Compton Scattering of Gamma Rays

Introduction:

The Compton Effect is the quantum theory of the scattering of electromagnetic waves by a charged particle in which a portion of the energy of the electromagnetic wave is given to the charged particle in an elastic, relativistic collision. Compton scattering was discovered in 1922 by Arthur H. Compton (1892-1962) while conducting research on the scattering of x-rays by light elements and received the Nobel Prize in 1927 for this discovery. His theoretical explanation of what is now known as Compton scattering required the use of special relativity and quantum mechanics, both of which were hardly understood at the time. When first reported, his results were controversial, but his work quickly triumphed and had a powerful effect on the future development of quantum theory.

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Thomson scattering, the classical theory of an electromagnetic wave scattered by charged particles, cannot explain low intensity shifts in wavelength (Classically, light of sufficient intensity for the electric field to accelerate a charged particle to a relativistic speed will cause radiation-pressure recoil and an associated Doppler shift of the scattered light, but the effect would become arbitrarily small at sufficiently low light intensities regardless of wavelength.) Light must behave as if it consists of particles in order to explain the low-intensity Compton scattering. Compton's experiment convinced physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the frequency.

Compton scattering is the main focus of this experiment, but it is necessary to understand the interactions of high energy, electromagnetic photon radiation with materials in general. Gamma rays are high energy photons emitted from radioactive sources. When they interact with matter, there are three primary ways their energies can be absorbed by materials. These are the photoelectric effect, Compton scattering, and pair production. In addition to these primary processes, there are several lesser ways such as x-ray production and Bremsstrahlung. The Compton Effect is studied with the measurement of a gamma ray energy spectrum using a scintillator, photomultiplier tube, and multichannel analyzer. The gamma rays interact with the scintillator producing all three primary interaction processes so that the very phenomenon that is
being studied in a sample is also taking place in the detector itself along with several other effects that mask the process of interest.

**Objectives:**

1. To investigate Compton scattering inside and outside the detector crystal.
2. Experimentally determine the energy position of the Compton edge in the $^{137}$Cs spectrum. The energy is compared with the theoretically calculated value.
3. By scattering the gamma radiation from one scatterer, the energy of the scattered radiation is studied as a function of scattering angle.

**Apparatus:**

- NaI detector with amplifier and accessories
- Scatterers of aluminum, copper and steel
- Lead sheets or lead bricks
- A rotational stage to move the detector at various angles with respected to collimated beam of incident gamma rays and scatterer
- $^{137}$Cs calibration source
- A multichannel analyzer system, which is inbuilt with the GDM amplifier and ADC unit

**Theory:**

Compton scattering involves the scattering of photons by charged particles where both energy and momentum are transferred to the charged particle while the photon moves off with a reduced energy and a change of momentum. Generally, the charged particle is an electron considered to be at rest and the photon is usually considered to be an energetic photon such as an x-ray photon or gamma ray photon. In this experiment gamma rays from a cesium-137 source are used for the source of photons that are scattered and each photon has energy of 0.662 MeV, when incident on the target scatterer. The charged particle is assumed to be an electron at rest in the target. While the theory here is applied to gamma rays and electrons, the theory works just as well for less energetic photons such as found in visible light and other particles.

The theory of Compton scattering uses relativistic mechanics for two reasons. First, it involves the scattering of photons that are massless, and secondly, the energy transferred to the electron is comparable to its rest energy. As a result the energy and momentum of the photons and electrons must be expressed using their relativistic values. The laws of conservation of
energy and conservation of momentum are then used with these relativistic values to develop the theory of Compton scattering.

Figure 1 illustrates the scattering of an incident photon of energy $E = h\nu$ moving to the right in the positive x direction with a momentum $p = h\nu/c = h/\lambda$ and interacting with an electron at rest with momentum $P_e = 0$ and energy equal to its rest energy, $2E_0 = m_0c^2$. The symbols $h$, $\nu$, and $\lambda$ are the standard symbols used for Planck’s constant, the photon’s frequency, its wavelength, and $m_0$ is the rest mass of the electron. In the interaction, the gamma ray is scattered in the positive x and y directions at an angle $\theta$ with momentum of magnitude $p' = h\nu'/c = h/\lambda'$ and energy $E' = h\nu'$. The electron is scattered in the positive x-direction and negative y-direction at an angle $\phi$ with respect to the positive x-direction with momentum $p_e' = 1/c\sqrt{(E_e^2 - E_0^2)^{1/2}}$ and energy $E_e = mc^2$ where $m$ is the relativistic mass of the electron after the interaction.

![Compton scattering diagram](image)

Figure 1: Compton scattering diagram showing the relationship of the incident photon and electron initially at rest to the scattered photon and electron given kinetic energy.

From the law of conservation of energy, the energy of the incident gamma ray, $h\nu$, and the rest mass of the electron, $E_0$, before scattering is equal to the energy of the scattered gamma ray, $h\nu'$, and the total energy of the electron, $E_e$, after scattering

$$h\nu + E_0 = h\nu' + E_e$$  \hspace{1cm} (1)
The relationship between the total energy, \( E_e \), of the electron after scattering, its rest mass, \( E_0 \), and its relativistic momentum, \( p_e \), is given by
\[
E_e^2 = (p'_e c)^2 + E_0^2
\]  
(2)

From the law of conservation of energy, eq. (1) and relativistic momentum, eq. (2) one can derive the equation,
\[
\frac{1}{E'} - \frac{1}{E} = \frac{1}{E_0} [1 - \cos \theta]
\]  
(3)

that relates the energy of a scattered photon \( E' \) to the energy of the incident photon \( E \) and the scattering angle \( \theta \).

Equation (3) is a simple equation that can be used to verify the theory for the Compton Effect. The energy of incident gamma rays \( E \) can be easily measured with a scintillator photomultiplier detector and multichannel analyzer system. The energy of the scattered gamma rays \( E' \) as a function of \( \theta \) can also be easily measured with the same system.

A plot of measurements of \( \frac{1}{E'} - \frac{1}{E} \) versus measurements of \( 1 - \cos \theta \) should result in a linear graph whose slope is the inverse of the electron’s rest energy \( \frac{1}{E_0} \).

If one considered that \( E = E_{in} \) and \( E' = E_{out} \) to calculate both the energy eq. 3 can be modified as
\[
E_{out} = \frac{E_{in}}{1 + \frac{E_{in}}{E_0} [1 - \cos \theta]}
\]  
(4)

The energy difference \( E_{in} - E_{out} \) denotes how much energy has been deposited to the electrons in the crystal by scattering. It is that energy that is registered in the spectrum.

The above equation (3), can be written in term of wavelength of the gamma-quantum as
\[
\Delta \lambda = \lambda - \lambda' = \lambda_0 [1 - \cos \theta]
\]  
(5)

This is called as Compton wavelength.

**Procedure:**

(a) **Energy calibration of a gamma-ray spectrum**

To perform any of the analysing procedures described below, one must first start the analyzing programme ‘WinDas’ on the respective computer. An explanation of the commands and the function keys can be obtained on the screen by the command **Help** or in WinDAS User’s Guide
chapter 4. Before a spectrum can be analyzed the spectrum must be read into primary memory. This is done with the command **File Open**.

**(b) The relation between the channel scale (the channel number) and the energy of the radiation**

Determine the channel position of the photo peak in the spectrum of $^{137}$Cs by using the centroid routine, which is prepared by first placing the cross of the lower marker on the left edge of the photo peak and the cross of the upper marker on the right edge of the same peak. This is most easily done by using the right and the left button of the mouse. See figure 2.

Now use the command **Calculate Centroid**, which gives the channel position of the centre of mass of the photo peak. Zero the spectrum on the screen (**File Clear**) and read the $^{40}$K spectrum. Repeat the same procedure for the photo peak in the $^{40}$K spectrum. Note that the number of pulses and the number of pulses per second in the photo peak are also given.

Draw a diagram of the energies of the two photo peaks as a function of the corresponding channel position, i.e. the energy along the vertical axis and the channel number along the horizontal axis. The energies of the gamma quanta of $^{60}$Co (two photo peaks) and $^{137}$Cs are 1.33 MeV, 1.17 MeV and 0.66 MeV, respectively.

Since the energy is very close to a linear function of the channel number you have now obtained an energy calibration. This means that each channel position corresponds to a definite energy. Draw a straight line through the calibration points. Use the calibration to determine the size of the energy region (energy window) of your spectrum, by reading from the diagram the energies which correspond to the beginning and the end of the spectrum. A suitable energy window is in most cases about 0.07 - 2.00 MeV.

Note down your energy windows: ..........................

![Figure 1: The standard spectrum of $^{137}$Cs for calibration](image-url)
(c) **Determination of the Compton edge**

Put the $^{137}$Cs source in front of the opening (as shown in Fig. 3) to the detector at a distance of about 5 cm to make sure that the count rate is not too high. Collect a spectrum for about 5 minutes. Use this spectrum to experimentally determine the energy of the Compton edge. The value is compared to the calculated value in the theory section.

The energy of the Compton edge was determined to be ................................ MeV

In the spectrum the position of the Compton edge is defined as the mid-point of the slope of the Compton edge.

![Figure 3: The schematic setup for Compton scattering experiment.](image)

(d) **Compton scattering with different scattering angle:**

1. Arrange the setup according to Fig. 3. To achieve a better collimation of the scattered gamma quanta, a 2 cm thick lead shield with a 2 cm wide slit can be placed in front of the detector. This option is not available in the current version of experimental setup.

2. By measuring the scattering angle $\theta$ according to figure 3, one can calculate the energy of the gamma quanta scattered into the detector opening with the help of the scattering equation. This energy is compared with the energy obtained for the photo peak of the scattered radiation.

3. Insert the scatterer Aluminum (Al). The scattering angle $\theta$ is chosen to be 90 degrees. A spectrum is collected for about 10 minutes. The spectrum is saved. The scatterer is now removed without moving the rest of the setup and a new spectrum is collected for about the same amount of time, i.e. 10 minutes.
4. Since the gamma radiation from $^{137}$Cs also is scattered from objects surrounding the detector opening and from the detector itself and since it would be very difficult to shield against this unwanted Compton scattering, one collects a spectrum without a scatterer. By subtracting the unwanted contribution the desired effect is observed more easily.

5. The subtracted spectrum is energy calibrated and the energy of the photo peak is determined. This value is compared with the theoretically calculated value and tabulate in Table.1.

6. Now choose a scattering angle between 20° to 180°. Repeat the measurement procedure for 3-4 times and take the mean value for final comparison.

7. Plot $\theta$ verses $E_{\text{out}}$ and explain the nature of curve

8. Repeat point 3-7 for another scatterer Cu.

### Table 1: Verification of Compton effect

<table>
<thead>
<tr>
<th>Scattering angle ($\theta$)</th>
<th>Incident energy ($E_{\text{in}}$)</th>
<th>Outer energy ($E_{\text{out}}$)</th>
<th>$\Delta E=E_{\text{in}}-E_{\text{out}}$</th>
<th>$\Delta \lambda$</th>
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**Observations:**
- Calculate the difference in Compton energy and Compton wavelength of the Compton scattering and compare with the literature value ($2.426 \times 10^{-12}$ m).

**Sources of errors:**
- What are the sources of error in this experiment?

**Precautions:**
- Handle the radioactive sources with care. Don’t touch in bare hand to the center of samples.
- While handling the liquid radioactive samples please use hand gloves.
- Don’t put your mobile near to the detector. It may add some counts to the signal.
• Amplifier and detector are always running at high voltage. So please don't switch off any part during the data acquisition, it may damage the amplifier and detector.

Questions and considerations:
1. Why is a plastic or other low-Z target preferred in this experiment? Would an Al or Pb rod work well as a target?
2. What is the maximum energy a backscattered gamma ray can have when it is backscattered from a material into a detector, regardless of the initial energy of the incident gamma ray photon?
3. Analysis of scattering angles is simplified by using the average scattering angle. Under what assumptions will the average scattering angle equal the geometrical angle? Consider the validity of the assumptions.
4. What influence does the finite angular extent of your target and detector have on your measurements? Can you substantiate any such influences quantitatively?
5. How far would the target need to have been displaced from its ideal position to substantially affect your measurements?

References:
6. GDM 10, Handbook and other related manuals.