Resolving Power of a telescope with a rectangular aperture

Objective:

- To study the resolving power of a telescope with a rectangular aperture

Apparatus:

Telescope, variable rectangular slit, Na lamp, source slits

Theoretical background:

A simple telescope consists of a large aperture objective lens with high focal length and an eye piece with a lower focal length and smaller aperture. An eye piece consists of two lenses separated a distance. The lens towards the objective is called the field lens and other which is near to the eye is known as eye lens.

The resolving power of an optical instrument, say a telescope or microscope, is its ability to produce separate images of two closely spaced objects/ sources. The plane waves from each source after passing through an aperture from diffraction pattern characteristics of the aperture. It is the overlapping of diffraction patterns formed by two sources sets a theoretical upper limit to the resolving power.

Consider two narrow slit sources, d distance apart, kept at a distance D away from the aperture, i.e. objective, of a telescope. In the following we will stick to rectangular aperture. Then the angular separation α of the slits at the aperture is α = d/D. Each slit will produce its own single slit diffraction pattern, for which the intensity distribution is given by,

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}, \text{ where } \beta = \frac{\pi a \sin\theta}{\lambda} \qquad (1)$$

And a is the slit width, θ is the angle of diffraction and λ is the wave length of light from the sources. The principal maximum of each slit corresponds to θ = 0 ⇒ $\beta = 0$ and the position of the minima, which are points of zero intensity, corresponds to $\beta = \pm\pi,\ \pm 2\pi$ etc. The angular separation of the two principal maxima is equal to the angular separation of the sources, i.e. α.

The Rayleigh's criterion for resolution of two diffraction patterns states that two sources or their diffraction patterns are resolved when the principal maximum of one falls exactly on the first minimum of the other. Since the first minimum is formed at $\beta = \pi$, then angular separation α of the maxima is equal to the corresponding θ_1,

$$\pi = \frac{\pi a \sin\theta_1}{\lambda} \rightarrow \qquad \sin\theta_1 \approx \theta_1 = \frac{\lambda}{a} \Rightarrow \boxed{\alpha = \theta_1 = \frac{\lambda}{a} = \frac{d}{D}} \qquad (2)$$

The angle $\theta_1$is known as minimum angle of resolution while $1/_{\theta_1}$is sometimes called the resolving power of the aperture a. For circular aperture, Rayleigh's criterion is modified to $\theta_1 = 1.22\,\lambda/_a$. To find the intensity at the centre of the resultant minimum for the overlapping diffraction fringes separated by θ_1, we note the curves of principal maxima cross at $\beta = \pi/_2$for either pattern. Therefore intensity at the centre, relative to the maximum, is sum of the intensity of either at $\beta = \pi/_2$,

$$\frac{I(\theta_1)}{I_0} = 2 \times \frac{\sin^2 \beta}{\beta^2} = \frac{8}{\pi^2} = 0.8106 \qquad (3)$$

Procedure:

1. Make the axis of the telescope horizontal by adjusting it with a spirit level and its height is to be adjusted that the images of the pair of slits are symmetrical with respect to the cross point of the cross wires.
2. The images are brought into sharp focus by adjusting the telescope while keeping the variable aperture wide open.
3. Reduce the width of the aperture gradually so that at first the two images appear out and ultimately their separation vanishes. Measure the width of the aperture at this critical position. Reducing aperture further and note the reading when the illumination (light) just disappears altogether. The difference of these two readings gives width of the aperture required.
4. Repeat the operation 3 for three times.
5. Measure the distance between the slits (sources) and the objective of the telescope by means of measuring tape.
6. Observe that as you increase the distance D between the telescope aperture and source for a fixed d and λ, you need larger aperture to resolve the two sources. It is needless to mention that larger aperture implies larger light gathering capacity.

For fun, you can test how good the resolving power of your eyes are by looking at the second star from the tip of the handle of Big Dipper or *saptarshi mandal* in the constellation of *Ursa major*. The name of the bright star is *Mizar (vasistha)*, but it has a faint optical companion called *Alcor (Arundhati)* and the ability to resolve the two stars with naked eye is often quoted as a test of eyesight.

Observe the construction of telescope. What are the components used in the telescope? Pull out the eye piece and observe the location of cross wires. Which type of eye piece is used in the telescope? Ramsden's or Huyghen's? What is the difference between these two types of eyepieces?

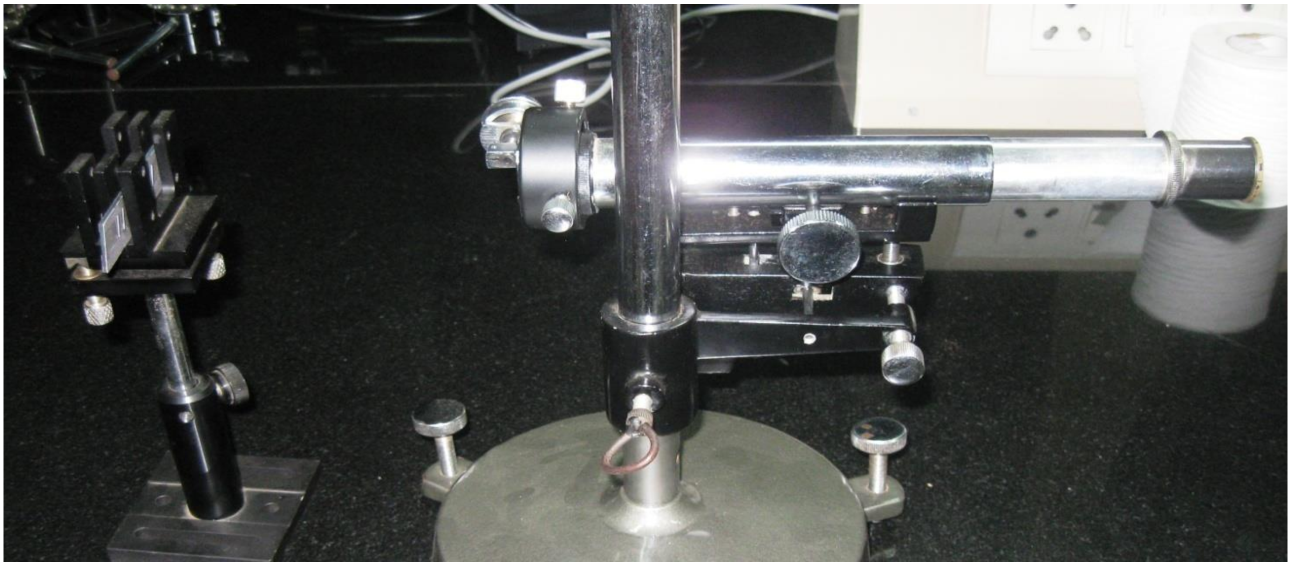

Fig. 1 Experimental setup

Observations

Determination of least count of the traveling microscope

Value of smallest main scale division (MSD) = ……………………………

……………………vernier scale division = ………………………..main scale division

Hence, 1 vernier scale division = …………………………main scale division (VSD)

Vernier Constant (least count) = (1 – VSD) x MSD = …………………………….cm

Table-I. Determination of the source slits separation d

Slit edge	obs	Left slit					Right slit					d (cm)	Mean d
		Main Scale Reading (MSR)	Verneir coincidence (VC)	Vernier scaLe-reading (VCXLC)	T = M SR+ VSR	Mean T	Main Scale Reading (MSR)	Verneir coincidence (VC	Vernier scaLe-reading (VCXLC)	T = M SR + VSR	Mean T		
L	1 2 3					α_l=					$\beta_l =$	$\alpha_l \sim \beta_l$	………
R	1 2 3					α_r=					$\beta_r =$	$\alpha_r \sim \beta_r$	

Least count of screw gauge attached to the aperture

Value of smallest main scale division (MSD) = ……………………………

Number of circular scale division (CD) = …………………………………….

Screw Pitch (P) = 1/CD = ……………………Least Count (LC) = P x MSD = ………....cm

Table II. Determination of minimum angle of resolution θ_1 at varying D

Wavelength of light used $\lambda = 589.3 \times 10^{-9}$m. Average source slits separation d = ……….cm

Obs	D (cm)	$\frac{D}{d}$	At the critical position (C_0)				Illumination (light) just disappears (C_1)				$a = C_0 - C_1$ cm	$\theta_1 = \frac{\lambda}{a}$ (radian)	$\frac{a}{\lambda}$
			MSR	Circular Coincidence	CSR	Total(cm)	MSR	Circular Coincidence	CSR	Total (cm)			
1a 1b 1c													
. . .	. . .	. . .	. . .				. . .				. . .	. . .	. . .
6a 6b 6c													

Plot the graph between distance 'D' vs Aperture 'a' and determine wavelength 'λ' of the given light source and compared with given wavelength.

Plot the $\theta_1 - a$ graph, explain why it gets difficult to achieve increasingly low θ_1?

Questions

1. How the minimum angle of resolution θ_1 changes with the wavelength of the light λ?
2. Which one of the two telescopes, optical and radio has more resolving power for a given aperture?
3. What is the θ_1 for human eye for red (700nm), yellow (600nm), and blue (400nm) light, assuming dark-adapted average pupil size is 5mm? (Use the Rayleigh's criterion for circular aperture.)