Diffraction of light using single and double slit

Objective

• To determine the wavelength of laser light from single-slit diffraction pattern.

• To determine the thickness of a fine wire from its diffraction pattern.

• Compare the thickness of the wire with the single-slit width that forms the same diffraction pattern as the wire and hence verify Babinet's principle.

• To understand the diffraction pattern due to the double slit and determine the slit width and the width of the opaque gap between the two slits.

Theoretical background[1]

When light passes through a small opening or around an edge, secondary waves from different portions of the emerging wavefront will, in general, travel different distances before reaching a screen. Although the waves from secondary sources are all in phase to start with, they will be out of phase by the time they reach the screen. The interference of these secondary wavefronts lead to the phenomenon of diffraction. We will study only Fraunhofer diffraction, where the light source, screen and the object causing diffraction are effectively at infinite distances from each other.

Single-slit diffraction

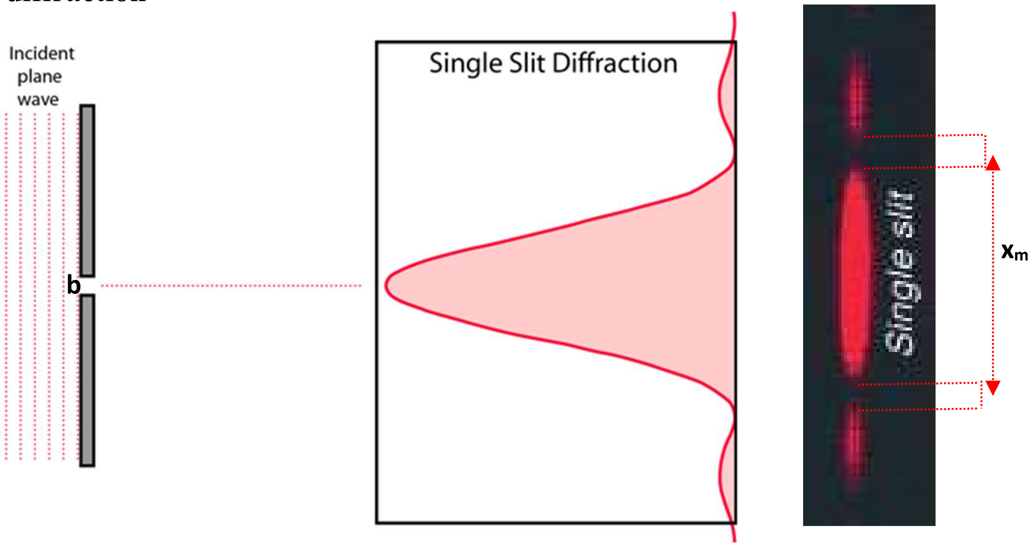

Figure 1. Schematics for single-slit diffraction. Distance between minima x_m is calculated from the average minima position on either side of principal maxima.

When a light of wavelength λ is incident normally on a narrow slit of width b as shown in Fig.1, the resultant intensity of the transmitted light is given by,

$$\boldsymbol{I = I_0 \frac{\sin^2 \beta}{\beta^2}}, \textbf{ with } \boldsymbol{\beta = \frac{\pi b \ \sin \theta}{\lambda}} \qquad (1)$$

where, θ being the angle of diffraction. The diffraction pattern consists of a principal maximum for β = 0, where all the secondary wavelets arrive in phase, and several secondary maxima of diminishing intensity with equally spaced points of zero intensity at β = mπ. The positions of the minima of a single-slit diffraction pattern are,

mλ = b sin θ, m = ±1, ±2, ±3, ...

$$\boldsymbol{m\lambda = b \sin \theta}, \qquad \textbf{m = ±1, ±2, ±3} \qquad (2)$$

If θ is small i.e. the slit to screen distance D is large compared to the distance x_m between two m-th order minima (on either side of principal maximum), then

$$\boldsymbol{\sin \theta \approx \theta = \frac{x_m}{2D} \Rightarrow m\lambda = \frac{b x_m}{2D}} \qquad (3)$$

The above equation (3) can be used to determine the wavelength of the monochromatic light source, laser in present case, by measuring b, D and x_m for various m. The positions of the minima can be obtained by averaging the two extremities of the zero intensity region, as shown in Fig. 1. A real photographic image of the pattern is shown in Fig.2.

Figure 2. Photographic image of a single-slit diffraction pattern

Diffraction of a thin wire

If the single-slit is replaced by a thin wire obstacle, which blocks as much laser light as a single-slit will allow to pass, the resulting diffraction pattern will be identical to that of a single-slit. Knowing the wavelength λ of the laser light, the equation (3) can be used to determine the thickness of the wire b as,

$$\boldsymbol{b = \frac{2m\lambda D}{x_m}} \qquad (4)$$

A typical diffraction pattern of a wire obstacle is shown in Fig.3. Here too, the positions of the minima are calculated by averaging the two ends of the spread of zero intensity regions as shown in Fig. 1.

Figure 3. **Photographic image of diffraction pattern from a thin wire**– similarity with single-slit pattern is what Babinet's principle asserts.

Babinet's principle[2]: One of the implications of Babinet's principle is that the locations of maxima and minima in the diffraction patterns of an opaque body are identical to that of an aperture of the same size and shape. This principle can be verified by replacing once again the wire with a single- slit and varying the slit-width until the pattern matches exactly. The slit width can then be compared with the wire thickness.

Double-slit diffraction and interference

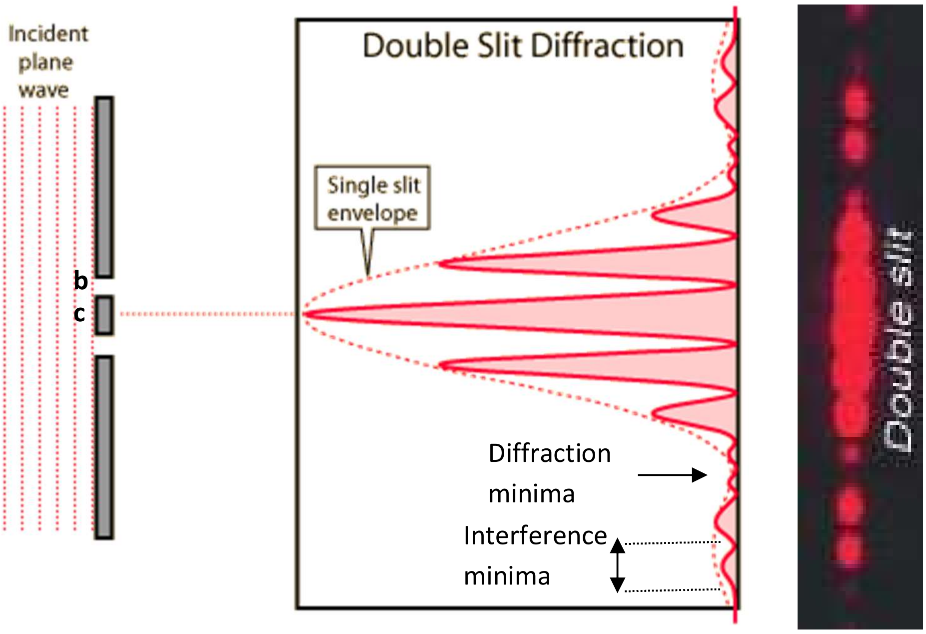

Figure 4. Schematics for Double-slit diffraction

If instead of single-slit, we have two parallel slits each of width b separated by an opaque space of width c, the corresponding intensity distribution of the Fraunhofer pattern formed is (see Fig.4) given as,

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma \qquad (5)$$

where θ being the angle of diffraction,

$$\beta = \frac{\pi b \sin\theta}{\lambda}, \qquad \gamma = \frac{\pi d \sin\theta}{\lambda}, \qquad d = b + c, \tag{6}$$

The intensity distribution is a product of two terms: the first term ($\sin^2 \beta/\beta^2$) represents the diffraction pattern produced by a single-slit (eqn.1) and the second term $\cos^2 \gamma$ is characteristic of interference produced by two beams of equal intensity and phase difference γ. The overall pattern, therefore, consists of single-slit diffraction fringes each broken into narrow maxima and minima of interference fringes. This interference of light from two narrow slits close together was first demonstrated by Thomas Young in 1801 and helped establish the wave nature of light.

The minima for the interference fringes are at $\gamma = (2p + 1)\pi/2$ with p = 0, 1, 2, . . . and those for diffraction fringes are at $\beta = m\pi$ where m = 1, 2, 3, ... The conditions for minima are,

$$d \sin\theta = \left(p + \frac{1}{2}\right) \lambda \tag{7}$$

$$b \sin\theta = m\lambda \tag{8}$$

Figure 5. Photographic image of double-slit diffraction pattern – each diffraction maxima is broken up into interference fringes

A photographic image of the double-slit Fraunhofer pattern obtained with a laser beam is shown in Fig 5. The intensity of the laser may render viewing the pattern difficult without photographing. It is evident from the Fig 5 that the positions of interference and diffraction minima hardly show any spread at all, hence there would be no need to average for x_p and x_m. Further, in this case both x_p and x_m are distances from the central principal maxima (and not the distance between two p-th or m-th order minima on either side of principal maxima). Another problem here might be to determine and distinguish order numbers p for interference and m for diffraction. It would, therefore, be better to consider differences in positions between n consecutive minima, $\Delta x_p = x_{p+n} - x_p$ and $\Delta x_m = x_{m+n} - x_m$. Assuming as before, the distance D of the screen from the double-slit is large compared to the x_p and x_m, we can approximate $\sin\theta \approx \theta = x_p/D$ or x_m/D and use the equations (7) and (8) to determine both the b and d as,

$$\sin\theta \approx \theta = \frac{x_p}{D} \implies \boxed{d = \frac{n\,\lambda\,D}{\Delta x_p}} \quad (9)$$

$$\sin\theta \approx \theta = \frac{x_m}{D} \implies \boxed{b = \frac{n\,\lambda\,D}{\Delta x_m}} \quad (10)$$

Apparatus

Laser source (and safety goggles), screen & ruled-paper for measurement, thin-wire, variable single-slit and double-slit, measuring tape, travelling microscope and (if available) digital camera

Figure 6. Experimental set up for laser diffraction

Procedure

WARNING: The laser beam can cause real damage to your eyes if you look into the beam either directly or by reflection from shiny objects.

1. Determine the vernier constant of the travelling microscope and measure the thickness b of the wire, slit-width b for single and double slit, and slit plus opaque space d = b + c of double-slit.

2. Arrange the screen at least 2 meter away from the laser source. On the screen, attach a ruled-paper with clips such that the ruled scale is horizontal. You may use graph paper in place of ruled-paper, if you consider it convenient.

3. Turn the laser on and be extremely careful not to let your eyes in the direct or reflected line of the laser. Do not turn the laser off and on too frequently; instead use something to block the laser when it is not in use.

4. Adjust the height of the laser (and also the screen) such that the laser spot is directly on the ruled line in the middle of the paper.

5. To record the pattern that will be produced on the screen, you can do either or both of the following:

(a) For the single-slit and wire, on the ruled paper mark off the two extremities of the dark region $\alpha_1^{l,r}$ and $\alpha_2^{l,r}$, where l, r indicate on which side of principal maxima the minima belong. To obtain the positions of the minima, do the averaging $\alpha^{l,r} = (\alpha_1^{l,r} + \alpha_2^{l,r})/2$.

(b) Photograph the pattern using a digital camera with flash off and set in "Auto" mode. To shoot, keep the camera at about the same height as the pattern but taking care not to interfere with the pattern formation. Transfer the image to a computer and read off the extrema positions as above.

6. First place the single-slit apparatus close in front of the laser and observe the diffraction pattern on the screen. Adjust the laser and slit so as to obtain a bright, crisp pattern. Adjust the slit-width such that at least 10 to 12 closely spaced maxima and minima are obtained. Record the observation either directly by marking off or photographing as explained above. Measure the slit to the screen distance D with a measuring tape. Using the data and equation (3), calculate the wavelength of the laser from an appropriate graph.

7. Next replace the single-slit with a thin wire and adjust the position of the wire to get a sharp diffraction pattern on the screen. Record the observation along with the wire-screen distance as before. Using the data and equation (4), calculate the thickness of the wire from an appropriate graph. You may use the supplied value of wavelength of the laser for this calculation. Compare the calculated value with that obtained using the travelling microscope.

8. To study Babinet's principle, bring back the single-slit in place of the wire and adjust the slit-width until the observed diffraction pattern is the same as the wire. Measure the slit width using the travelling microscope and verify that its value is almost the same as the thickness of the wire. No graph required for this part of the observation.

9. To explore the double-slit pattern, proceed exactly the same way as single-slit, but this time around it may be difficult to mark off the diffraction minima directly on the screen although the interference minima are fairly easy to spot. You may take a photograph of the pattern and mark positions later. Do not forget to measure D. Using appropriate graphs, calculate d and b using equations (9) and (10) respectively and verify their values against those measured directly. Determine the value of c.

Observations and results

Least count for travelling microscope = ……

Table I. Determination of the single-slit width and wire thickness using travelling microscope

Object	Obs	Left edge				Right edge				b (m)	Mean b (m)
		Main Scale (M)	Vernier (V)	T = M + VCxV	Mean T	Main Scale (M)	Vernier (V)	T = M + VCxV	Mean T		
Single slit	1 2 3				α_l =				α_r=	$\alpha_l \sim \alpha_r$	
Wire	1 2 3				α_l=				α_r=	$\alpha_l \sim \alpha_r$	

Table-II. Determination of wavelength of the laser light using diffraction pattern

Slit-width b =…….   Slit screen distance D=……

Order m	Left fringes			Right fringes			$x_m = \alpha^l \sim \alpha^r$ (cm)
	α_1^l (cm)	α_2^l (cm)	$\alpha^l =$ (cm)	α_1^r (cm)	α_2^r (cm)	$\alpha^r =$ (cm)	
1 2 3 ..							

From graph, λ = …..

Table –III. Thickness of the thin wire b using diffraction pattern

Wavelength =……….. wire-screen distance D=…………

Order m	Left fringes			Right fringes			$x_m = \alpha^l \sim \alpha^r$ (cm)
	α_1^l (cm)	α_2^l (cm)	$\alpha^l =$ (cm)	α_1^r (cm)	α_2^r (cm)	$\alpha^r =$ (cm)	
1 2 3 ..							

From graph, b = …..

Table-IV. Verifying Babinet's principle-comparing slit-width and wire thickness

Wavelength =……. Slit-screen distance D=……..

Order m	Left fringes			Right fringes			$x_m = \alpha^l \sim \alpha^r$ (cm)	slit b (nm)	Average slit b (nm)	Wire b (cm)
	α_1^l (cm)	α_2^l (cm)	$\alpha^l =$ (cm)	α_1^r (cm)	α_2^r (cm)	$\alpha^r =$ (cm)				
1 2 3										

Table V: Determination of b and d using travelling microscope

Object	Obs	Left edge: Main scale (M)	Left edge: Vernier (V)	Left edge: T=M+VC×V	Left edge: Mean T	Right edge: Main scale (M)	Right edge: Vernier (V)	Right edge: T=M+VC×V	Right edge: Mean T	Width (cm)	Mean width (cm)
slit-1	1	⋯	⋯	⋯		⋯	⋯	⋯		$b_1 =$	$b =$
	2	⋯	⋯	⋯	$\alpha_l =$	⋯	⋯	⋯	$\alpha_r =$	$\alpha_l \sim \alpha_r$	
slit-2	1	⋯	⋯	⋯		⋯	⋯	⋯		$b_2 =$	
	2	⋯	⋯	⋯	$\alpha_l =$	⋯	⋯	⋯	$\alpha_r =$	$\alpha_l \sim \alpha_r$	
slit-1 + wire	1	⋯	⋯	⋯		⋯	⋯	⋯		$d_1 =$	$d =$
	2	⋯	⋯	⋯	$\alpha_l =$	⋯	⋯	⋯	$\alpha_r =$	$\alpha_l \sim \alpha_r$	
slit-2 + + wire	1	⋯	⋯	⋯		⋯	⋯	⋯		$d_2 =$	
	2	⋯	⋯	⋯	$\alpha_l =$	⋯	⋯	⋯	$\alpha_r =$	$\alpha_l \sim \alpha_r$	

c = d – b = ….

Table VI: Determination of b and d using diffraction pattern

Wavelength =……. Slit screen distance D=……..

Interference Order $p+n$	Left fringes x_p (cm)	Δx_p (cm)	d (cm)	Right fringes x_p (cm)	Δx_p (cm)	d (cm)	Average d (cm)
p	…			…			
$p+1$	…	…	…	…	…	…	
$p+2$	…	…	…	…	…	…	…
⋮	…	…	…	…	…	…	

Diffraction Order $m+n$	x_m (cm)	Δx_m (cm)	b (cm)	x_m (cm)	Δx_m (cm)	b (cm)	Average b (cm)
m	…			…			
$m+1$	…	…	…	…	…	…	
$m+2$	…	…	…	…	…	…	…
⋮	…	…	…	…	…	…	

c = d – b = ….

Questions:

1.What are the phenomena of interference and diffraction?

2. In what way interference and diffraction differ?

3. What would you expect if ordinary sodium lamp (supposing it to be monochromatic) is used instead of laser? What if white light is used?

4. Why the positions for minima are measured instead of maxima in the cases of single-slit, wire and double-slit pattern? [Hint: The secondary maxima are not located precisely halfway between the minima.]

5. What is missing order? Do you expect to get one in the present experimental setup? Explain yourself.

References:

1. Fundamentals of Optics, Jenkins & White
2. http://web.mit.edu/8.03-esg/watkins/8.03/Babinet.pdf