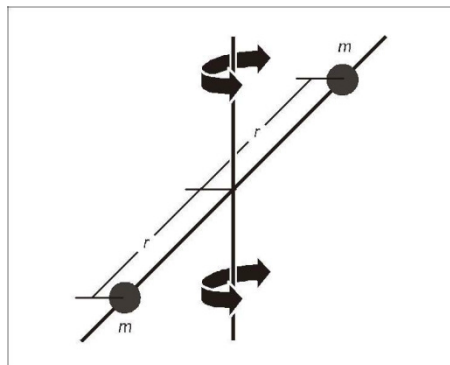


Fig. 1 Schematic illustration

In both cases, the moment of inertia is proportional to the square of the distance r .

In the experiment, the rigid connection between the two masses is established by means of a thin rod whose middle is fixed to the torsion axle (see Fig. 1). After deflection from the equilibrium position, the system oscillates with the period of oscillation T . From Eq. (I) it follows that

$$J = D \cdot \left(\frac{T}{2\pi}\right)^2 \dots\dots\dots (IV)$$

However, the moment of inertia is composed of the moment of inertia J_2 of the two weights and the moment of inertia J_0 of the rod:

$$J = 2 \cdot m \cdot r^2 + J_0 \dots\dots (V).$$

Thus,

$$D \frac{T^2}{4\pi^2} = 2 \cdot m \cdot r^2 + J_0$$

Therefore, the period of oscillation T_0 of the rod without weights is measured in another measurement, which leads to

$$D \cdot \left(\frac{T}{2\pi}\right)^2 = 2 \cdot m \cdot r^2 + D \cdot \left(\frac{T_0}{2\pi}\right)^2$$

Or

$$T^2 = \frac{8 \cdot m \cdot \pi^2}{D} \cdot r^2 + T_0^2 \dots\dots\dots (VI)$$

Thus a linear relation between the square of the period of oscillation T and the square of the distance r is obtained. From the slope of the straight line,

$$a = \frac{8 \cdot m \cdot \pi^2}{D} \dots\dots\dots (VII)$$

the restoring torque D can be calculated if the mass m is known.

Setup and carrying out the experiment

The experimental setup is illustrated in Fig. 2.

- Fix the middle of the transverse rod to the torsion axle and arrange the weights symmetrically at a distance of 30 cm from the torsion axle.
- Mark the equilibrium position on the table.

- Rotate the transverse rod to the right in between 10° - 180° and release it.
- Start the time measurement as soon as the transverse rod passes through the equilibrium position and stop the measurement after five oscillations.
- Repeat the measurement four times, alternately deflecting the rod to the left and to the right.
- Calculate the period of oscillation T from the mean value of the five measured values.
- One after another reduce the distance to 25 cm, 20 cm, 15 cm, 10 cm and 5 cm, each time repeating the measurement.
- Remove the weights, and repeat the measurement.

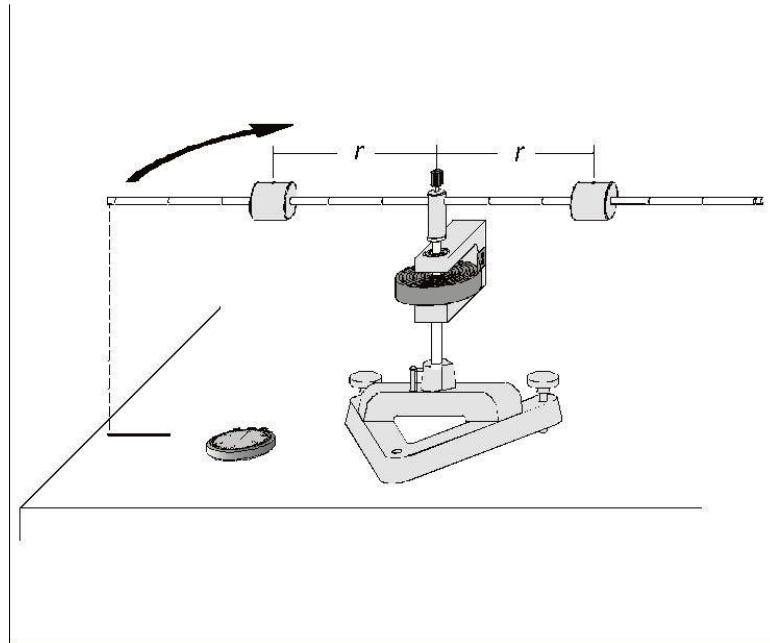


Fig. 2. Experimental setup for determining the period of oscillation.

Observations:

Measured duration of five oscillations for various distances r between the torsion axle and the weights and mean value of the period of oscillation T calculated from the measured value.

Table-1

No. of obs.	r cm	r^2 cm ²	$5T$					Average of 5T	T s	T^2 s ²
			s							
	Without weights									

Calculation

From the Table 1 we obtain $T_0 = \dots\dots\dots$ s and therefore $T_0^2 = \dots\dots\dots$ s².

The squares of the oscillation periods of the system with weights

Plot the graph between T^2 and r^2 .

As the mass 'm' is known, the restoring torque 'D' can be calculated from the slope according to (VII).

We obtain $D = \dots\dots\dots$ N.m



Objectives of the experiment (Part 2)

- Determining the moments of inertia of rotationally symmetric bodies from their period of oscillation on a torsion axle.
- Comparing the periods of oscillation of two bodies having different masses, but the same moment of inertia.
- Comparing the periods of oscillation of hollow bodies and solid bodies having the same mass and the same dimensions.
- Comparing the periods of oscillation of two bodies having the same mass and the same body shape, but different dimensions.

Theory: The moment of inertia is a measure of the resistance of a body against a change of its rotational motion and it depends on the distribution of its mass relative to the axis of rotation. For a calculation of the moment of inertia J , the body is subdivided into sufficiently small mass elements Δm_i with distances r_i from the axis of rotation and a sum is taken over all mass elements:

$$J = \sum_i \Delta m_i \cdot r_i^2 \dots\dots\dots(1)$$

For bodies with a continuous mass distribution, the sum can be converted into an integral. If, in addition, the mass distribution is homogeneous, the integral reads

$$J = M \cdot \frac{1}{V} \int_V r^2 dV \dots\dots\dots (2)$$

The calculation of the integral is simplified when rotationally symmetric bodies are considered which rotate around their axis of symmetry. The simplest case is that of a hollow cylinder with radius R . As all mass elements have the distance R from the axis of rotation, the moment of inertia of the hollow cylinder is

$$J_{HC} = M \cdot R^2 \dots\dots\dots (3)$$

In the case of a solid cylinder with equal mass M and equal radius R , Eq. (II) leads to the formula

$$J_{SC} = M \cdot \frac{1}{V} \int_0^R r^2 \cdot 2\pi \cdot r \cdot H \cdot dr \text{ with } V = \pi \cdot R^2 \cdot H$$

And the result is

$$J_{SC} = \frac{1}{2} M \cdot R^2 \dots\dots\dots (4)$$

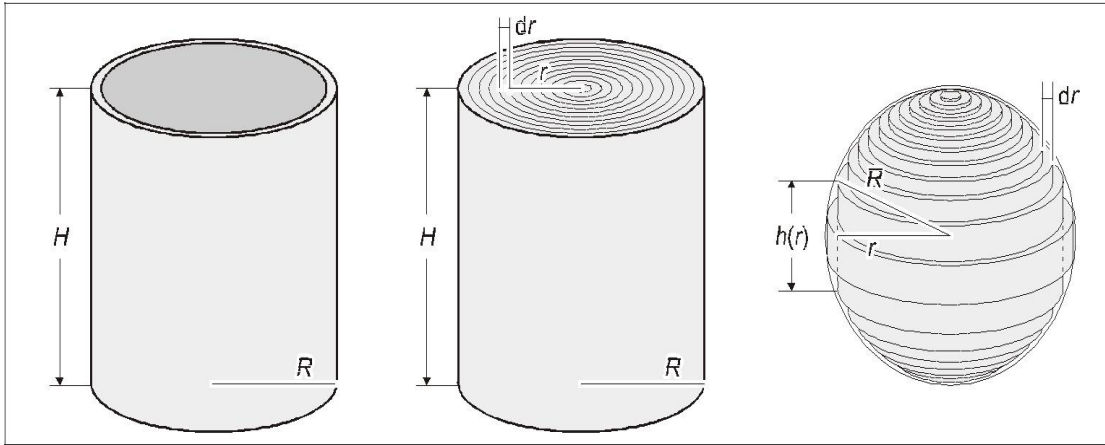


Fig. 3: Calculation of the moments of inertia of a hollow cylinder, a solid cylinder and a sphere

That means, the moment of inertia of a solid cylinder is smaller than that of the hollow cylinder as the distances of the mass elements from the axis of rotation are between 0 and R .

An even smaller value is expected for the moment of inertia of a solid sphere with radius R (see Fig. 1). In this case, Eq. (2) leads to the formula

$$J_{SS} = M \cdot \frac{1}{V} \int_0^R r^2 \cdot 2\pi \cdot r \cdot 2 \cdot \sqrt{R^2 - r^2} \cdot dr \text{ with } V = \frac{4\pi}{3} R^3$$

and the result is $J_{SS} = \frac{2}{5} \cdot M \cdot R^2$ (5)

Thus, apart from the mass M and the radius R of the bodies under consideration a dimensionless factor enters the calculation of the moment of inertia, which depends on the shape of the respective body.

The moment of inertia is determined from the period of oscillation of a torsion axle, on which the test body is fixed and which is connected elastically to the stand via a helical spring. The system is excited to perform harmonic oscillations. If the restoring torque D is known, the moment of inertia of the test body is calculated from the period of oscillation T according to

$$J = D \cdot \left(\frac{T}{2\pi}\right)^2 \text{ (6)}$$

Setup and carrying out the experiment

The experimental setup is illustrated in Fig 2.

- Put the sphere on the torsion axle, and mark the equilibrium position on the table.
- Rotate the sphere to the right in between $10-180^\circ$ and release it.
- Start the time measurement as soon as the sphere passes through the equilibrium position and stop the measurement after five oscillations ($5T$).
- Calculate the period of oscillation T .
- Replace the sphere with the disk, and repeat the measurement.
- Replace the disk with the supporting plate.
- Repeat the measurement with the solid cylinder and then with the hollow cylinder.
- Finally carry out the measurement with the empty supporting plate.

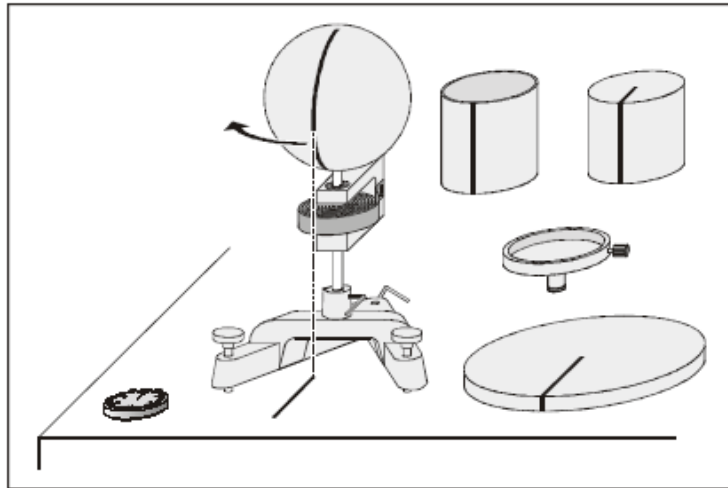


Fig.4: Setup for measuring time period

Data

Table 1: list of the bodies under consideration and the measured oscillation periods

Body	\underline{M} (gm)	$\underline{2R}$ (cm)	$\underline{5T}$ (sec)	\underline{T} (sec)
Solid sphere				
Flat solid cylinder (disk)				
Long solid cylinder				
Hollow cylinder				
Empty supporting plate	NA	NA		

Evaluation

a) Qualitative comparison:

Bodies having different masses, but the same moment of inertia:

The sphere and the flat solid cylinder (disk) have different shapes and different masses. They oscillate at approximately the same period, i.e. they have the same moment of inertia.

Hollow body and solid body:

The hollow cylinder and the solid cylinder have approximately the same mass and the same diameter. The periods of oscillation are clearly different, i.e. they have different moments of inertia.

Bodies having the same mass and the same shape but different dimensions:

The flat solid cylinder (disk) and the long solid cylinder have approximately the same mass, but different diameters. They oscillate at clearly different periods, i.e. the moments of inertia are different.

b) Quantitative comparison:

With Eq. (6), the moments of inertia J can be calculated from the periods T listed in Table 1.

Calculate dimensionless factors of Eqs. (3), (4) and (5) and compare with the values calculated from the measuring data.

Body	MR^2 (gm.cm ²)	$J = D \cdot \left(\frac{T}{2\pi}\right)^2$	Measurement (J/MR ²)	Theory (J/MR ²)	Error
Solid sphere					
disk					
Solid cylinder					
Hallow cylinder					