

## Measurement of acceleration due to gravity (g) by a compound pendulum

**Aim:** (i) To determine the acceleration due to gravity (g) by means of a compound pendulum.

(ii) To determine radius of gyration about an axis through the center of gravity for the compound pendulum.

**Apparatus and Accessories:** (i) A bar pendulum, (ii) a knife-edge with a platform, (iii) a spirit level, (iv) a precision stop watch, (v) a meter scale and (vi) a telescope.

**Theory:**

A simple pendulum consists of a small body called a “bob” (usually a sphere) attached to the end of a string the length of which is great compared with the dimensions of the bob and the mass of which is negligible in comparison with that of the bob. Under these conditions the mass of the bob may be regarded as concentrated at its center of gravity, and the length of the pendulum is the distance of this point from the axis of suspension. When the dimensions of the suspended body are not negligible in comparison with the distance from the axis of suspension to the center of gravity, the pendulum is called a compound, or physical, pendulum. A rigid body mounted upon a horizontal axis so as to vibrate under the force of gravity is a compound pendulum.

In Fig.1 a body of irregular shape is pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle  $\theta$ . In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is  $l$  and the mass of the body is  $m$ . The restoring torque for an angular displacement  $\theta$  is

$$\tau = - mgl \sin\theta \quad \dots(1)$$

For small amplitudes ( $\theta \approx 0$ ),

$$I \frac{d^2\theta}{dt^2} = -mgl\theta, \quad \dots(2)$$

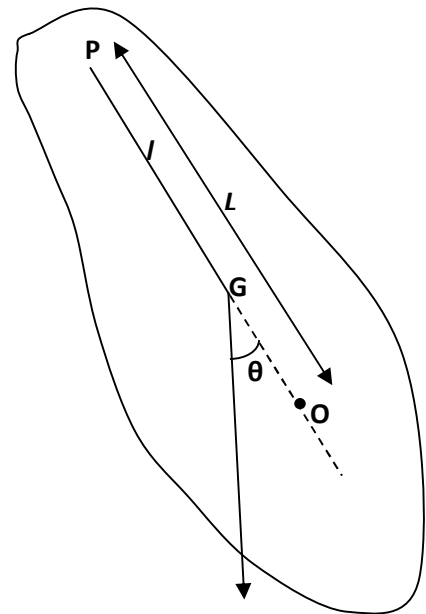
where  $I$  is the moment of inertia of the body through the axis P. Eq. (2) represents a simple harmonic motion and hence the time period of oscillation is given by

$$T = 2\pi \sqrt{\frac{I}{mgl}} \quad \dots(3)$$

Now  $I = I_G + ml^2$ , where  $I_G$  is the moment of inertia of the body about an axis parallel with axis of oscillation and passing through the center of gravity G.

$$I_G = mK^2 \quad \dots(4)$$

where  $K$  is the radius of gyration about the axis passing through G. Thus,



**Fig. 1**

$$T = 2\pi \sqrt{\frac{mK^2 + ml^2}{mgl}} = 2\pi \sqrt{\frac{\frac{K^2}{l} + l}{g}} \quad \dots(5)$$

The time period of a simple pendulum of length  $L$ , is given by

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \dots(6)$$

Comparing with Eq. (5) we get

$$L = l + \frac{K^2}{l} \quad \dots(7)$$

This is the length of “equivalent simple pendulum”. If all the mass of the body were concentrated at a point O (See Fig.1) such that  $OP = \frac{K^2}{l} + l$ , we would have a simple pendulum with the same time period. The point O is called the ‘Centre of Oscillation’. Now from Eq. (7)

$$l^2 - Ll + K^2 = 0 \quad \dots(8)$$

i.e. a quadratic equation in  $l$ . Equation 6 has two roots  $l_1$  and  $l_2$  such that

$$\text{and} \quad \begin{aligned} l_1 + l_2 &= L \\ l_1 l_2 &= K^2 \end{aligned} \quad \dots(9)$$

Thus both  $l_1$  and  $l_2$  are positive. This means that on one side of C.G there are two positions of the centre of suspension about which the time periods are the same. Similarly, there will be a pair of positions of the centre of suspension on the other side of the C.G about which the time periods will be the same. Thus there are four positions of the centers of suspension, two on either side of the C.G, about which the time periods of the pendulum would be the same. The distance between two such positions of the centers of suspension, asymmetrically located on either side of C.G, is the length  $L$  of the simple equivalent pendulum. Thus, if the body was supported on a parallel axis through the point O (see Fig. 1), it would oscillate with the same time period  $T$  as when supported at P. Now it is evident that on either side of G, there are infinite numbers of such pair of points satisfying Eq. (9). If the body is supported by an axis through G, the time period of oscillation would be infinite. From any other axis in the body the time period is given by Eq. (5). From Eq.(6) and (9), the value of  $g$  and  $K$  are given by

$$g = 4\pi^2 \frac{L}{T^2} \quad \dots(10)$$

$$K = \sqrt{l_1 l_2} \quad \dots(11)$$

By determining  $L$ ,  $l_1$  and  $l_2$  graphically for a particular value of  $T$ , the acceleration due to gravity  $g$  at that place and the radius of gyration  $K$  of the compound pendulum can be determined.

## Description:

The bar pendulum consists of a metallic bar of about one-meter long. A series of circular holes each of approximately 5 mm in diameter are made along the length of the bar. The bar is suspended from a horizontal knife-edge passing through any of the holes (Fig. 2). The knife-edge, in turn, is fixed in a platform provided with the screws. By adjusting the rear screw, the platform can be made horizontal.

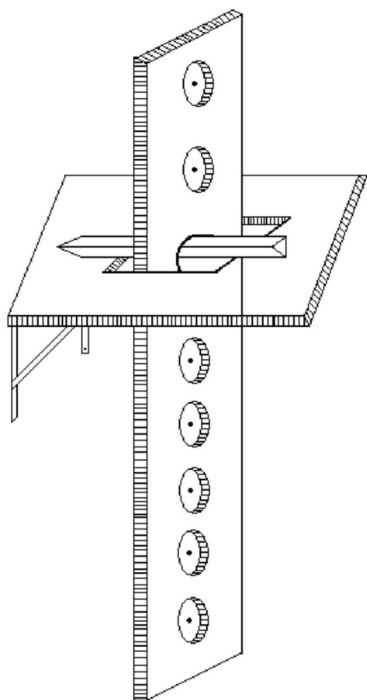


Fig. 2

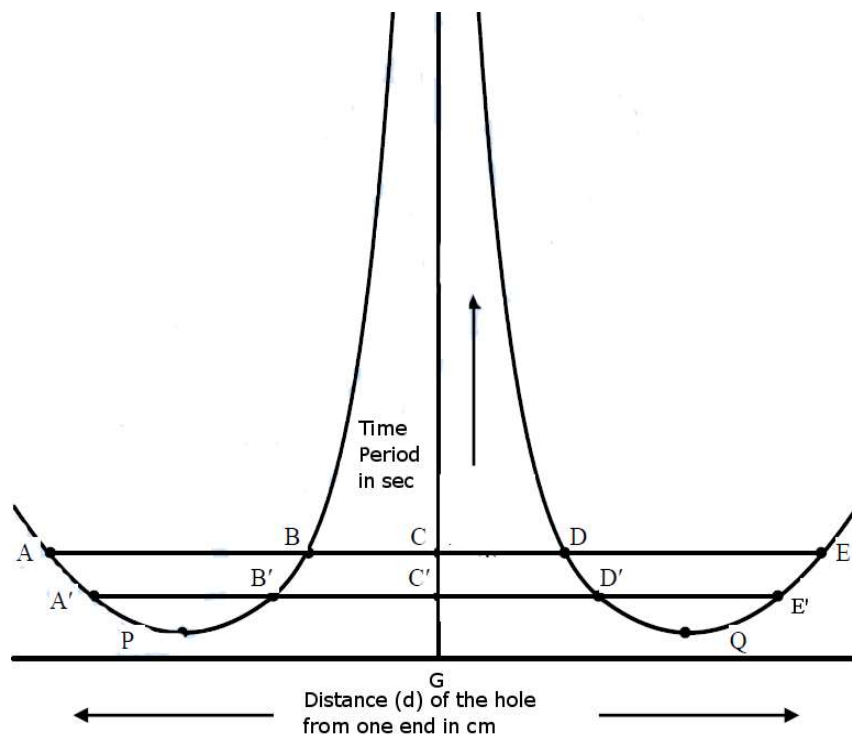


Fig. 3

## Procedure:

- (i) Suspend the bar using the knife edge of the hook through a hole nearest to one end of the bar. With the bar at rest, focus a telescope so that the vertical cross-wire of the telescope is coincident with the vertical mark on the bar.
- (ii) Allow the bar to oscillate in a vertical plane with small amplitude (within  $4^{\circ}$  of arc).
- (iii) Note the time for 20 oscillations by a precision stop-watch by observing the transits of the vertical line on the bar through the telescope. Make this observation three times and find the mean time  $t$  for 20 oscillations. Determine the time period  $T$ .
- (iv) Repeat operation (i) to (iii) for the other holes till C.G. of the bar is approached where the time period becomes very large.
- (v) Invert the bar and repeat operations (i) to (iv) for each hole starting from the extreme top.
- (vi) Draw a graph with the distance  $d$  of the holes as abscissa and the time period  $T$  as ordinate. The nature of graph will be as shown in Fig. 3.

Draw the horizontal line ABCDE parallel to the X-axis. Here A, B, D and E represent the point of intersections of the line with the curves. Note that the curves are symmetrical about a vertical line which meets the X-axis at the point G, which gives the position of the C.G of the bar. This vertical line intersects with the line ABCDE at C. Determine the length AD and BE and find the length L of the equivalent simple pendulum from  $L = \frac{AD+BE}{2} = \frac{L_x}{2}$ .

Find also the time period T corresponding to the line ABCDE and then compute the value of  $g$ . Draw several horizontal lines parallel to X-axis and adopting the above procedure find the value of  $g$  for each horizontal line. Calculate the mean value of  $g$ . Alternatively, for each horizontal line obtain the values of L and T and draw a graph with  $T^2$  as abscissa and L as ordinate. The graph would be a straight line. By taking a convenient point on the graph,  $g$  may be calculated.

Similarly, to calculate the value of  $K$ , determine the length AC, BC or CD, CE of the line ABCDE and compute  $\sqrt{AC \times BC}$  or  $\sqrt{CD \times CE}$ . Repeat the procedure for each horizontal line. Find the mean of all  $K$ .

**Observations: Table 1-Data for the T versus d graph**

Serial no of holes from one end		Distance d of the hole from C.G (cm)	Time for 20 oscillations (sec)	Time period T = t/20 (sec)	
One side of C.G	1	.....	.... .... ....	.....	
	2	.....	... ... ....	.....	
	3	.....	.... .... ...	.....	
	...				
	...				
	...				
	9				
	Other side of C.G	1	.....	.... .... ....	.....
		2	.....	..... ..... .....	.....
3		.....	..... ..... .....	.....	
...					
...					
...					
9					

**TABLE 2- The value of  $g$  and  $K$  from T vs. d graph**

No. of obs.	$L$ (cm)	T (sec)	$g = 4\pi^2 \frac{L}{T^2}$ (cm/sec <sup>2</sup> )	Mean ' $g$ ' (cm/sec <sup>2</sup> )	$K$ (cm)	Mean ' $K$ ' (cm)
1. ABCDE	(AD+BE)/2	..	..		$\sqrt{AC \times BC}$ or $\sqrt{CD \times CE}$	
2.	..	..	..		..	
3.	..	..	..		..	

**Computation of proportional error:**

We have from Eq. (10)

$$g = 4\pi^2 \frac{(L_x/2)}{\left(\frac{t}{20}\right)^2} \quad \dots(12)$$

Since  $L = L_x/2$  ( $L_x = AD+BE$ ) and  $T = t/20$ , therefore, we can calculate the maximum proportional error in the measurement of ' $g$ ' as follows

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta L_x}{L_x}\right)^2 + \left(2 \frac{\delta t}{t}\right)^2} \quad \dots (13)$$

$\delta L_x$  = the value of the smallest division of the X- axis of the plotted graph.

$\delta t$  = the value of the smallest division of the stop-watch

**Precautions and Discussions:**

- (i) Ensure that the pendulum oscillates in a vertical plane and that there is no rotational motion of the pendulum.
- (ii) The amplitude of oscillation should remain within  $4^\circ$  of arc.
- (iii) Use a precision stop-watch and note the time accurately as far as possible.
- (iv) Make sure that there is no air current in the vicinity of the pendulum.

**References:**

1. Fundamentals of Physics: Resnick & Halliday
2. Practical physics: R.K. Shukla, Anchal Srivatsava, New Age International (P) Ltd, New Delhi
3. Eric J. Irons, American Journal of Physics, Vol. 15, Issue 5, pp.426 (1947)