

Determination of Young's modulus of material by Cornu's apparatus

Objective

To determine Young's modulus and Poisson's ratio of a glass plate using Cornu's method.

Theoretical Background

Young's modulus, also known as modulus of elasticity is an important characteristic of a material and is defined to be ratio of longitudinal stress and longitudinal strain and is given by

$$Y = \frac{F.L_0}{A.\Delta L} \quad \dots (1)$$

Which has unit of Pressure (Pascal) and F , A , ΔL and L_0 are force, area, extension and initial length respectively. Young's modulus can be used to predict elongation or compression of an object as long as the stress is less than the yield strength of the material. Another important elastic constant is Poisson's ratio. When a sample of material is stretched in one direction it tends to get thinner in the other two directions. Poisson's ratio is a measure of this tendency and is defined as the ratio of the strain in the direction of applied load to the strain in the transverse direction. A perfectly incompressible material has Poisson's ratio $\sigma = 0.5$. Most practical engineering materials have $0 \leq \sigma \leq 0.5$. For example, Poisson's ratio for cork, steel and rubber is 0, 0.3 and 0.5 respectively. Polymer foams have negative Poisson's ratio, when it is stretched it gets thicker in other direction.

In an elegant experiment, Marie Alfred Cornu in the year 1869 first showed that the interference phenomenon in optics could be used for measuring deformation of a solid under load. At that time, it was very interesting to find that counting of interference fringes could provide information about Young's modulus and Poisson's ratio for a transparent material. The method proposed by Cornu employs a glass plate placed on top

of a glass beam. When load is applied on both the sides of the glass beam, it gets deformed due to strain along the longitudinal direction (X-axis). Since Poisson's ratio $\sigma \neq 0$, the glass beam will bend in the transverse direction (Y-axis). Thus the beam deforms into the shape of horse saddle forming a thin film of air between them. When the film is illuminated by monochromatic light, interference occurs between the light reflected from the bottom of the glass plate and the top of the beam as shown in Fig. 1.

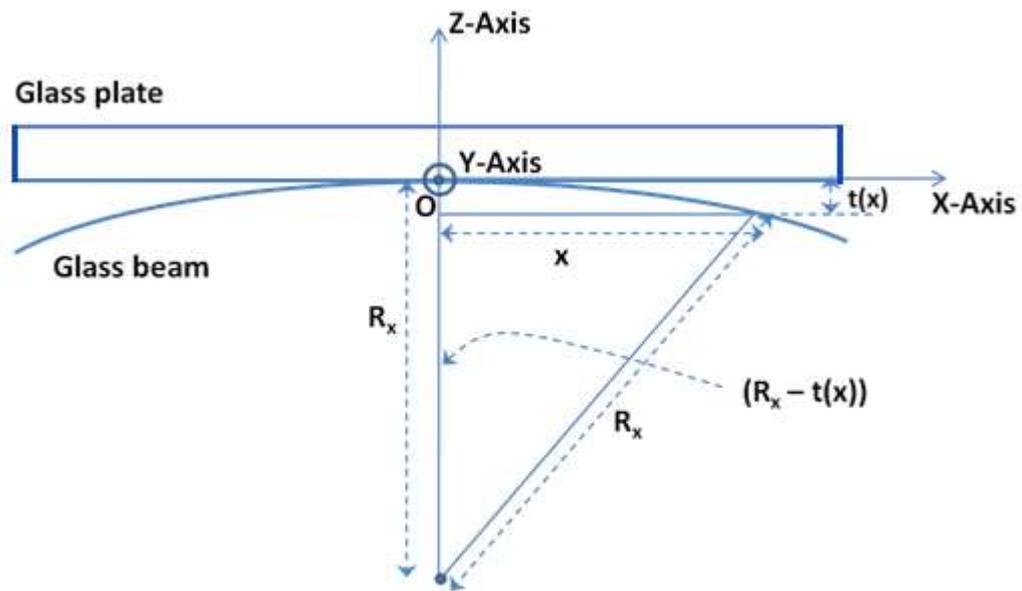


Fig. 1 Geometry for obtaining interference fringes

Let 'x' and 'y' represent coordinates along longitudinal and transverse direction with the middle point being the origin (O). Also, let R_x and R_y be the radius of curvature in longitudinal (X) and transverse (Y) directions respectively. In order to obtain the shape of the interference fringes, consider that the thickness of air film between the glass plate and the beam to be ' $t(x,y)$ ' at appoint (x,y) in the XY-plane. First, let us consider only the X-dependence of air film i.e. $t \equiv t(x)$. The width ' $t(x)$ ' of the air film inside the glass beam and the X-axis through the origin at a coordinate 'x' along X-axis can be obtained from

$$(R_x - t(x))^2 = R_x^2 - x^2 \quad \dots (2)$$

Assuming ' $t(x)$ ' to be very small we can solve it to get

$$t(x) = \frac{x^2}{2R_x} \quad \dots (3)$$

Similarly, the width $t(y)$ of the air film inside the glass beam and the XY-plane through the origin 'O' at a coordinate 'y' along Y-axis can be obtained from

$$t(y) = -\frac{y^2}{2R_y} \quad \dots (4)$$

It is to be noted that the sign is negative because along Y-axis the glass beams bends upward. Therefore width of air film between parallel plate and glass beam at a coordinate (x, y) is given by

$$t(x, y) = t(x) + t(y) = \frac{x^2}{2R_x} - \frac{y^2}{2R_y} \quad \dots (5)$$

The shapes of the fringes are determined by the locus of all points that have identical path difference. In the present case, the path difference will be identical for points with a constant value of thickness ' $t(x, y)$ '. Thus the shape of the fringe will be given by,

$$\frac{x^2}{2R_x} - \frac{y^2}{2R_y} = a^2 \quad \dots (6)$$

where 'a' is a constant and this is an equation of hyperbola. Therefore, the fringes will be hyperbolic.

It is important to note that the light waves passing through glass plate will be divided into two parts. One component would comprise the reflection from the bottom of the glass plate-air interface and the second one would be from the top of air film-glass beam interface. These two components would interfere and produce the fringe pattern. The latter one would undergo a phase change of π because of reflection at air film-glass beam interface. Also, this component traverses the width of the air film twice; therefore the optical phase difference between these two waves (for almost normal incidence) is given by,

$$\Delta\phi = \frac{2\pi}{\lambda} [2\mu(t(x, y))] + \pi \quad \dots (7)$$

Where ' μ ' is the refractive index of the film, λ is the free-space wavelength.

Let us consider the fringes along the X-axis and take into account that the air-film has a refractive index $\mu = 1$. If the distance of N -th dark fringe from the origin is x_N , then the interfering waves are essentially out-of-phase i.e.

$$\Delta\varphi = (2N + 1)\pi \quad \dots (8)$$

$$2t_N(x) = \frac{x_N^2}{R_x} = N\lambda \quad \dots (9)$$

It is to be noted that in the case of grating this is precisely the condition for bright fringes.

Therefore, if x_{N+s} is the distance of $(N+s)$ -th dark fringe (along X-axis), we get

$$2t_{N+s}(x) = \frac{x_{N+s}^2}{R_x} = (N + s)\lambda \quad \dots (10)$$

Subtracting Eq. (9) from Eq. (10), we get

$$R_x = \frac{x_{N+s}^2 - x_N^2}{s\lambda} \quad \dots (11)$$

For convenience, we define

$$\rho_x(s) = x_{N+s}^2 - x_N^2 \quad \dots (12)$$

Thus, measuring the distance of different fringes from the origin, squaring them and subtracting we get the radius of curvature of the bent beam along X direction. Since, it is difficult to find the origin it is convenient to measure the ‘*diameter*’ (D) of the fringe which is related by $D_{N_x} = 2x_N$ and is the distance between N -th dark fringe on left side of the origin and the N -th dark fringe on right side of the origin.

Once we obtain the radius of curvature along X-direction we can calculate the bending moment from it. This is given by the following relation

$$G_x = Y \cdot \frac{bd^3}{12} \frac{1}{R_x} \quad \dots (13)$$

where ‘ b ’ and ‘ d ’ are the width and thickness of the glass beam respectively while ‘ Y ’ is the Young’s modulus. The factors involving ‘ b ’ and ‘ d ’ comes from the moment of inertia of the glass beam about an axis which is at a distance of ‘ R_x ’ from the origin ‘ O ’ (see Fig. 1) and parallel to Y-axis. This internal bending moment should be equal to the external bending moment applied by the loads hanging from the glass beam. If l is the distance between the knife-edge (the points where the glass beam is supported to the

base) and the suspension point of the load $W (= mg)$ then $G_x = W.l$ and therefore we can have,

$$W.l = Y \cdot \frac{bd^3}{12} \frac{s\lambda}{\rho_x(s)} \quad \dots (14)$$

If we carry out the measurement for two different loads, then we obtain

$$(m_1 - m_2)gl = Y \frac{bd^3}{12} s\lambda \left(\frac{1}{\rho_x^1(s)} - \frac{1}{\rho_x^2(s)} \right) \quad \dots (15)$$

Equation (15) could be used for calculation of Young's modulus.

In order to calculate the Poisson's ratio, it is required to obtain the ratio of radius of curvature in the longitudinal direction to that in the transverse direction. In analogy with the argument leading to Eq. (11), we can obtain R_y by counting fringes along the Y-direction as,

$$R_y = \frac{y_{N+s}^2 - y_N^2}{s\lambda} \quad \dots (16)$$

where y_N is the distance of the N -th dark fringe from the center along Y-axis. Also, $\rho_y(s)$ is defined as

$$\rho_y(s) = y_{N+s}^2 - y_N^2$$

Therefore Poisson's ratio is given by

$$\sigma = \frac{R_x}{R_y} = \frac{x_{N+s}^2 - x_N^2}{y_{N+s}^2 - y_N^2} \quad \dots (17)$$

Apparatus

1. Optically plane glass beam
2. Cornu's apparatus (Stage microscope fitted with XY movement micrometers and a vertical stand fitted with adjustable knife edges)
3. Sodium lamp with Power supply
4. A square shaped glass plate
5. Slide caliper and screw gauge
6. Hangers, loads etc.

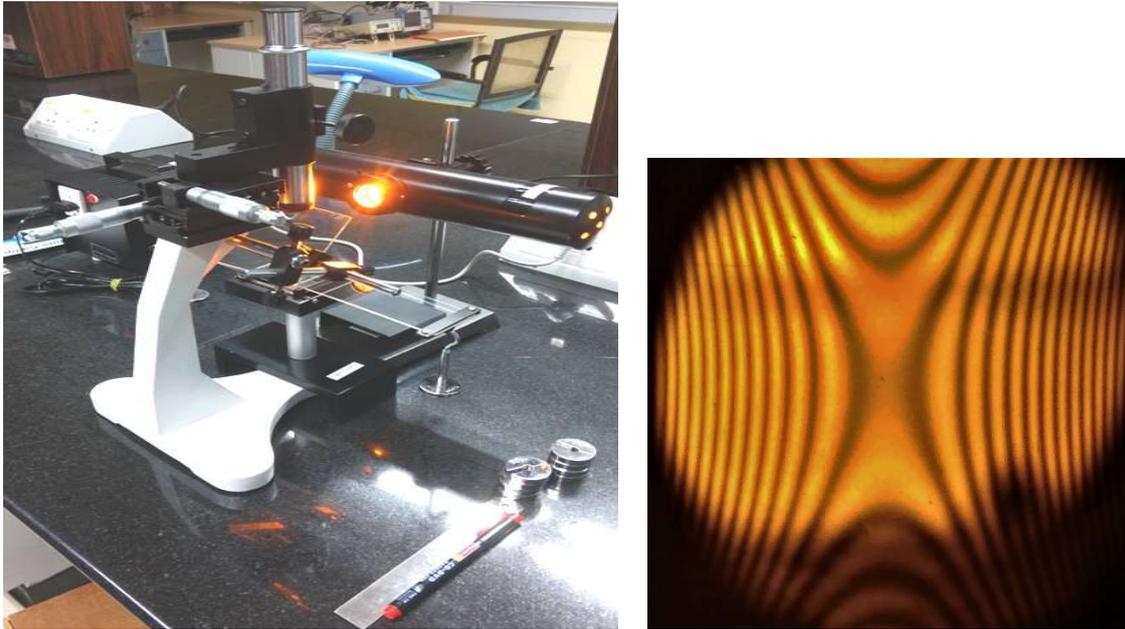


Fig. 2: Experimental set up and hyperbolic fringes observed through microscope

Procedure:

1. Measure the width and the depth of the glass beam using vernier caliper and screw gauge. Take at least three readings for avoiding any error.
2. The experimental set up is shown in Fig. 2. Place the glass beam on two knife-edges. Measure the distance between the knife-edge and point of suspension on either side. Hang the load (say 250 gm) with hanger on both sides.
3. Place the plane glass plate on the beam near the middle. Adjust the beam and glass plate so that the fringes can be observed as shown in Fig. 2.
4. Focus the microscope and adjust the beam and plate so that the fringes are symmetrical on both sides of horizontal cross-wire and tangential to the vertical cross-wire.
5. Turn the micrometer screw attached with microscope in longitudinal direction (along X) of every transverse fringe on both sides to measure longitudinal position (x). If necessary reduce the aperture size fitted with sodium lamp housing to improve contrast while taking reading.

6. Adjust the fringe position and micrometer in such a way that at least 8-10 fringes are visible on either side from center. Take readings for about 10 fringes on each sides of the center. To avoid backlash error start from one extreme position (say 10th fringe on the left or right side) from center.
7. Similarly measure transverse position (y) of longitudinal fringes by moving microscope in transverse direction.
8. Repeat the steps from 5-7 for another weight.
9. Plot an appropriate graph and determine values of Y and σ .

Observations:

Least Count of the Micrometer = _____

$m_1 = 250 \text{ g}$

Along X-

Order of the fringe	Fringes on the left (x)			Fringes on the right (x)						R_x (cm)
	MSR (cm)	CSR (cm)	Total (cm)	MSR (cm)	CSR (cm)	Total (cm)	D (cm)	D^2 (cm ²)	ρ_x (cm ²)	

Along Y-

Order of the fringe	Fringes towards top(y)			Fringes towards bottom (y)						R_y (cm)
	MSR (cm)	CSR (cm)	Total (cm)	MSR (cm)	CSR (cm)	Total (cm)	D (cm)	D^2 (cm ²)	ρ_y (cm ²)	

Calculations:

Calculate Y and σ and do the error analysis. Compare your results with the literature values.

Literature Value of Y for Glass = 50-90 GPa

Literature Value of σ for Glass = 0.2- 0.7

Precautions:

1. Handle the components carefully and make sure that loads $> 400\text{g}$ are not exerted on the Glass beams respectively.
2. Make sure that you get regular shaped fringes. Adjust the glass plate slowly to change the shape of fringes from any irregular pattern.
3. Be careful about backlash error while taking the readings.

References:

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