

Magnetic susceptibility of a paramagnetic material by Quincke's method

Objective

1. To determine the magnetic susceptibility χ of a given paramagnetic solution for a specific concentration.
2. Calculate mass susceptibility χ' , Molar susceptibility χ'' , Curie constant C and Magnetic dipole moment.

Theory

When a material is placed within a magnetic field, the magnetic forces of the material's electrons will be affected. This effect is known as Faraday's Law of Magnetic Induction. However, materials can react quite differently to the presence of an external magnetic field. This reaction is dependent on a number of factors, such as the atomic and molecular structure of the material, and the net magnetic field associated with the atoms. The magnetic moments associated with atoms have three origins. These are the electron motion, the change in motion caused by an external magnetic field, and the spin of the electrons. In most atoms, electrons occur in pairs with spins in opposite directions. These opposite spins cause their magnetic fields to cancel each other. Therefore, no net magnetic field exists. Alternately, materials with some unpaired electrons will have a net magnetic field and will react more to an external field. Most materials can be classified as diamagnetic, paramagnetic or ferromagnetic.

Although you might expect the determination of electromagnetic quantities such as susceptibility to involve only electrical and magnetic measurements, this practical shows how very simple measurements of *mechanical* phenomena, such as the displacement of a liquid column can be used instead. Quincke devised a simple method to determine the magnetic susceptibility, χ , of a *paramagnetic* solution by observing how the liquid rises up between the two pole pieces of an electromagnet, when a direct current is passed through the electromagnet coil windings. A material's magnetic susceptibility tells us how "susceptible" it is to becoming temporarily magnetised by an applied magnetic field and defined as the magnetization (M) produced per unit magnetic field (H).

$$\chi = M/H \quad (1)$$

Consider a paramagnetic medium in the presence of a uniform applied flux density B_0 . Loosely speaking, paramagnets are materials which are attracted to magnets. They contain microscopic magnetic dipoles of magnetic dipole moment \mathbf{m} which are randomly oriented. However, in the presence of a uniform field \mathbf{B} each dipole possesses a magnetic potential energy $U = -\mathbf{m} \cdot \mathbf{B}$, [1]. So they all tend to align up parallel to \mathbf{B} , which is the orientation in which their potential energy is minimum (i.e. most negative). Consequently, the liquid, which contains many such dipoles, will tend to be drawn into the region of maximum field since this will minimize its total magnetic potential energy. In other words, the liquid experiences an attractive magnetic force Fm pulling it into the region of strongest field. The dipoles in the liquid, FeCl_3 solution for this experiment, are due to Fe^{3+} ions which are paramagnetic in their ground-state. The “spins” of several outer electrons are aligned parallel to each other to give rise to a net magnetic moment \mathbf{m} which is not compensated by other electrons.

A region of empty space permeated by a magnetic field H possesses an energy whose density (energy per unit volume) is $u = \frac{1}{2}\mu_0 H^2$ [1], where μ_0 is the magnetic permeability of vacuum. In presence of a medium, this magnetic energy density may be written:

$$u = \frac{1}{2} \mu H^2 \quad (2)$$

where μ is the magnetic permeability of the medium and $H = |\mathbf{H}|$. For fields which are not too large, the magnetic permeability μ of a paramagnet can be treated as independent of the applied field; i.e. it is a “constant”. Note that $\mu > \mu_0$ for a paramagnet. The \mathbf{H} vector has the very useful property that its tangential component is continuous across a boundary, so that the value of H in the air above the meniscus is equal to that in the liquid. This is in contrast to the flux density, where B_0 in air is different (less, in this case) from the value B in the liquid:

$$H = \frac{B_0}{\mu_0} = \frac{B}{\mu} = \frac{B}{\mu_0(1 + \chi)} \quad (3)$$

Suppose that, when the field is turned on, the meniscus in the narrow tube rises by an amount h , relative to its zero-field position (see Fig. 1). A volume $\pi r^2 h$ of air in the narrow tube (with permeability μ_0) is, therefore, replaced by liquid. Hence, the magnetic potential energy of this volume of space increases by an amount:

$$\Delta U = \frac{1}{2} (\mu - \mu_a) H^2 \times (\pi r^2 h) \quad (4)$$

The work done by the upward magnetic force F_m in raising the liquid by an amount h is

$$F_m = \frac{\Delta U}{h} = \frac{1}{2} (\mu - \mu_a) H^2 \pi r^2 \quad (5)$$

When the liquid in one arm of the tube rises by h , it falls on the other arm by h . It continues to rise till the upward magnetic force is balanced by the weight of the head of liquid. The downward gravitational force on the head of liquid, of mass m , is given by

$$F_g = mg = 2\pi r^2 \rho hg \quad (6)$$

where ρ is the density of the liquid. However, there is also a very small additional upwards force on the liquid due to the buoyancy of the air, which, strictly, ought to be included (By the Principle of Archimedes, bodies immersed in any fluid, even air, experience this buoyancy; you are yourself very slightly lighter by virtue of the surrounding air, though this effect is extremely tiny compared to that which you experience when immersed in a much denser fluid, such as water). The liquid in the narrow column displaces a volume of air, while that in the wide column is replaced by air, and this leads to a net upwards buoyancy force on the narrow column given by

$$F_b = 2\pi r^2 \rho_a hg \quad (7)$$

where ρ_a is the density of the air. Combining all these forces, we have $F_m = F_g - F_b$, so that

$$\frac{1}{2}(\mu - \mu_a)H^2 \pi r^2 = 2\pi r^2 (\rho - \rho_a)hg \quad (8)$$

Using Eq. 3 in Eq. 8, we finally obtain the volume susceptibility, which is a dimensionless quantity:

$$\chi = [4g\mu_o(\rho - \rho_a)] \frac{h}{B^2} - \chi_a \quad (9)$$

Where χ_a is the susceptibility of air. In practice, the corrections due to air are negligible. There will also be a small but significant diamagnetic (i.e. *negative*) contribution to the susceptibility mainly due to water. The total susceptibility of the solution is given by $\chi = \chi_{Fe} + \chi_{water}$. In the present work you will correct χ to yield the true value of χ_{Fe} . Some other parameters are defined in terms of volume susceptibility as follows:

Mass Susceptibility is given by: $\chi' = \chi/\rho$ (10)

Molar Susceptibility is given by: $\chi'' = \chi' M$ (11)

where M= Molecular weight

Curie constant is given by: $C = \chi''T$ (12)

where T= Temperature of sample

Magnetic moment μ of dipole of the specimen by relation

$$\mu = 2.8241\sqrt{C} \quad (13)$$

where μ is expressed in Bohr magneton μ_B with a value of 9.272×10^{-24} A-m²

Literature value of molar susceptibility of FeCl₃ is $+1.69 \times 10^{-8}$ m³/mol [2].

Since the thermal effects tend to destroy the alignment of magnetic dipoles, so the susceptibility of a paramagnet decreases as the temperature T is increased. Using statistical mechanics, it may be shown that at high temperatures ($kT \gg mB$) the contribution χ_{Fe} of the paramagnetic Fe³⁺ ions to the volume susceptibility of the solution is given by,

$$\chi = \frac{\mu_o M}{B} = \frac{Nm^2 \mu_o}{3kT} = \frac{Np^2 \mu_B^2 \mu_o}{3kT} \quad (14)$$

where k is Boltzmann's constant and N is the number of Fe³⁺ ions per unit volume, p is the magneton number defined in Appendix A, μ_B is the Bohr magneton and $m = p\mu_B$. The $1/T$ dependence of χ_{Mn} is known as Curie's Law.

The above theory assumes that the magnetic field acting on each ion is just the applied field B ; field and contributions due to neighboring magnetic ions are neglected. For dilute paramagnetic materials these other contributions are very small and the approximation is valid. This is not so for concentrated magnetic materials and ferromagnets.

Apparatus:

1. Adjustable electromagnet with pole pieces
2. Constant power supply (0-16 V, 5A DC)
3. Digital Gauss meter
4. Hall probe for magnetic strength measurement
5. Traveling Microscope
6. Quincke's tube (an U tube)
7. Measuring cylinder (100ml), Pipette (5ml)/dropper, Wash bottle
8. Specific gravity bottle (25cc)
9. FeCl₃ for making solutions
10. Electronic balance (Least count = 0.01 gm)
11. Connecting cords

Experimental set-up

A schematic diagram of Quinck's method is shown in Fig 1. Quinck's tube is U shaped glass tube. One arm of the tube is placed between the pole-pieces of an electromagnet shown as N-S such that the meniscus of the

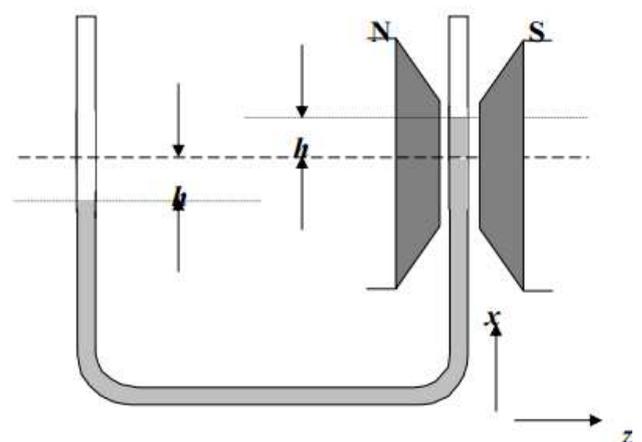


Fig. 1: Schematics of the set up

liquid lies symmetrically between N-S. The length of the limb is sufficient as to keep the other lower extreme end of this limb well outside the field H of the magnet. The rise or fall h is measured by means of a traveling microscope of least count of the order of 10^{-3} cm. The picture of the actual set up is given in Fig. 2.



Fig. 2 Experimental set up for Quincke's method

Procedure

1. Prepare the FeCl_3 solution of known mass (5-10gm) in 50 ml water.
2. Calculate the number of moles of Fe^{3+} ions per unit volume of the solution. 1 mole of a substance has a weight in grams equal to its molar weight, W_m . The molecular weight is found by adding up the atomic weights of the constituent atoms of the molecule. If X grams of FeCl_3 were dissolved in V m³ of the solution, the number of moles is X/W_m . Each mole contains N_A (Avagadro's number) of molecules. Thus the number of molecules in V m³ is

$$N = N_A X / W_m$$

3. Measure the density ρ of your solution using a specific gravity bottle. The method here is to (a) weigh the bottle + stopper when it is dry and empty, let it be w_1 . (b) fill it with distilled water and weigh it again, let it be w_2 . (c) dry it with a dryer and fill it with your solution and weigh it again. Let the weight be w_3 . The density ρ may be found, knowing the density of water ρ_{water} , from the following equation

$$\rho = \rho_{\text{water}} \frac{w_3 - w_1}{w_2 - w_1} \quad (16)$$

4. Adjust the pole pieces so that the gap between them is about 10 mm (Diameter of the U-tube is about 8mm).
5. Connect the electromagnet coils in series to the power supply and ammeter. The field between the pole pieces must be calibrated as a function of current over an appropriate range (1- 4A). The magnet may run continuously with a current of 5A (for precautions, we would avoid prolonged use at 5A, hence the range up to 4A) and for short periods with 10A. The Hall probe will be used to measure the magnetic field B (how does this work?). Position the Hall probe using the stand provided so that the same position is maintained throughout the calibration. With the U-tube removed, insert the Hall probe into the field region between the flats of the pole pieces.
6. Switch on the Gauss meter and rotate the zero adjustment knob till you get zero reading on it. Now, switch on the power supply and adjust the current at 1A. Adjust the probe's position and orientation until it registers a maximum positive field. Clamp the probe handle firmly in place so that it cannot move. Measure the flux density B . Slowly increase the current (I) in small steps and record corresponding values of B .
7. If you record your calibration data with sufficiently small increments of current this will provide the best definition of the entire curve, which will be linear in a certain range for smaller values of current and then the slope will decrease as magnetic saturation occurs in the material of the pole pieces. Note there may also be some magnetic hysteresis present and for a given current, the field may be slightly different, depending on whether the current is increasing or decreasing. The magnetic saturation means that the highest values of current do not produce an equivalent increase in the values of the magnetic field.
8. Bring the current in the power supply back to zero and switch off supply after you finish calibration.
9. Transfer some of the prepared solution to the U- tube so that the meniscus is at the centre of the pole-pieces. Focus the travelling microscope on the meniscus and note down the initial reading for $B = 0$.
10. Switch on the supply and slowly vary the current up to 4A in steps of 0.5A. The solution in the tube rises up. Note down the corresponding height of the liquid column for each value of current.

Observations

Table 1: Data for calibration

Sl#	I (A)	B (Gauss)

Table 2: Measurement of ρ

Wt. of empty specific gravity bottle (w_1) = ..

Wt. of specific gravity bottle filled with distilled water (w_2) = ..

Wt. of specific gravity bottle filled with test liquid (w_3) = ..

$$\rho = \rho_{water} \frac{w_3 - w_1}{w_2 - w_1}$$

Use $\rho_{water} = 1000 \text{ kg/m}^3$

Table 2. Measurement of $h \sim B$

Least count of travelling microscope =

For $B=0$, using travelling microscope,

Initial position of meniscus (a) = Main scale (M.S.) + Vernier Scale (V.S.)

Sl No	I	B	B^2	Meniscus Reading $B \neq 0$ (b)			Difference $h = (b-a)$
				M.S	V.S	T.R.	

Graph:

Plot a graph $h \sim B^2$ and do straight line fitting to determine slope.

Calculations: Use $\chi_a \sim 0$ and $\rho_a = 1.29 \text{ kg/m}^3$

Volume susceptibility, $\chi_{water} = -9.04 \times 10^{-6}$

$\chi_{Fe} = \dots$ $\chi_{Fe}' = \dots$ $\chi_{Fe}'' = \dots$ $C = \dots$ $\mu = \dots$

Conclusion and discussion:

Precautions:

1. Scrupulous cleanliness of the U-tube is essential. Thoroughly clean the tube and rinse it well with distilled water before starting and dry it.
2. Make several sets of measurements to ensure consistency; false readings can arise from liquid running down the tube or sticking to the sides.
3. Carefully swab down the inside of the U- tube with a cotton bud, to ensure that there are no droplets of liquid which might interfere with the plastic spacers on the rod which measures the height of the meniscus.
4. Do not use the U-tube for longer than one laboratory period without recleaning. After cleaning ask the laboratory technician to dry the tube for you with compressed air.
5. Try to avoid the backlash error of the travelling microscope. The small change of height may cause you more error in the calculation.

References

[1] I. S. Grant and W.R. Phillips, "*Electromagnetism*", (Wiley)

[2]<http://www.agoenvironmental.com/sites/default/files/pdf/diamagnetic%20element%20list.pdf>

Appendix A: Magnetic moment values

The magnetic susceptibility of a substance is related to the magnetic dipole moments of its individual atoms or ions. The total angular momentum of an atom or ion arises from both the orbital motion and the spin of the electrons. The magnetic dipole moment can be expressed in the form

$$m = p\mu_B,$$

where p , the magneton number, is the dipole moment in units of the quantity μ_B , which is known as the Bohr magneton. The Bohr magneton is the atomic unit of magnetic moment defined by,

$$\mu_B = eh / 4\pi m_e$$

where, in this equation, e and m_e are the electronic charge and mass and h is Planck's constant. The dimensionless magneton number p is usually between 1 and 10 for atomic systems.

The rules for calculating p can be summarised as follows,

- (i) the unfilled electron shells for any atom or ion can be found in standard tables.
- (ii) the quantum numbers of the individual electrons can be added

$$L = \sum_i l_i \text{ and } S = \sum_i s_i$$

to give the largest values of L and S consistent with the Pauli Exclusion Principle

- (iii) the total quantum number J can be found from

$$J = L - S \text{ first half of the electron shell}$$

$$J = L + S \text{ second half of the electron shell}$$

- (iv) the magneton number p is given by,

$$p = g\sqrt{J(J+1)} \quad \text{where } g \text{ is the Landé splitting factor}$$

$$g = \left\{ \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} \right\}$$

g takes into account that the spin effectively creates twice as much magnetic moment as the orbital motion.

(v) the result of these calculations are tabulated in most textbooks on condensed matter physics, See the Table 1.

Table 1 Magneton numbers p for some transition metals (TM²⁺ free ions)

No of electrons in 3d shell	Ion	S	L	J	P
0	Ca ²⁺	0	0	0	0
1	Sc ²⁺	1/2	2	3/2	1.55
2	Ti ²⁺	1	3	2	1.63
3	V ²⁺	3/2	3	3/2	0.77
4	Cr ²⁺	2	2	0	0
5	Mn ²⁺	5/2	0	5/2	5.92
6	Fe ²⁺	2	2	4	6.71
7	Fe ³⁺	5/2	0	5/2	5.92
8	Co ²⁺	3/2	3	9/2	6.63
9	Ni ²⁺	1	3	4	5.59
10	Cu ²⁺	1/2	2	5/2	3.55
11	Zn ²⁺	0	0	0	0