SYLLABUS

Integrated M.Sc. in Mathematics

Applicable from the Academic Year 2017–2018

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

BHUBANESWAR

Course Structure for Integrated M.Sc. in Mathematics

Semester	Course No.		Credits		Course Name
Semester I	M101	-	3	-	Mathematics-I
	CS141	-	2	-	Computation Laboratory-I
	B101	-	3	-	Biology-I
	B141	-	2	-	Biology Laboratory-I
	C101	-	3	-	Chemistry-I
	C141	-	2	-	Chemistry Laboratory-I
	P101	-	3	-	Physics-I
	P141	-	2	-	Physics Laboratory-I
	H109	-	2	-	Technical Communication-I
	H125	-	2	-	Introduction to Psychology
Semester II	M102	-	3	-	Mathematics-II
	CS142	-	2	-	Computation Laboratory-II
	B102	-	3	-	Biology-II
	B142	-	2	-	Biology Laboratory-II
	C102	-	3	-	Chemistry-II
	C142	-	2	-	Chemistry Laboratory-II
	P102	-	3	-	Physics-II
	P142	-	2	-	Physics Laboratory-II
	H110	-	2	-	Technical Communication -II
	H133	-	2	-	Introduction to Sociology
Semester III	M201	-	4	-	Real Analysis
	M202	-	4	-	Group Theory
	M203	-	4	-	Discrete Mathematics
	M207	-	4	-	Number Theory
	****	-	4	-	Elective-I
	****	-	4	-	Elective-II
Semester IV	M204	-	4	-	Metric Spaces
	M205	-	4	-	Linear Algebra
	M206	-	4	-	Probability Theory
	M208	-	4	-	Graph Theory
	****	-	4	-	Elective-III
	****	-	4	-	Elective-IV
Semester V	M306	-	4	-	Calculus of Several Variables
	M302	-	4	-	Rings and Modules
	M303	-	4	-	Differential Equations
	M304	-	4	-	Topology
	M305	-	4	-	Statistics
	****	-	4	-	Elective-V

Semester	Course No.		Credits		Course Name
	M301	-	4	-	Lebesgue Integration
	M307	-	4	-	Field Theory
Semester VI	M308	-	4	-	Complex Analysis
	M310	-	4	-	Geometry of Curves and Surfaces
	M311	-	4	-	Numerical Analysis
	****	-	4	-	Elective-VI
	M401	-	4	-	Functional Analysis
	M403	-	4	-	Commutative Algebra
Semester VII	M498	-	4	-	Project-I
	****	-	4	-	Elective-VII
	****	-	4	-	Elective-VIII
	****	-	4	-	Elective-IX
	M404	-	4	-	Algebraic Topology
	M402	-	4	-	Representations of Finite Groups
Semester VIII	M499	-	4	-	Project-II
	****	-	4	-	Elective-X
	****	-	4	-	Elective-XI
	****	-	4	-	Elective-XII
Semester IX	M598	-	20	-	Dissertation
	****	-	4	-	Elective-XIII
Semester X	M599	-	20	-	Dissertation
	****	-	4	-	Elective-XIV

Out of 56 credits as Electives, at least 8 credits from the School of Humanities and Social Sciences and at least 16 credits from other science schools must be taken.

Course Structure for a Minor in Mathematics

All the following courses are compulsory to get a Minor in Mathematics.

1. M101 - Mathematics-I
2. M102 - Mathematics-II
3. M201 - Real Analysis
4. M202 - Group Theory
5. M204 - Metric Spaces
6. M205 - Linear Algebra
7. M206 - Probability Theory
8. M303 - Differential Equations

A student may have the following two options to cover the last six courses from the above list of courses during his/her stay at NISER based on the present syllabus.

1. - Semester 3: Group Theory
   - Semester 4: Linear Algebra
   - Semester 5: Real Analysis
   - Semester 6/8/10: Metric Spaces/ Probability Theory
   - Semester 7/9: Differential Equations
2. - Semester 3: Real Analysis
   - Semester 4/6/8/10: Metric Spaces/ Linear Algebra/ Probability Theory
   - Semester 5/7/9: Group Theory/ Differential Equations

List of Courses from School of Mathematical Sciences

Compulsory Courses

Course No.		Credits		Course Name
M101	-	3	-	Mathematics-I
M102	-	3	-	Mathematics-II
M201	-	4	-	Real Analysis
M202	-	4	-	Group Theory
M203	-	4	-	Discrete Mathematics
M207	-	4	-	Number Theory
M204	-	4	-	Metric Spaces
M205	-	4	-	Linear Algebra
M206	-	4	-	Probabilty Theory
M208	-	4	-	Graph Theory
M306	-	4	-	Calculus of Several Variables
M302	-	4	-	Rings and Modules
M303	-	4	-	Differential Equations
M304	-	4	-	Topology
M305	-	4	-	Statistics
M301	-	4	-	Lebesgue Integration
M307	-	4	-	Field Theory
M308	-	4	-	Complex Analysis
M310	-	4	-	Geometry of Curves and Surfaces
M311	-	4	-	Numerical Analysis
M401	-	4	-	Functional Analysis
M403	-	4	-	Commutative Algebra
M404	-	4	-	Algebraic Topology
M402	-	4	-	Representations of Finite Groups

Elective Courses

Course No.		Credits		Course Name
M451	-	4	-	Advanced Complex Analysis
M452	-	4	-	Advanced Functional Analysis
M453	-	4	-	Advanced Linear Algebra
M454	-	4	-	Partial Differential Equations
M455	-	4	-	Introduction to Stochastic Processes
M456	-	4	-	Algebraic Geometry
M457	-	4	-	Algebraic Graph Theory
M458	-	4	-	Algebraic Number Theory
M460	-	4	-	Algorithm
M462	-	4	-	Cryptology
M463	-	4	-	Finite Fields
M464	-	4	-	Information and Coding Theory
M465	-	4	-	Mathematical Logic
M466	-	4	-	Measure Theory
M467	-	4	-	Nonlinear Analysis
M468	-	4	-	Operator Theory
M469	-	4	-	Theory of Computation
M470	-	4	-	Abstract Harmonic Analysis
M471	-	4	-	Advanced Number Theory
M472	-	4	-	Advanced Probability
M473	-	4	-	Algebraic Combinatorics
M474	-	4	-	Foundations of Cryptography
M475	-	4	-	Incidence Geometry
M476	-	4	-	Lie Algebras
M477	-	4	-	Optimization Theory
M478	-	4	-	Advanced Partial Differential Equations
M479	-	4	-	Random Graphs
M480	-	4	-	Randomized Algorithms and Probabilistic Methods
M481	-	4	-	Statistical Inference I
M482	-	4	-	Multivariate Statistical Analysis
M483	-	4	-	Introduction to Manifolds

Course No.		**Credits**		**Course Name**
M551	-	4	-	Algebraic Computation
M552	-	4	-	Analytic Number Theory
M553	-	4	-	Classical Groups
M554	-	4	-	Ergodic Theory
M555	-	4	-	Harmonic Analysis
M556	-	4	-	Lie Groups and Lie Algebras-I
M557	-	4	-	Operator Algebras
M558	-	4	-	Representations of Linear Lie Groups
M559	-	4	-	Harmonic Analysis on Compact Groups
M560	-	4	-	Modular Forms of One Variable
M561	-	4	-	Elliptic Curves
M562	-	4	-	Brownian Motion and Stochastic Calculus
M563	-	4	-	Differentiable Manifolds and Lie Groups
M564	-	4	-	Lie Groups and Lie Algebras-II
M565	-	4	-	Mathematical Foundations for Finance
M566	-	4	-	Designs and Codes
M567	-	4	-	Statistical Inference II

Program outcome: Integrated M.Sc. in Mathematics

The Integrated M.Sc. Program in Mathematics aims to provide comprehensive training to the students so that they will be able to build a carrier in Mathematics for themselves. The program aims to train people who are oriented towards research and teaching in both basic and advanced areas of Mathematical sciences. The core courses of the program provide a basic understanding in all areas of Mathematics which will be a foundation for further study of advanced topics. The electives courses provide knowledge in specialized topics and interconnection between different areas of Mathematics. Projects/Dissertation under the guidance of the faculty members give students exposure to current research in different areas of Mathematics and imbibe effective scientific and/or technical communication in both oral and writing. After successful completion of this program students will be able to apply knowledge of Mathematics in different fields of science and technology.

Syllabus of Compulsory Courses

M101: Mathematics-I

L	P	T	C
2	0	1	3

Outcome: Expects students to learn how to prove theorems, expressing mathematical objects, and understand the construction of natural numbers and symmetry of plane figures.

Contents: Method of Mathematical Proofs: Induction, Construction, Contradiction, Contrapositive. Set: Union and Intersection of sets, Distributive laws, De Morgan's Law, Finite and infinite sets. Relation: Equivalence relation and equivalence classes. Function: Injections, Surjections, Bijections, Composition of functions, Inverse function, Graph of a function. Countable and uncountable sets, Natural numbers via Peano arithmetic, Integers, Rational numbers, Real Numbers and Complex Numbers. Matrices, Determinant, Solving system of linear equations, Gauss elimination method, Linear mappings on $\mathbb{R}^2$ and $\mathbb{R}^3$, Linear transformations and Matrices. Symmetry of Plane Figures: Translations, Rotations, Reflections, Glide-reflections, Rigid motions.

References:

1. G. Polya, "How to Solve It", Princeton University Press, 2004.
2. K. B. Sinha et. al., "Understanding Mathematics", Universities Press (India), 2003.
3. M. Artin, "Algebra", Prentice-Hall of India, 2007 (Chapters 1, 4, 5).
4. J. R. Munkres, "Topology", Prentice-Hall of India, 2013 (Chapter 1).

M102: Mathematics-II

L	P	T	C
2	0	1	3

Outcome: Upon successful completion of the course students will become aware of some basic properties of real line and real valued functions.

Contents: Concept of ordered field, Bounds of a set, ordered completeness axiom and characterization of $\mathbb{R}$ as a complete ordered field. Archimedean property of real numbers. Modulus of real numbers, Intervals, Neighbourhood of a point. Sequences of Real Numbers: Definition and examples, Bounded sequences, Convergence of sequences, Uniqueness of limit, Algebra of limits, Monotone sequences and their convergence, Sandwich rule. Series: Definition and convergence, Telescopic series, Series with non-negative terms. Tests for convergence [without proof]: Cauchy condensation test, Comparison test, Ratio test, Root test, Absolute and conditional convergence, Alternating series and Leibnitz test. Limit of a function at a point, Sequential criterion for the limit of a function at a point. Algebra of limits, Sandwich theorem, Continuity at a point and on intervals, Algebra of continuous functions. Discontinuous functions, Types of discontinuity. Differentiability: Definition and examples, Geometric and physical interpretations, Algebra of differentiation, Chain rule, Darboux Theorem, Rolle's Theorem, Mean Value Theorems of Lagrange and Cauchy. Application of derivatives: Increasing and decreasing functions, Maxima and minima of functions. Higher order derivatives, Leibnitz rule, L'Hopital rule.

Text Book:

1. R. G. Bartle, D. R. Sherbert, “Introduction to Real Analysis”, John Wiley & Sons, 1992.

References:

1. K. A. Ross, “Elementary Analysis”, Undergraduate Texts in Mathematics, Springer, 2013.
2. S. K. Berberian, “A First Course in Real Analysis”, Undergraduate Texts in Mathematics, Springer-Verlag, 1994.

M201: Real Analysis

Prerequisites: M102

L	P	T	C
3	0	1	4

Outcome: Knowledge on Continuity differentiable and Riemann integration theory. Sequence and series and it’s application to numerical analysis.

Contents: Countability of a set, Countability of rational numbers, Uncountability of real numbers. Limit point of a set, Bolzano-Weirstrass theorem, Open sets, Closed sets, Dense sets. Subsequence, Limit superior and limit inferior of a sequence, Cauchy criterion for convergence of a sequence, Monotone subsequence. Tests of convergence of series, Abel’s and Dirichlet’s tests for series, Riemann rearrangement theorem. Continuous functions on closed and bounded intervals, Intermediate value theorem, Monotone functions, Continuous monotone functions and their invertibility, Discontinuity of monotone functions. Uniform continuity, Equivalence of continuity and uniform continuity on closed and bounded intervals, Lipschitz condition, Other sufficient condition for uniform continuity. Riemann Integration: Darboux’s integral, Riemann sums and their properties, Algebra of Riemann integrable functions, Class of Riemann integrable functions, Mean value theorem, Fundamental theorems of calculus, Change of variable formula (statement only), Riemann-Stieltjes integration (definition). Taylor’s theorem and Taylor’s series, Elementary functions. Improper integral, Beta and Gamma functions.

Text Books:

1. R. G. Bartle, D. R. Sherbert, “Introduction to Real Analysis”, John Wiley & Sons, 1992.
2. K. A. Ross, “Elementary Analysis”, Undergraduate Texts in Mathematics, Springer, 2013.

References:

1. T. M. Apostol, “Calculus Vol. I”, Wiley-India edition, 2009.
2. S. K. Berberian, “A First Course in Real Analysis”, Undergraduate Texts in Mathematics, Springer-Verlag, 1994.

M202: Group Theory

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will be able to understand the notion of symmetries in the language of groups. Furthermore, students will become aware of various properties of groups and subgroups.

Contents: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, isomorphism theorems, automorphisms, permutation groups, group actions, Sylow’s theorem, direct products, finite abelian groups, semidirect products, free groups.

Text Book:

1. D. S. Dummit, R. M. Foote, “Abstract Algebra”, Wiley-India edition, 2013.

References:

1. I. N. Herstein, “Topics in Algebra”, Wiley-India edition, 2013.
2. M. Artin, “Algebra”, Prentice-Hall of India, 2007.

M203: Discrete Mathematics

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Learning different combinatorial techniques to solve many counting problems and understanding some mathematical structures

Contents: Pigeonhole principle, Counting principles, Binomial coefficients, Principles of inclusion and exclusion, recurrence relations, generating functions, Catalan numbers, Stirling numbers, Partition numbers, Schröder numbers, Block designs, Latin squares, Partially ordered sets, Lattices, Boolean algebra.

Text Books:

1. R. A. Brualdi, “Introductory Combinatorics”, Pearson Prentice Hall, 2010.
2. J. P. Tremblay, R. Manohar, “Discrete Mathematical Structures with Application to Computer Science”, Tata McGraw-Hill Edition, 2008.

References:

1. J. H. van Lint, R. M. Wilson, “A Course in Combinatorics”, Cambridge University Press, 2001.
2. I. Anderson, “A First Course in Discrete Mathematics”, Springer Undergraduate Mathematics Series, 2001.
3. R. P. Stanley, “Enumerative Combinatorics Vol. 1”, Cambridge Studies in Advanced Mathematics, 49, Cambridge University Press, 2012.

M204: Metric Spaces

Prerequisites: M201

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will become aware about generalisation of euclidean distance on arbitrary sets and various properties of functions defined on them.

Contents: Metric spaces, open balls and open sets, limit and cluster points, closed sets, dense sets, complete metric spaces, completion of a metric space, Continuity, uniform continuity, Banach contraction principle, Compactness, Connectedness, pathconnected sets. Sequences of functions, Pointwise convergence and uniform convergence, Arzela-Ascoli Theorem, Weierstrass Approximation Theorem, power series, radius of convergence, uniform convergence and Riemann integration, uniform convergence and differentiation, Stone-Weierstrass theorem for compact metric spaces.

Text Books:

1. G. F. Simmons, “Introduction to Topology and Modern Analysis”, Tata McGraw-Hill, 2013.
2. S. Kumaresan, “Topology of Metric Spaces”, Narosa Publishing House, 2005.

References:

1. R. R. Goldberg, “Methods of Real Analysis”, John Wiley & Sons, 1976.
2. G. B. Folland, “Real Analysis”, Wiley-Interscience Publication, John Wiley & Sons, 1999.

M205: Linear Algebra

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will learn the relation between linear transformations and matrices. Moreover, student will also learn various fundamental results of matrices, namely, diagonalisation, triangulation and primary decomposition theorem.

Contents: System of Linear Equations, Matrices and elementary row operations, Row-reduced echelon form of matrices, Vector spaces, subspaces, quotient spaces, bases and dimension, direct sums, Linear transformations and their matrix representations, Dual vector spaces, transpose of a linear transformation, Polynomial rings (over a field), Determinants and their properties, Eigenvalues and eigenvectors, Characteristic polynomial and minimal polynomial, Triangulation and Diagonalization, Simultaneous Triangulation and diagonalization, Direct-sum decompositions, Primary decomposition theorem.

Text Book:

1. K. Hoffman, R. Kunze, "Linear Algebra", Prentice-Hall of India, 2012.

References:

1. S. H. Friedberg, A. J. Insel, L. E. Spence, "Linear Algebra", Prentice Hall, 1997.
2. A. Ramachandra Rao, P. Bhimasankaram, "Linear Algebra", Texts and Readings in Mathematics, 19. Hindustan Book Agency, New Delhi, 2000.
3. M. Artin, "Algebra", Prentice-Hall of India, 2007.

M206: Probability Theory

Prerequisites: M102

L	P	T	C
3	0	1	4

Outcome: Students will be introduced to the basic theory of probability starting from axiomatic definition of probability up to limit theorems of probability.

Contents: Combinatorial probability and urn models; Conditional probability and independence; Random variables – discrete and continuous; Expectations, variance and moments of random variables; Transformations of univariate random variables; Jointly distributed random variables; Conditional expectation; Generating functions; Limit theorems; Simple symmetric random walk.

Text Books:

1. S. Ross, "A First Course in Probability", Pearson Education, 2012.
2. D. Stirzaker, "Elementary Probability", Cambridge University Press, Cambridge, 2003.

References:

1. K. L. Chung, F. AitSahlia, "Elementary Probability Theory", Undergraduate Texts in Mathematics. Springer-Verlag, 2003.
2. P. G. Hoel, S. C. Port, C. J. Stone, "Introduction to Probability Theory", The Houghton Mifflin Series in Statistics. Houghton Mifflin Co., 1971.
3. W. Feller, "An Introduction to Probability Theory and its Applications Vol. 1 and Vol. 2", John Wiley & Sons, 1968, 1971.

M207: Number Theory

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Expects students to learn elementary properties of rings of integers including divisibility, congruences, continued fractions and Gauss reciprocity laws.

Contents: Divisibility, Primes, Fundamental theorem of arithmetic, Congruences, Chinese remainder theorem, Linear congruences, Congruences with prime-power modulus, Fermat's little theorem, Wilson's theorem, Euler function and its applications, Group of units, primitive roots, Quadratic residues, Jacobi symbol, Binary quadratic form, Arithmetic functions, Möbius Inversion formula, Dirichlet product, Sum of squares, Continued fractions and rational approximations.

Text Book:

1. I. Niven, H. S. Zuckerman, H. L. Montgomery, "An Introduction to the Theory of Numbers", Wiley-India Edition, 2008.

References:

1. T. M. Apostol, "Introduction to Analytic Number Theory", Springer International Student Edition, 2000.
2. G. A. Jones, J. M. Jones, "Elementary Number Theory", Springer Undergraduate Mathematics Series. Springer-Verlag, 1998.

M208: Graph Theory

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Understanding the fundamentals of graph theory and learning the structure of graphs and techniques used to analyze different problems

Contents: Graphs, subgraphs, graph isomorphisms, degree sequence, paths, cycles, trees, bipartite graphs, Hamilton cycles, Euler tours, directed graphs, matching, Tutte's theorem, connectivity, Menger's theorem, planar graphs, Kuratowski's theorem, vertex and edge colouring of graphs, network flows, max-flow min-cut theorem, Ramsey theory for graphs, matrices associated with graphs.

Text Book:

1. R. Diestel, "Graph Theory", Graduate Texts in Mathematics, 173. Springer, 2010.

References:

1. B. Bollobás, "Modern Graph Theory", Graduate Texts in Mathematics, 184. Springer-Verlag, 1998.
2. F. Harary, "Graph Theory", Addison-Wesley Publishing Co., 1969.
3. J. A. Bondy, U. S. R. Murty, "Graph Theory", Graduate Texts in Mathematics, 244. Springer, 2008.

M301: Lebesgue Integration

Prerequisites: M201

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will learn the concept of measures and measurable functions. Students also learn Lebesgue integration and their various properties

Contents: Outer measure, measurable sets, Lebesgue measure, measurable functions, Lebesgue integral, Basic properties of Lebesgue integral, convergence in measure, differentiation and Lebesgue measure. L_p Spaces, Holder and Minkowski inequalities, Riesz-Fisher theorem, Radon-Nykodin theorem, Riesz representation theorem. Fourier series, L_2-convergence properties of Fourier series, Fourier transform and its properties.

Text Books:

1. H. L. Royden, "Real Analysis", Prentice-Hall of India, 2012.
2. G. B. Folland, "Real Analysis", Wiley-Interscience Publication, John Wiley & Sons, 1999.

References:

1. G. de Barra, "Measure Theory and Integration", New Age International, New Delhi, 2003.
2. W. Rudin, "Principles of Mathematical Analysis", Tata McGraw-Hill, 2013.

M302: Rings and Modules

Prerequisites: M202, M205

L	P	T	C
3	0	1	4

Outcome: Expects students to learn structure and various properties of rings and modules, structure of finitely generated modules over PID.

Contents: Rings, ideals, quotient rings, ring homomorphisms, isomorphism theorems, prime ideals, maximal ideals, Chinese remainder theorem, Field of fractions, Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains, Polynomial rings, Gauss lemma, irreducibility criteria.
Modules, submodules, quotients modules, module homomorphisms, isomorphism theorems, generators, direct product and direct sum of modules, free modules, finitely generated modules over a PID, Structure theorem for finitely generated abelian groups, Rational form and Jordan form of a matrix, Tensor product of modules.

Text Book:

1. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

References:

1. I. N. Herstein, "Topics in Algebra", Wiley-India edition, 2013.
2. M. Artin, "Algebra", Prentice-Hall of India, 2007.

M303: Differential Equations

Prerequisites: M201, M205

L	P	T	C
3	0	1	4

Outcome: This course starts with the origin and applications of differential equations and discusses many solution techniques such as separation of variable, variation of parameter, annihilator method and Frobenious method, etc. Then it introduces basic theory of existence and uniqueness for the system of first order ODEs which is essential for many branches of mathematics. This course also gives a glimpse how to analyze the behavior of solutions (maximum principle, stability, asymptotic stability, etc.). This course ends with an introduction to partial differential equations and method of characteristics, a technique to solve first order partial differential equations. Upon successful completion of this course the student will be able to model some practical situations into ordinary differential equations or partial differential equations and analyze the solution to get information about the parameters involved in the model.

Contents: Classifications of Differential Equations: origin and applications, family of curves, isoclines. First order equations: separation of variable, exact equation, integrating factor, Bernoulli equation, separable equation, homogeneous equations, orthogonal trajectories, Picard's existence and uniqueness theorems. Second order equations: variation of parameter, annihilator methods. Series solution: power series solutions about regular and singular points. Method of Frobenius, Bessel's equation and Legendre equations. Wronskian determinant, Phase portrait analysis for 2nd order system, comparison and maximum principles for 2nd order equations. Linear system: general properties, fundamental matrix solution, constant coefficient system, asymptotic behavior, exact and adjoint equation, oscillatory equations, Green's function. Sturm-Liouville theory. Partial Differential Equations: Classifications of PDE, method of separation of variables, characterstic method.

Text Books:

1. S. L. Ross, "Differential Equations", Wiley-India Edition, 2009.
2. E. A. Coddington, "An Introduction to Ordinary Differential Equations", Prentice-Hall of India, 2012.

References:

1. G. F. Simmons, S. G. Krantz, "Differential Equations", Tata Mcgraw-Hill Edition, 2007.
2. B. Rai, D. P. Choudhury, "A Course in Ordinary Differential Equation", Narosa Publishing House, New Delhi, 2002.
3. R. P. Agarwal, D. OŔegan, "Ordinary and Partial Differential Equations", Universitext. Springer, 2009.

M304: Topology

Prerequisites: M204

L	P	T	C
3	0	1	4

Outcome: This course builds the foundations of point set topology and also covers basic algebraic topology (basics of covering spaces and fundamental group). After taking this course the students will be proficient in the abstract notion of a topological space, where continuous functions are defined in terms of open sets (and not the traditional $\varepsilon - \delta$ definition used in analysis). The students will appreciate some of the most important concepts in analysis from a topological perspective. For example, they will realize Intermediate value theorem is a statement about connectedness, Bolzano Weirstrass Theorem is a statement about compactness and so on. They will also get a solid grasp of quotient topology (which is a fundamentally new concept that is not an extension of the things already taught in analysis). The students will then be taught deeper concepts such as Ursysohn Lemma, Tietze extension theorem and Tychonoff Theorem. These topics will be of great use to anyone pursuing further studies in Topology, Functional Analysis, PDE and Probability.
The students will also learn the basics of fundamental group and covering spaces; they will be able to compute the fundamental group of a circle (but not much more beyond that in this course). After this course, they will be fully ready to study a more advanced course in Algebraic Topology that gets into the intricate details of fundamental group and singular homology. This course will also be very useful to anyone pursuing further studies in Differential Geometry (theory of manifolds).

Contents: Topological Spaces, Open and closed sets, Interior, Closure and Boundary of sets, Basis for Topology, Product Topology, Subspace Topology, Metric Topology, Compact Spaces, Locally compact spaces, Continuous functions, Open map, Homeomorphisms, Function Spaces, Separation Axioms: T1, Hausdorff, regular, normal spaces; Uryshon's lemma, Tietze

Extension Theorem, One point compactification, Connected Spaces, Path Connected Spaces, Quotient Topology, Homotopic Maps, Deformation Retract, Contractible Spaces, Fundamental Group, The Brouwer fixed-point theorem.

Text Books:

1. J. R.Munkres, "Topology", Prentice-Hall of India, 2013.
2. M. A. Armstrong, "Basic Topology", Undergraduate Texts in Mathematics, Springer-Verlag, 1983.

References:

1. J. L. Kelley, "General Topology", Graduate Texts in Mathematics, No. 27. Springer-Verlag, New York-Berlin, 1975.
2. K. Jänich, "Topology", Undergraduate Texts in Mathematics. Springer-Verlag, 1984.

M305: Statistics

Prerequisites: M206

L	P	T	C
3	0	1	4

Outcome: Students will be introduced to the discipline of statistics, learn about descriptive statistics of data sets including graphical representation using some statistical software. The focus is to learn about basic theory of point estimation, interval estimation, hypothesis testing and linear regression.

Contents: Descriptive Statistics, Graphical representation of data, Curve fittings, Simple correlation and regression, Multiple and partial correlations and regressions, Sampling, Sampling distributions, Standard error. Normal distribution and its properties, The distribution of $\overline{X}$ and S^2 in sampling from a normal distribution, Exact sampling distributions: χ^2, t, F. Theory and Methods of Estimation: Point estimation, Criteria for a good estimator, Properties of estimators: Unbiasedness, Efficiency, Consistency, Sufficiency, Robustness. A lower bound for a variance of an estimate, Method of estimation: The method of moment, Least square method, Maximum likelihood estimation and its properties, UMVU Estimator, Interval estimation. Test of Hypothesis: Elements of hypothesis testing, Unbiased test, Neyman-Pearson Theory, MP and UMP tests, Likelihood ratio and related tests, Large sample tests, Test based on χ^2, t, F.

Text Books:

1. H. J. Larson, "Introduction to Probability Theory and Statistical Inference", John Wiley & Sons, 1982.
2. V. K. Rohatgi, "Introduction to Probability Theory and Mathematical Statistics", John Wiley & Sons, 1976.

References:

1. I. Miller, M. Miller, "John E. Freund's Mathematical Statistics with Applications", Pearson, 2013.

M306: Calculus of Several Variables

Prerequisites: M201, M204

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will learn the notion of limits, continuity, differentiation and integration in the higher dimensional euclidean spaces.

Contents: Differentiability of functions from an open subset of $\mathbb{R}^n$ to $\mathbb{R}^m$ and properties, chain rule, partial and directional derivatives, Continuously differentiable functions, Inverse function theorem, Implicit function theorem, Interchange of order of differentiation, Taylor's series, Extrema of a function, Extremum problems with constraints, Lagrange multiplier method with applications, Integration of functions of several variables, Change of variable formula (without proof) with examples of applications of the formula, spherical coordinates, Stokes theorem (without proof), Deriving Green's theorem, Gauss theorem and Classical Stokes theorem.

Text Books:

1. W. Fleming, "Functions of Several Variables", Undergraduate Texts in Mathematics. Springer-Verlag, 1977.
2. T. M. Apostol, "Calculus Vol. II", Wiley-India edition, 2009.

References:

1. W. Kaplan, "Advanced Calculus", Addison-Wesley Publishing Company, 1984.
2. T. M. Apostol, "Mathematical Analysis", Narosa Publishing House, 2013.

M307: Field Theory

Prerequisites: M205, M302

L	P	T	C
3	0	1	4

Outcome: Expects students to learn basic properties of fields including the fundamental theorem of Galois theory.

Contents: Field extensions, algebraic extensions, Ruler and compass constructions, splitting fields, algebraic closures, separable and inseparable extensions, cyclotomic polynomials and extensions, automorphism groups and fixed fields, Galois extensions, Fundamental theorem of Galois theory, Fundamental theorem of algebra, Finite fields, Galois group of polynomials, Computations of Galois groups over rationals, Solvable groups, nilpotent groups, Solvability by radicals, Transcendental extensions.

Text Book:

1. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

References:

1. I. N. Herstein, "Topics in Algebra", Wiley-India edition, 2013.
2. M. Artin, "Algebra", Prentice-Hall of India, 2007.
3. J. Rotman, "Galois Theory", Universitext, Springer-Verlag, 1998.

M308: Complex Analysis

Prerequisites: M306

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will learn the concept of (complex) differentiation and integration of functions defined on the complex plane and their properties.

Contents: Algebraic and geometric representation of complex numbers; elementary functions including the exponential functions and its relatives (log, cos, sin, cosh, sinh, etc.); concept of holomorphic (analytic) functions, complex derivative and the Cauchy-Riemann equations; harmonic functions. Conformal Mapping, Linear Fractional Transformations, Complex line integrals and Cauchy Integral formula, Representation of holomorpic functions in terms of power series, Morera's theorem, Cauchy estimates and

Liouville's theorem, zeros of holomorphic functions, Uniform limits of holomorphic functions. Behaviour of holomorphic function near an isolated singularity, Laurent expansions, Counting zeros and poles, Argument principle, Rouche's theorem, Calculus of residues and evaluation of integrals using contour integration. The Open Mapping theorem, Maximum Modulus Principle, Schwarz Lemma.

Text Books:

1. J. B. Conway, "Functions of One Complex Variable", Narosa Publishing House, 2002.
2. R. E. Greene, S. G. Krantz, "Function Theory of One Complex Variable", American Mathematical Society, 2011.

References:

1. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.
2. L. V. Ahlfors, "Complex Analysis", Tata McGraw-Hill, 2013.
3. T. W. Gamelin, "Complex Analysis", Undergraduate Texts in Mathematics, Springer, 2006.
4. E. M. Stein, R. Shakarchi, "Complex Analysis", Princeton University Press, 2003.

M310: Geometry of Curves and Surfaces

Prerequisites: M306

L	P	T	C
3	0	1	4

Outcome: Knowledge on curve and surfaces, manifold and vector field some application on geometry of surfaces.

Contents: Curves in two and three dimensions, Curvature and torsion for space curves, Existence theorem for space curves, Serret-Frenet formula for space curves, Jacobian theorem, Surfaces in $\mathbb{R}^3$ as 2-dimensional manifolds, Tangent spaces and derivatives of maps between manifolds, Geodesics, First fundamental form, Orientation of a surface, Second fundamental form and the Gauss map, Mean curvature, Gaussian Curvature, Differential forms, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

Text Books:

1. M. P. Do Carmo, "Differential Geometry of Curves and Surfaces", Prentice Hall, 1976.
2. Andrew Pressley, "Elementary Differential Geometry", Springer, 2010.

References:

1. M. P. Do Carmo, "Differential Forms and Applications", Springer, 1994.
2. J. A. Thorpe, "Elementary Topics in Differential Geometry", Undergraduate texts in mathematics, Springer, 2011.

M311: Numerical Analysis

Prerequisites: M201, M303

L	P	T	C
2	1	1	4

Outcome: Upon successful completion of the course students will learn practical use of some important results from real analysis and linear algebra.

Contents: Errors in computation: Representation and arithmetic of numbers, source of errors, error propagation, error estimation. Numerical solution of non-linear equations: Bisection method, Secant method, Newton-Raphson method, Fixed point methods, Muller's method. Interpolations: Lagrange interpolation, Newton divided differences, Hermite interpolation, Piecewise polynomial interpolation. Approximation of functions: Weierstrass and Taylor expansion, Least square approximation. Numerical Integration: Trapezoidal rule, Simpson's rule, Newton-Cotes rule, Guassian quadrature. Numerical solution of ODE: Euler's method, multi-step methods, Runge-Kutta methods, Predictor-Corrector methods. Solutions of systems of linear equations: Gauss elimination, pivoting, matrix factorization, Iterative methods – Jacobi and Gauss-Siedel methods. Matrix eigenvalue problems: power method.

Text Book:

1. K. E. Atkinson, "An Introduction to Numerical Analysis" Wiley-India Edition, 2013.

References:

1. S. D. Conte, C. De Boor, "Elementary Numerical Analysis, Tata McGraw-Hill, 2006.
2. W. H. Press et. al., "Numerical Recipes - The Art of Scientific Computing", Cambridge University Press, 2007.

M401: Functional Analysis

Prerequisites: M204, M205

L	*P*	*T*	*C*
3	0	1	4

Outcome: Upon successful completion of the course students will learn the concept of normed linear space and various properties of operators defined on them.

Contents: Normed linear spaces and continuous linear transformations, Hahn-Banach theorem (analytic and geometric versions), Baire's theorem and its consequences – three basic principles of functional analysis (open mapping theorem, closed graph theorem and uniform boundedness principle), Computing the dual of wellknown Banach spaces, Hilbert spaces, Riesz representation theorem, Adjoint operator, Compact operators, Spectral theorem for self adjoint compact operators.

Text Books:

1. J. B. Conway, "A Course in Functional Analysis", Graduates Texts in Mathematics 96, Springer, 2006.
2. B. Bollobás, "Linear Analysis", Cambridge University Press, 1999.

References:

1. G. F. Simmons, "Introduction to Topology and Modern Analysis", Tata McGraw-Hill, 2013.

M402: Representations of Finite Groups

Prerequisites: M202, M205, M302

L	*P*	*T*	*C*
3	0	1	4

Outcome: This course gives an introduction to the representation of finite groups via character theory.

Contents: Group representations, Maschke's theorem and completely reducibility, Characters, Inner product of Characters, Orthogonality relations, Burnside's theorem, induced characters, Frobenius reciprocity, induced representations, Mackey's Irreducibility Criterion, Character table of some well-known groups, Representation theory of the symmetric group: partitions and tableaux, constructing the irreducible representations.

Text Book:

1. G. James, M. Liebeck, "Representations and Characters of Groups", Cambridge University Press, 2010.

References:

1. J. L. Alperin, R. B. Bell, "Groups and Representations", Graduate Texts in Mathematics 162, Springer, 1995.
2. B. Steinberg, "Representation Theory of Finite Groups", Universitext, Springer, 2012.
3. J-P. Serre, "Linear Representations of Finite Groups", Graduate Texts in Mathematics 42, Springer-Verlag, 1977.
4. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.

M403: Commutative Algebra

Prerequisites: M302

L	*P*	*T*	*C*
3	0	1	4

Outcome: Expects students to understand various properties of commutative rings, various class of commutative rings, and dimension theory.

Contents: Commutative rings, ideals, operations on ideals, prime and maximal ideals, nilradicals, Jacobson radicals, extension and contraction of ideals, Modules, free modules, projective modules, exact sequences, tensor product of modules, Restriction and extension of scalars, localization and local rings, extended and contracted ideals in rings of fractions, Noetherian modules, Artinian modules, Primary decompositions and associate primes, Integral extensions, Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals, Completion, Dimension theory.

Text Book:

1. M. F. Atiyah, I. G. Macdonald, "Introduction to Commutative Algebra", Addison-Wesley Publishing Co., 1969.

References:

1. R. Y. Sharp, "Steps in Commutative Algebra", London Mathematical Society Student Texts, 51. Cambridge University Press, 2000.
2. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

M404: Algebraic Topology

Prerequisites: M302, M304

L	*P*	*T*	*C*
3	0	1	4

Outcome: This course lays down the foundations of fundamental group (π_1) and singular homology. The students will get a good in-depth knowledge of covering spaces. To begin with, they will study covering spaces as a tool to compute fundamental group (such as the circle, torus etc). Later on, they will study covering spaces in much greater depth; they will get an understanding of the correspondence between conjugacy classes of π_1 and the different covering spaces they correspond to. They will also learn that this correspondence is bijective if and only if the space is reasonable (path connected, locally path connected and semi-locally simply connected). Students will also learn different techniques to compute the fundamental group such as homotopy invariance and Van-Kampen Theorem.
The students will also learn about the basics of singular homology. They will learn different techniques to compute singular homology of a space, including homotopy invariance, Mayer-Vietoris, excision, long exact sequence etc. The students will also learn about the degree of a map. They will be able to use these concepts to prove non-trivial theorems such as invariance of domain, hairy ball theorem etc.

Contents: Homotopy Theory: Simply Connected Spaces, Covering Spaces, Universal Covering Spaces, Deck Transformations, Path lifting lemma, Homotopy lifting lemma, Group Actions, Properly discontinuous action, free groups, free product with amalgamation, Seifert-Van Kampen Theorem, Borsuk-Ulam Theorem for sphere, Jordan Separation Theorem. Homology Theory: Simplexes, Simplicial Complexes, Triangulation of spaces, Simplicial Chain Complexes, Simplicial Homology, Singular Chain Complexes, Cycles and Boundary, Singular Homology, Relative Homology, Short Exact Sequences, Long Exact Sequences, Mayer-Vietoris sequence, Excision Theorem, Invariance of Domain.

Text Books:

1. J. R. Munkres, "Topology", Prentice-Hall of India, 2013.
2. A. Hatcher, "Algebraic Topology", Cambridge University Press, 2009.

References:

1. G. E. Bredon, "Topology and Geometry", Graduates Texts in Mathematics 139, Springer, 2009.

Syllabus of Elective Courses

M451: Advanced Complex Analysis

Prerequisites: M308

L	P	T	C
3	0	1	4

Outcome: Students will learn some important theorems in complex analysis such as Riemann mapping theorem, Weirstrass factorization theorem, Runges theorem, Hardamard factorization theorem, Little Picards theorem and Great Picards theorem. They will also learn some basic techniques of harmonic functions and characterization of Dirichlet Region. These results are very useful in many branches of mathematics such as Number Thoery, Differential Geometry, Operator theory, Partal Differential Equations etc.

Contents: Review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Space of continuous functions, Arzela's theorem, Spaces of analytic functions, Spaces of meromorphic functions, Riemann mapping theorem, Weierstrass Factorization theorem, Runge's theorem, Simple connectedness, Mittag-Leffler's theorem, Analytic continuation, Schwarz reflection principle, Mondromy theorem, Jensen's formula, Genus and order of an entire function, Hadamard factorization theorem, Little Picard theorem, Great Picard theorem, Harmonic functions.

References:

1. L. V. Ahlfors, "Complex Analysis", Tata McGraw-Hill, 2013.
2. J. B. Conway, "Functions of One Complex Variable II", Graduate Texts in Mathematics 159, Springer-Verlag, 1996.
3. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.
4. R. Remmert, "Theory of Complex Functions", Graduate Texts in Mathematics 122, Springer, 2008.

M452: Advanced Functional Analysis

Prerequisites: M401

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will become aware of the concept of topological vector space, as a generalisation of normed linear spaces, and various properties of operators defined on them.

Contents: Definition and examples of topological vector spaces (TVS) and locally convex spaces (LCS); Linear operators; Hahn-Banach Theorems for TVS/ LCS (analytic and geometric forms); Uniform boundedness principle; Open mapping theorem; Closed graph thoerem; Weak and weak* vector topologies; Bipolar theorem; dual of LCS spaces; Krein-Milman theorem for TVS; Krien-Smulyan theorem for Banach spaces; Inductive and projective limit of LCS.

References:

1. W. Rudin, "Functional Analysis", Tata McGraw-Hill, 2007.
2. A. P. Robertson, W. Robertson, "Topological Vector Spaces", Cambridge Tracts in Mathematics 53, Cambridge University Press, 1980.
3. J. B. Conway, "A Course in Functional Analysis", Graduates Texts in Mathematics 96, Springer, 2006.

M453: Advanced Linear Algebra

Prerequisites: M205

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will become aware of various decomposition results of matrices and their applications.

Contents: Rational and Jordan canonical forms, Inner product spaces, Unitary and Normal operators, Forms on inner product spaces, Spectral theorems, Bilinear forms, Matrix decomposition theorems, Courant- Fischer minimax and related theorems, Nonnegative matrices, Perron-Frobenius theory, Generalized inverse, Matrix Norm, Perturbation of eigenvalues.

References:

1. R. A. Horn, C. R. Johnson, "Matrix Analysis", Cambridge University Press, 2010.
2. K. Hoffman, R. Kunze, "Linear Algebra", Prentice-Hall of India, 2012.
3. S. Roman, "Advanced Linear Algebra", Graduate Texts in Mathematics 135, Springer, 2008.

M454: Partial Differential Equations

Prerequisites: M301, M303, M306

L	P	T	C
3	0	1	4

Outcome: Students will learn explicit representations of solutions of four important classes of PDEs, namely, Transport equations, Heat equation, Laplace equation and wave equation for initial value problems. They will study the properties of solutions of these equations such as mean value property, maximum principles and regularity. They will also study Cauchy-Kowalevski Theorem and uniqueness theorem of Holmgreen for quasilinear equations.

Contents: Classification of Partial Differential Equations, Cauchy Problem, Cauchy-Kowalevski Theorem, Lagrange-Green identity, The uniquness theorem of Holmgren, Transport equation: Initial value problem, nonhomogeneous problem. Laplace equation: Fundamental solution, Mean Value formula, properties of Harmonic functions, Green's function, Energy methods, Harnack's inequality. Heat Equation: Fundamental solution, Mean value formula, properties of solutions. Wave equation: Solution by spherical means, Nonhomogeneous problem, properties of solutions.

References:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. F. John, "Partial Differential Equations", Springer International Edition, 2009.
3. G. B. Folland, "Introduction to Partial Differential Equations", Princeton University Press, 1995.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.

M455: Introduction to Stochastic Processes

Prerequisites: M206

L	P	T	C
3	0	1	4

Outcome: Students will be introduced to the theory of both discrete time and continuous time Markov chains.

Contents: Discrete Markov chains with countable state space; Classification of states: recurrences, transience, periodicity. Stationary distributions, reversible chains, Several illustrations including the Gambler's Ruin problem, queuing chains, birth and death chains etc. Poisson process, continuous time Markov chain with countable state space, continuous time birth and death chains.

References:

1. P. G. Hoel, S. C. Port, C. J. Stone, "Introduction to Stochastic Processes", Houghton Mifflin Co., 1972.
2. R. Durrett, "Essentials of Stochastic Processes", Springer Texts in Statistics, Springer, 2012.
3. G. R. Grimmett, D. R. Stirzaker, "Probability and Random Processes", Oxford University Press, 2001.
4. S. M. Ross, "Stochastic Processes", Wiley Series in Probability and Statistics: Probability and Statistics, John Wiley & Sons, 1996

M456: Algebraic Geometry

Prerequisites: M205, M302

L	P	T	C
3	0	1	4

Outcome: This course will introduce the students to the fundamentals of classical algebraic geometry. They will learn about the theory of Riemann surfaces, divisors, line bundles, Chern Classes and the Riemann Roch Theorem.

Contents: Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert's Nullstellensatz, Affine and Projective varieties, Zariski Topology, Rational functions and morphisms, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Riemann-Roch for curves, Line bundles on Projective spaces.

References:

1. K. Hulek, "Elementary Algebraic Geometry", Student Mathematical Library 20, American Mathematical Society, 2003.
2. I. R. Shafarevich, "Basic Algebraic Geometry 1: Varieties in Projective Space", Springer, 2013.
3. J. Harris, "Algebraic geometry", Graduate Texts in Mathematics 133, Springer-Verlag, 1995.
4. M. Reid, "Undergraduate Algebraic Geometry", London Mathematical Society Student Texts 12, Cambridge University Press, 1988.
5. K. E. Smith et. al., "An Invitation to Algebraic Geometry", Universitext, Springer-Verlag, 2000.
6. R. Hartshorne, "Algebraic Geometry", Graduate Texts in Mathematics 52, Springer-Verlag, 1977.

M457: Algebraic Graph Theory

Prerequisites: M205, M208

L	P	T	C
3	0	1	4

Outcome: Learning the different algebraic techniques used in the study of the graphs

Contents: Adjacency matrix of a graph and its eigenvalues, Spectral radius of graphs, Regular graphs and Line graphs, Strongly regular graphs, Cycles and Cuts, Laplacian matrix of a graph, Algebraic connectivity, Laplacian spectral radius of graphs, Distance matrix of a graph, General properties of graph automorphisms, Transitive and Arc-tranisitve graphs, Symmetric graphs.

References:

1. N. Biggs, "Algebraic Graph Theory", Cambridge University Press, 1993.

2. C. Godsil, G. Royle, “Algebraic Graph Theory”, Graduate Texts in Mathematics 207, Springer-Verlag, 2001.
3. R. B. Bapat, “Graphs and Matrices”, Universitext, Springer, Hindustan Book Agency, New Delhi, 2010.

M458: Algebraic Number Theory

Prerequisites: M207, M307, M403

L	P	T	C
3	0	1	4

Outcome: This course gives an introduction to the basic properties of number fields, computation of class numbers and zeta functions.

Contents: Number Fields and Number rings, prime decomposition in number rings, Dedekind domains, Ideal class group, Galois theory applied to prime decomposition, Gauss reciprocity law, Cyclotomic fields and their ring of integers, finiteness of ideal class group, Dirichlet unit theorem, valuations and completions of number fields, Dedekind zeta function and distribution of ideal in a number ring.

References:

1. D. A. Marcus, “Number Fields”, Universitext, Springer-Verlag, 1977.
2. G. J. Janusz, “Algebraic Number Fields”, Graduate Studies in Mathematics 7, American Mathematical Society, 1996.
3. S. Alaca, K. S. Williams, “Introductory Algebraic Number Theory”, Cambridge University Press, 2004.
4. S. Lang, “Algebraic Number Theory”, Graduate Texts in Mathematics 110, Springer-Verlag, 1994.
5. A. Frohlich, M. J. Taylor, “Algebraic Number Theory”, Cambridge Studies in Advanced Mathematics 27, Cambridge University Press, 1993.
6. J. Neukirch, “Algebraic Number Theory”, Springer-Verlag, 1999.

M460: Algorithm

Prerequisites: M208

L	P	T	C
3	0	1	4

Outcome: Students will learn (i) Data structure, (ii) design and analysis algorithms and (iii) some important algorithms like sortings, graph theoretics, polynomial related and optimization related.

Contents: Algorithm analysis, asymptotic notation, probabilistic analysis; Data Structure: stack, queues, linked list, hash table, binary search tree, red-black tree; Sorting: heap sort, quick sort, sorting in linear time; Algorithm design: divide and conquer, greedy algorithms, dynamic programming; Algebraic algorithms: Winograd’s and Strassen’s matrix multiplication algorithm, evaluation of polynomials, DFT, FFT, efficient FFT implementation; Graph algorithms: breadth-first and depth-first search, minimum spanning trees, single-source shortest paths, all-pair shortest paths, maximum flow; NP-completeness and approximation algorithms.

References:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, “The Design and Analysis of Computer Algorithms”, Addison-Wesley Publishing Co., 1975.
2. T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, “Introduction to Algorithms”, MIT Press, Cambridge, 2009.
3. E. Horowitz, S. Sahni, “Fundamental of Computer Algorithms”, Galgotia Publication, 1987.
4. D. E. Knuth, “The Art of Computer Programming Vol. 1, Vol. 2, Vol 3”, Addison-Wesley Publishing Co., 1997, 1998, 1998.

M462: Cryptology

Prerequisites: M202, M207

L	P	T	C
3	0	1	4

Outcome: It introduces the basics of Cryptography and cryptanalysis. Students learn theory and design of cryptographic schemes like stream ciphers, block ciphers and public key ciphers like RSA, El-Gamal, elliptic curve cryptosystem. Further, they learn about data authentication, integrity and secret sharing.

Contents: Overview of Cryptography and cryptanalysis, some simple cryptosystems (e.g., shift, substitution, affine, knapsack) and their cryptanalysis, classification of cryptosystems, classification of attacks; Information Theoretic Ideas: Perfect secrecy, entropy; Secret key cryptosystem: stream cipher, LFSR based stream ciphers, cryptanalysis of stream cipher (e.g., correlation attack, algebraic attacks), block cipher, DES, linear and differential cryptanalysis, AES; Public-key cryptosystem: Implementation and cryptanalysis of RSA, ElGamal public-key cryptosystem, Discrete logarithm problem, elliptic curve cryptography; Data integrity and authentication: Hash functions, message authentication code, digital signature scheme, ElGamal signature scheme; Secret sharing: Shamir's threshold scheme, general access structure and secret sharing.

References:

1. D. R. Stinson, "Cryptography: Theory And Practice", Chapman & Hall/CRC, 2006.
2. A. J. Menezes, P. C. van Oorschot, S. A. Vanstone, "Handbook of Applied Cryptography", CRC Press, 1997.

M463: Finite Fields

Prerequisites: M307

L	P	T	C
3	0	1	4

Outcome: This course gives a structure of finite fields, factorization of polynomials, some applications towards cryptography, coding theory and combinatorics.

Contents: Structure of finite fields: characterization, roots of irreducible polynomials, traces, norms and bases, roots of unity, cyclotomic polynomial, representation of elements of finite fields, Wedderburn's theorem; Polynomials over finite field: order of polynomials, primitive polynomials, construction of irreducible polynomials, binomials and trinomials, factorization of polynomials over small and large finite fields, calculation of roots of polynomials; Linear recurring sequences: LFSR, characteristic polynomial, minimal polynomial, characterization of linear recurring sequences, Berlekamp-Massey algorithm; Applications of finite fields: Applications in cryptography, coding theory, finite geometry, combinatorics.

References:

1. R. Lidl, H. Neiderreiter, "Finite Fields", Cambridge university press, 2000.
2. G. L. Mullen, C. Mummert, "Finite Fields and Applications", American Mathematical Society, 2007.
3. A. J. Menezes et. al., "Applications of Finite Fields", Kluwer Academic Publishers, 1993.
4. Z-X. Wan, "Finite Fields and Galois Rings", World Scientific Publishing Co., 2012.

M464: Information and Coding Theory

Prerequisites: M205, M307

L	P	T	C
3	0	1	4

Outcome: It introduces information theory and coding theory. In information theory, students learn how to measure information and encoding of information. In coding theory, students learn theory and techniques of error correcting codes like Reed-Muller codes, BCH codes, Reed-Solomon codes, Algebraic codes.

Contents: Information Theory: Entropy, Huffman coding, Shannon-Fano coding, entropy of Markov process, channel and mutual information, channel capacity;
Error correcting codes: Maximum likelihood decoding, nearest neighbour decoding, linear codes, generator matrix and parity-check matrix, Hamming bound, Gilbert-Varshamov bound, binary Hamming codes, Plotkin bound, nonlinear codes, Reed-Muller codes, Cyclic codes, BCH codes, Reed-Solomon codes, Algebraic codes.

References:

1. R. W. Hamming, "Coding and Information Theory", Prentice-Hall, 1986.
2. N. J. A. Sloane, F. J. MacWilliams, "Theory of Error Correcting Codes", North-Holland Mathematical Library 16, North-Holland, 2007.
3. S. Ling, C. Xing, "Coding Theory: A First Course", Cambridge University Press, 2004.
4. V. Pless, "Introduction to the Theory of Error-Correcting Codes", Wiley-Interscience Publication, John Wiley & Sons, 1998.
5. S. Lin, "An Introduction to Error-Correcting Codes", Prentice-Hall, 1970.

M465: Mathematical Logic

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: Students will learn Mathematical logic. It starts from the propositional logic and then first order theory. Then introduces the completeness and compactness theorems with Godels incompleteness theorem.

Contents: Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem. First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g., theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godels first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

References:

1. J. R. Shoenfield, "Mathematical logic", Addison-Wesley Publishing Co., 1967.
2. E. Mendelson, "Introduction to Mathematical Logic", Chapman & Hall, 1997.

M466: Measure Theory

Prerequisites: M301

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will learn the concept of measures and measurable functions. Students also learn integration and their various properties.

Contents: $\sigma-$algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions, L_p-spaces, Riesz representation theorem for the space $C[0,1]$.

References:

1. G. De Barra, "Measure theory and integration".
2. J. Nevue, "Mathematical foundations of the calculus of probability", Holden-Day, Inc., 1965.
3. I. K. Rana, "An introduction to measure and integration", Narosa Publishing House.
4. P. Billingsley, "Probability and measure", John Wiley & Sons, Inc., 1995.
5. W. Rudin, "Real and complex analysis", McGraw-Hill Book Co., 1987.
6. K. R. Parthasarathy, "Introduction to probability and measure", The Macmillan Co. of India, Ltd., 1977.

M467: Nonlinear Analysis

Prerequisites: M306, M401

L	P	T	C
3	0	1	4

Outcome: Students will learn Calculus in Banach Spaces and degree theory. As an application of degree theory, they will study fixed point theorems of Brouwer and Schauder. Students will also learn homotopy, homotopy extension and invariance theorems and its applications. This course is very useful for the students who want to specialize in Partial Differential Equations.

Contents: Calculus in Banach spaces, inverse and multiplicit function theorems, fixed point theorems of Brouwer, Schauder and Tychonoff, fixed point theorems for nonexpansive and set-valued maps, predegree results, compact vector fields, homotopy, homotopy extension, invariance theorems and applications.

References:

1. S. Kesavan, "Nonlinear Functional Analysis", Texts and Readings in Mathematics 28, Hindustan Book Agency, 2004.

M468: Operator Theory

Prerequisites: M401

L	P	T	C
3	0	1	4

Outcome: Upon successful completion of the course students will become familiar with concepts of C*-algebra, von-Neuman algebra and toeplitz operators and the notion of index for Fredholm operators.

Contents: Compact operators on Hilbert Spaces. (a) Fredholm Theory (b) Index, C*-algebras - noncommutative states and representations, Gelfand-Neumark representation theorem, Von-Neumann Algebras; Projections, Double Commutant theorem, L^∞ functionalCalculus, Toeplitz operators.

References:

1. W. Arveson, "An invitation to C*-algebras", Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, "Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space", Interscience Publishers John Wiley i& Sons 1963.
3. R. V. Kadison and J. R. Ringrose, "Fundamentals of the theory of operator algebras. Vol. I. Elementary theory", Pure and Applied Mathematics, 100, Academic Press, Inc., 1983.

4. V. S. Sunder, "An invitation to von Neumann algebras", Universitext, Springer-Verlag, 1987.

M469: Theory of Computation

Prerequisites: M101

L	P	T	C
3	0	1	4

Outcome: It introduces the theory of computer science. Here, the students learn (i) Automata and Language Theory by studying automata and context free language (ii) Computability theory by studying Turing machine and halting problem (iii) Complexity theory by studying P and NP class problems

Contents: Automata and Language Theory: Finite automata, regular expression, pumping lemma, context free grammar, context free languages, Chomsky normal form, push down automata, pumping lemma for CFL; Computability: Turing machines, Churh-Turing thesis, decidability, halting problem, reducibility, recursion theorem; Complexity: Time complexity of Turing machines, Classes P and NP, NP completeness, other time classes, the time hierarchy.

References:

1. J. E. Hopcroft, R. Motwani, J. D. Ullman, "Introduction to Automata Theory, Languages, and Computation", Addison-Wesley, 2006.
2. H. Lewis, C. H. Papadimitriou, "Elements of the Theory of Computation", Prentice-Hall, 1997.
3. M. Sipser, "Introduction to the Theory of Computation", PWS Publishing, 1997.

M470: Abstract Harmonic Analysis

Prerequisites: M301, M308, M401

L	P	T	C
3	0	1	4

Outcome: Knowledge on Haar measure, convolution structure on Lie group with emphasize to harmonic analysis on the groups Circle and real line.

Contents: Topological Groups: Basic properties of topological groups, subgroups, quotient groups. Examples of various matrix groups. Connected groups. Haar measure: Discussion of Haar measure without proof on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$ and simple matrix groups, Convolution, the Banach algebra $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$. Basic Representation Theory: Unitary representation of groups, Examples and General properties, The representations of Group and Group algebras, C^*-algebra of a group, GNS construction, Positive definite functions, Schur's Lemma. Abelian Groups: Fourier transform and its properties, Approximate identities in $L^1(G)$, Classical Kernels on $\mathbb{R}$, The Fourier inversion Theorem, Plancherel theorem on $\mathbb{R}$, Plancherel measure on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$. Dual Group of an Abelian Group: The Dual group of a locally compact abelian group, Computation of dual groups for $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$, Pontryagin's Duality theorem.

References:

1. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
2. H. Helson, "Harmonic Analysis", Texts and Readings in Mathematics, Hindustan Book Agency, 2010.
3. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
4. L. H. Loomis, "An Introduction to Abstract Harmonic Analysis", Dover Publication, 2011.
5. E. Hewitt, K. A. Ross, "Abstract Harmonic Analysis Vol. I", Springer-Verlag, 1979.

6. W. Rudin, “Real and Complex Analysis”, Tata McGraw-Hill, 2013.

M471: Advanced Number Theory

Prerequisites: M207, M307, M308

L	P	T	C
3	0	1	4

Outcome: This advanced course gives a brief introduction to p-adic numbers, quadratic forms, Dirichlet series and modular forms.

Contents: Review of Finite fields, Gauss Sums and Jacobi Sums, Cubic and biquadratic reciprocity, Polynomial equations over finite fields, Theorems of Chevally and Warning, Quadratic forms over prime fields. Ring of p-adic integers, Field of p-adic numbers, completion, p-adic equations, Hensel's lemma, Hilbert symbol, Quadratic forms with p-adic coefficients. Dirichlet series: Abscissa of convergence and absolute convergence, Riemann Zeta function and Dirichlet L-functions. Dirichlet's theorem on primes in arithmetic progression. Functional equation and Euler product for L-functions. Modular Forms and the Modular Group, Eisenstein series, Zeros and poles of modular functions, Dimensions of the spaces of modular forms, The j-invariant L-function associated to modular forms, Ramanujan τ function.

References:

1. J.-P. Serre, “A Course in Arithmetic”, Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
2. K. Ireland, M. Rosen, “A Classical Introduction to Modern Number Theory”, Graduate Texts in Mathematics 84, Springer-Verlag, 1990.
3. H. Hasse, “Number Theory”, Classics in Mathematics, Springer-Verlag, 2002.
4. W. Narkiewicz, “Elementary and Analytic Theory of Algebraic Numbers”, Springer Monographs in Mathematics, Springer-Verlag, 2004.
5. F. Q. Gouvêa, “p-adic Numbers”, Universitext, Springer-Verlag, 1997.

M472: Advanced Probability

Prerequisites: M206, M301

L	P	T	C
3	0	1	4

Outcome: Students will learn about measure theoretic probability starting from probability spaces to theory of martingales.

Contents: Probability spaces, Random Variables, Independence, Zero-One Laws, Expectation, Product spaces and Fubini's theorem, Convergence concepts, Law of large numbers, Kolmogorov three-series theorem, Levy-Cramer Continuity theorem, CLT for i.i.d. components, Infinite Products of probability measures, Kolmogorov's Consistency theorem, Conditional expectation, Discrete parameter martingales with applications.

References:

1. A. Gut, “Probability: A Graduate Course”, Springer Texts in Statistics, Springer, 2013.
2. K. L. Chung, “A Course in Probability Theory”, Academic Press, 2001.
3. S. I. Resnick, “A Probability Path”, Birkhäuser, 1999.
4. P. Billingsley, “Probability and Measure”, Wiley Series in Probability and Statistics, John Wiley & Sons, 2012.
5. J. Jacod, P. Protter, “Probability Essentials”, Universitext, Springer-Verlag, 2003.

M473: Algebraic Combinatorics

Prerequisites: M202, M203

L	P	T	C
3	0	1	4

Outcome: Learning the use of different algebraic technique to study the combinatorial problems

Contents: Catalan Matrices and Orthogonal Polynomials, Catalan Numbers and Lattice Paths, Combinatorial Interpretation of Catalan Numbers, Symmetric Polynomials and Functions, Schur Functions, Jacobi-Trudi identity, RSK Algorithm, Standard Tableaux, Young diagrams and q-binomial coefficients, Plane Partitions, Group actions on boolean algebras, Enumeration under group action, Walks in graphs, Cubes and the Radon transform, Sperner property, Matrix-Tree Theorem.

References:

1. R. P. Stanley, "Algebraic Combinatorics", Undergraduate Texts in Mathematics, Springer, 2013.
2. M. Aigner, "A Course in Enumeration", Graduate Texts in Mathematics 238, Springer, 2007.
3. R. P. Stanley, "Enumerative Combinatorics Vol. 2", Cambridge Studies in Advanced Mathematics 62, Cambridge University Press, 1999.

M474: Foundations of Cryptography

Prerequisites: M102, M206

L	P	T	C
3	0	1	4

Outcome: The theoretical study of cryptography which puts foundation for the study and design of real-life cryptography.

Contents: Introduction to cryptography and computational model, computational difficulty, pseudorandom generators, zero-knowledge proofs, encryption schemes, digital signature and message authentication schemes, cryptographic protocol.

References:

1. O. Goldreich, "Foundations of Cryptography - Vol. I and Vol. II", Cambridge University Press, 2001, 2004.
2. S. Goldwasser, Mihir Bellare, "Lecture Notes on Cryptography", 2008, available online from http://cseweb.ucsd.edu/ mihir/papers/gb.html

M475: Incidence Geometry

Prerequisites: M205

L	P	T	C
3	0	1	4

Outcome: Understanding different kinds of incidence structures such as projective spaces, affine spaces, generalized quadrangles, polar spaces and quadratic sets.

Contents: Definitions and Exampleas, projective planes, affine planes, projective spaces, affine spaces, collineations of projective and affine spaces, fundamental theorem of projective and affine spaces, polar spaces, generalized quadrangles, quadrics and quadratic sets.

References:

1. J. Ueberberg, "Foundations of Incidence Geometry", Springer Monographs in Mathematics, Springer, 2011.
2. L. M. Batten, "Combinatorics of Finite Geometries", Cambridge University Press, 1997.
3. E. E. Shult, "Points and Lines", Universitext, Springer, 2011.
4. L. M. Batten, A. Beutelspacher, "The Theory of Finite Linear Spaces: Combinatorics of points and lines", Cambridge University Press, 1993.
5. G. E. Moorhouse, "Incidence Geometry", 2007, available online from http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf

M476: Lie Algebras

Prerequisites: M202, M205, M307

L	P	T	C
3	0	1	4

Outcome: This course serves two purposes. (i) To introduce the basics of Lie algebras to the students who are interested in algebra and pursue further in the studies of infinite dimensional Lie algebras like Kac-Moody Lie algebras on one hand and finite dimensional Lie algebras and their representations over any field on the other hand. (ii) As Lie algebras play infinitesimal part of Lie groups, they play important role in understanding Lie groups. The theory of semisimple Lie algebras is extremely rich thanks to Cartan, Weyl.. without which, one can not understand the geometry of semisimple Lie groups and their representations and also compact Lie groups.
After having done this course, one can pursue the studies on either Lie algebras or Representation theory of Lie groups.

Contents: Definitions and Examples, Derivations, Ideals, Homomorphisms, Nilpotent Lie Algebras and Engel's theorem, Solvable Lie Algebras and Lie's theorem, Jordan decomposition and Cartan's criterion, Semisimple Lie algebras, Casimir operator and Weyl's theorem, Representations of $sl(2, F)$, Root space decomposition, Abstract root systems, Weyl group and Weyl chambers, Classification of irreducible root systems, Abstract theory of weights, Isomorphism and conjugacy theorems, Universal enveloping algebras and PBW theorem, Representation theory of semi-simple Lie algebras, Verma modules and Weyl character formula.

References:

1. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
2. K. Erdmann, M. J. Wildon, "Introduction to Lie Algebras", Springer Undergraduate Mathematics Series, Springer-Verlag, 2006.
3. J.-P. Serre, "Complex Semisimple Lie Algebras", Springer Monographs in Mathematics, Springer-Verlag, 2001.
4. N. Jacobson, "Lie Algebras", Dover Publications, 1979.

M477: Optimization Theory

Prerequisites: M102, M205

L	P	T	C
3	0	1	4

Outcome: Understanding the different techniques used to solve the linear and non-linear programming problem

Contents: Linear programming problem and its formulation, convex sets and their properties, Graphical method, Simplex method, Duality in linear programming, Revised simplex method, Integer programming, Transportation problems, Assignment problems, Games and strategies, Two-person (non) zero-sum games, Introduction to non-linear programming and techniques.

References:

1. J. K. Strayer, "Linear Programming and its Applications", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
2. P. R. Thie, G. E. Keough, "An Introduction to Linear Programming and Game Theory", John Wiley & Sons, 2008.
3. L. Brickman, "Mathematical Introduction to Linear Programming and Game Theory", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
4. D. G. Luenberger, Y. Ye, "Linear and Nonlinear Programming", International Series in Operations Research & Management Science 116, Springer, 2008.

M478: Advanced Partial Differential Equations

Prerequisites: M401, M454

L	P	T	C
3	0	1	4

Outcome: Students will learn basics of distribution Theory, Sobolev Spaces and their properties. Using Sobolev space Theory, students will learn existence theory of solutions for Dirichlet, Neuman and oblique derivative problems for second order elliptic partial differential equations. They will also learn weak and strong maximum principles, Hopf Maximum Principle and Alexandrof-Bakelmann-Pucci estimate for the solutions. This course is very useful for the students who want to specialize in Partial Differential Equations.

Contents: Distribution Theory, Sobolev Spaces, Embedding theorems, Trace theorem. Dirichlet, Neumann and Oblique derivative problem, Weak formulation, Lax–Milgram, Maximum Principles– Weak and Strong Maximum Principles, Hopf Maximum Principle, Alexandroff-Bakelmann-Pucci Estimate.

References:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Universitext, Springer, 2011.
3. R. A. Adams, J. J. F. Fournier, "Sobolev Spces", Pure and Applied Mathematics 140, Elsevier/Academic Press, 2003.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.
5. M. Renardy, R. C. Rogers, "An Introduction to Partial Differential Equations", Springer, 2008.

M479: Random Graphs

Prerequisites: M206, M208

L	P	T	C
3	0	1	4

Outcome: The aim is to learn random graphs and its applications.

Contents: Models of random graphs and of random graph processes; illustrative examples; random regular graphs, configuration model; appearance of the giant component small subgraphs; long paths and Hamiltonicity; coloring problems; eigenvalues of random graphs and their algorithmic applications; pseudo-random graphs.

References:

1. N. Alon, J. H. Spencer, "The Probabilistic Method", John Wiley & Sons, 2008
2. B. Bollobás, "Random Graphs", Cambridge Studies in Advanced Mathematics 73, Cambridge University Press, 2001.
3. S. Janson, T. Luczak, A. Rucinski, "Random Graphs", Wiley-Interscience, 2000.
4. R. Durrett, "Random Graph Dynamics", Cambridge University Press, 2010.
5. J. H. Spencer, "The Strange Logic of Random Graphs", Springer-Verlag, 2001.

M480: Randomized Algorithms and Probabilistic Methods

Prerequisites: M206

L	P	T	C
3	0	1	4

Outcome: The aim is to learn how to use probabilistic techniques to different areas of mathematics and computer science.

Contents: Inequalities of Markov and Chebyshev (median algorithm), first and second moment method (balanced allocation), inequalities of Chernoff (permutation routing) and Azuma (chromatic number), rapidly mixing

Markov chains (random walk in hypercubes, card shuffling), probabilistic generating functions (random walk in d-dimensional lattice)

References:

1. R. Motwani, P. Raghavan, "Randomized Algorithms", Cambridge University Press, 2004.
2. M. Mitzenmacher, E. Upfal, "Probability and Computing: Randomized algorithms and probabilistic analysis", Cambridge University Press, 2005.

M481: Statistical Inference I

Prerequisites: M206 and M305 or equivalent courses

L	P	T	C
3	0	1	4

Outcome: The outcome of this course is to learn about parametric statistical inference to be applicable to almost all branches of statistics. Students will learn various methods of estimation and hypothesis testing and their large sample and small sample properties.

Contents: Review: joint and conditional distributions, order statistics, group family, exponential family. Introduction to parametric inference, sufficiency principle and data reduction, factorization theorem, minimal sufficient statistics, Fisher information, ancillary statistics, complete statistics, Basu's theorem. Unbiasedness, best unbiased and linear unbiased estimator, Rao-Blackwell theorem, Lehmann- Scheffe theorem and UMVUE, Cramer-Rao lower bound and UMVUE, multi-parameter cases. Location and scale invariance, principle of equivarience. Methods of estimation: method of moments, likelihood principle and maximum likelihood estimation, properties of MLE: invariance, consistency, asymptotic normality. Hypothesis testing: error probabilities and power, most powerful tests, Neyman-Pearson lemma and its applications, p-value, uniformly most powerful (UMP) test via Neyman- Pearson lemma, UMP test via monotone likelihood ratio property, existence and nonexistence of UMP test for two sided alternative, unbiased and UMP unbiased tests. Likelihood (generalized) ratio tests and its properties, invariance and most powerful invariant tests. Introduction to confidence interval estimation, methods of fining confidence intervals: pivotal quantity, inversion of a test, examples such as confidence interval for mean, variance, difference in means, optimal interval estimators, uniformly most accurate confidence bound, large sample confidence intervals.

References:

1. E. L. Lehmann and G. Casella, "Theory of Point Estimation", 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann and J. P. Romano, "Testing Statistical Hypothesis", 3rd edition, Springer, 2005.
3. N. Mukhopadhyay, "Probability and Statistical Inference", Marcel Dekker, New York. 2000.
4. G. Casella and R. L. Berger, "Statistical Inference", 2nd edition, Cengage Learning, 2001.
5. A. M. Mood, F. A. Graybill and D. C. Boes, "Introduction to the theory of Statistics", 3rd edition, McGraw Hill, 1974.

M482: Multivariate Statistical Analysis

Prerequisites: M305, M306, and M205 or equivalent courses

L	P	T	C
3	0	1	4

Outcome: Students will learn about various modern statistical tools to analyze and draw inference from multivariate data sets. Starting from multivariate normal distribution, students will learn inference about multivariate sample mean and variance, techniques of dimension reduction, introductory factor analysis, cluster analysis and statistical pattern recognition.

Contents: Review of matrix algebra (optional), data matrix, summary statistics, graphical representations. Distribution of random vectors, moments and characteristic functions, transformations, some multivariate distributions: multivariate normal, multinomial, Dirichlet distribution, limit theorems. Multivariate normal distribution: properties, geometry, characteristics function, moments, distributions of linear combinations, conditional distribution and multiple correlation. Estimation of mean and variance of multivariate normal, theoretical properties, James-Stein estimator (optional), distribution of sample mean and variance, the Wishart distribution, large sample behavior of sample mean and variance, assessing normality. Inference about mean vector: testing for normal mean, Hotelling T^2 and likelihood ratio test, confidence regions and simultaneous comparisons of component means, paired comparisons and a repeated measures design, comparing mean vectors from two populations, MANOVA. Techniques of dimension reduction, principle component analysis: definition of principle components and their estimation, introductory factor analysis, multidimensional scaling. Classification problem: linear and quadratic discriminant analysis, logistic regression, support vector machine. Cluster analysis: non-hierarchical and hierarchical methods of clustering.

References:

1. K. V. Mardia, J. T. Kent and J. M. Bibby, "Multivariate Analysis", Academic Press, 1980.
2. T. W. Anderson, "An introduction to Multivariate Statistical Analysis", Wiley, 2003.
3. C. Chatfield and A. J. Collins, "Introduction to Multivariate Analysis", Chapman & Hall, 1980.
4. R. A. Johnson and D. W. Wichern, "Applied Multivariate Statistical Analysis", 6th edition, Pearson, 2007.
5. Brian Everitt and Torsten Hothorn, "An Introduction to Applied Multivariate Analysis with R", Springer, 2011.
6. M. L. Eaton, "Multivariate Statistics", John Wiley, 1983.

M483: Introduction to Manifolds

Prerequisites: M304

L	*P*	*T*	*C*
3	0	1	4

Outcome: This course lays the foundations of modern Differential Geometry. After taking this course, the students will get a good knowledge of smooth manifolds, tangent and cotangent spaces, vector bundles, (co)tangent bundles, vector fields, differential forms, exterior differentiation, De-Rham cohomology, integration on manifolds, homotopy invariance of De-Rham cohomology and the statement of Poincare Duality. After studying this course, students will be fully prepared to pursue further studies in (complex) algebraic geometry, theory of Riemann surfaces and Riemannian Geometry. Students will also be fully equipped to pursue further studies in analysis on manifolds, particularly the theory of Elliptic operators on smooth manifolds and Hodge Theory (which culminates in the proof of Poincare Duality). Students who are interested in either Topology, Differential Geometry, Algebraic Geometry and certain topics in analysis and PDE, with a geometric flavor (i.e. Geometric Analysis) will find this course very useful.

Contents: Differentiable manifolds and maps: Definition and examples, Inverse and implicit function theorem, Submanifolds, immersions and submersions. The tangent and cotangent bundle: Vector bundles, (co)tangent bundle as a vector bundle, Vector fields, flows, Lie derivative. Differential forms and Integration: Exterior differential, closed and exact forms, Poincaré lemma, Integration on manifolds, Stokes theorem, De Rham cohomology.

References:

1. Michael Spivak, "A comprehensive introduction to differential geometry", Vol. 1, 3rd edition, 1999.
2. Frank Warner, "Foundations of differentiable manifolds and Lie groups", Springer-Verlag, 2nd edition, 1983.
3. John Lee, "Introduction to smooth manifolds", Springer Verlag, 2nd edition, 2013.
4. Louis Auslander and Robert E. MacKenzie, "Introduction to differentiable manifolds", Dover, 2nd edition, 2009.

M551: Algebraic Computation

Prerequisites: M205, M307

L	P	T	C
3	0	1	4

Outcome: It is a unique style of course where the mathematics students having interest in computation can learn to compute different algebraic problems in computer. Here students will learn the computation of the problems related (i) linear algebra, (ii) non-linear system of equations like Grobner bases, (iii) polynomial, (iv) algebraic number theory and (v) elliptic curve.

Contents: Linear algebra and lattices: Asymptotically fast matrix multiplication algorithms, linear algebra algorithms, normal forms over fields, Lattice reduction; Solving system of non-linear equations: Gröbner basis, Buchberger's algorithms, Complexity of Gröbner basis computation; Algorithms on polynomials: GCD, Barlekamp-Massey algorithm, factorization of polynomials over finite field, factorization of polynomials over $\mathbb{Z}$ and $\mathbb{Q}$; Algorithms for algebraic number theory: Representation and operations on algebraic numbers, trace, norm, characteristic polynomial, discriminant, integral bases, polynomial reduction, computing maximal order, algorithms for quadratic fields; Elliptic curves: Implementation of elliptic curve, algorithms for elliptic curves.

References:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, "The Design and Analysis of Computer Algorithms", Addison-Wesley Publishing Co., 1975.
2. H. Cohen, "A Course in Computational Algebraic Number Theory", Graduate Texts in Mathematics 138, Springer-Verlag, 1993.
3. D. Cox, J. Little, D. O'shea, "Ideals, Varieties and Algorithms: An introduction to computational algebraic geometry and commutative algebra", Undergraduate Texts in Mathematics, Springer-verlag, 2007.

M552: Analytic Number Theory

Prerequisites: M201, M207, M308

L	P	T	C
3	0	1	4

Outcome: Expects students to learn elementary properties of Dirichlet series and distribution of primes.

Contents: Arithmetic functions, Averages of arithmetical functions, Distribution of primes, finite abelian groups and characters, Gauss sums, Dirichlet series and Euler products, Reimann Zeta function, Dirichlet L-functions, Analytic proof of the prime number theorem, Dirichlet Theorem on primes in arithmetic progression.

References:

1. T. M. Apostol, "Introduction to Analytic Number Theory", Springer International Student Edition, 2000.
2. K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, 1968.
3. H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, 2004.

M553: Classical Groups

Prerequisites: M202, M205, M307

L	P	T	C
3	0	1	4

Outcome: Understanding the basic facts about classical groups defined over fields such as General Linear groups, Special Linear groups, Symplectic groups, Orthogonal groups and Unitary groups.

Contents: General and special linear groups, bilinear forms, Symplectic groups, symmetric forms, quadratic forms, Orthogonal geometry, orthogonal groups, Clifford algebras, Hermitian forms, Unitary spaces, Unitary groups.

References:

1. L. C. Grove, "Classical Groups and Geometric Algebra", Graduate Studies in Mathematics 39, American Mathematical Society, 2002.
2. E. Artin, "Geometric Algebra", John Wiley & sons, 1988.

M554: Ergodic Theory

Prerequisites:

L	P	T	C
3	0	1	4

Outcome: The origin and motivation of studies of Ergodic theory comes from the statistical physics. One of the main branches in Analysis, it aims to give a formal mathematical treatment of movements of particles in a measure space. The important application is to study the behaviours of atoms and molecules in the ambit of aggregate systems. So, naturally the probability theory lies in the undercurrent of Ergodic theory. This theory emerged as a bridge between Probability theory, Physics and Functional analysis. It has a lot of applications in Statistical physics and mathematical biology.

Contents: Measure preserving systems; examples: Hamiltonian dynamics and Liouvilles theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product, Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlins lemma. Recurrence in Topological Dynamics, Birkhoffs Recurrence theorem, Ergodicity, Weak-mixing and strong-mixing and their characterizations, Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyls theorem, The Isomorphism problem; conjugacy, spectral equivalence, Transformations with discrete spectrum, Halmosvon Neumann theorem, Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon McMillanBreiman Theorem, Flows. Birkhoffs ergodic Theorem and Wieners ergodic theorem for flows. Flows built under a function.

References:

1. Peter Walters, "An introduction to ergodic theory", Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
2. Patrick Billingsley, "Ergodic theory and information", Robert E. Krieger Publishing Co., 1978.
3. M. G. Nadkarni, "Basic ergodic theory", Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
4. H. Furstenberg, "Recurrence in ergodic theory and combinatorial number theory", Princeton University Press, 1981.
5. K. Petersen, "Ergodic theory", Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

M555: Harmonic Analysis

Prerequisites: M301

L	P	T	C
3	0	1	4

Outcome: knowldege on Fourier Series , Fourier transforms and celebrated differentiation theorem and important operators like Hilbert transform and Maximal function.

Contents: Fourier series and its convergences, Dirichlet kernel, Frejer kernel, Parseval formula and its applications. Fourier transforms,the Schwartz space, Distribution and tempered distribution, Fourier Inversion and Plancherel theorem. Fourier analysis on L_p-spaces. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem for distribution. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem.

References:

1. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
2. E. M. Stein, G. Weiss, "Introduction to Fourier Analysis on Euclidean Spaces", Princeton Mathematical Series 32, Princeton University Press, 1971.
3. G. B. Folland, "Fourier Analysis and its Applications", Pure and Applied Undergraduate Texts 4, America Mathematical Society, 2010.

M556: Lie Groups and Lie Algebras - I

Prerequisites: M205, M304, M306

L	P	T	C
3	0	1	4

Outcome: Understanding the technique used for constructing combinatorial designs and its relation with linear codes Outcomes: The aim of this course and M564- Lie groups and Lie algebras II is to give a strong foundation on the study of Lie groups and their infinitesimal version viz., Lie algebras. The prominent role played by Lie groups in the study of Geometry and theoritical physics needs no further emphasis. This course is tremendously beneficial for the mathematics students and physics students as well. It begins with the rudiments of Lie groups and finally ends with irreducible representations of compact Lie groups parametrised by Weyl Character formula.

Contents: General Properties: Definition of Lie groups, subgroups, cosets, group actions on manifolds, homogeneous spaces, classical groups. Exponential and logarithmic maps, Adjoint representation, Lie bracket, Lie algebras, subalgebras, ideals, stabilizers, center Baker-Campbell-Hausdorff formula, Lie's Theorems. Structure Theory of Lie Algebras: Solvable and nilpotent Lie algebras (with Lie/Engel theorems), semisimple and reductive algebras, invariant bilinear forms, Killing form, Cartan criteria, Jordan decomposition. Complex semisimple Lie algebras, Toral subalgebras, Cartan subalgebras, Root decomposition and root systems. Weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, BGG resolution, Weyl character formula.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
6. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
7. V. S. Varadharajan, "Lie Groups, Lie Algebras and their Representations", Springer-Verlag, 1984.

M557: Operator Algebras

Prerequisites: M401

L	*P*	*T*	*C*
3	0	1	4

Outcome: Upon successful completion of the course students will become familiar with concepts and various structure theorems of C*-algebra and von-Neuman algebra.

Contents: Banach algebras/C*–algebras: Definition and examples; Spectrum of a Banach algebra; Gelfand transform; Gelfand-Naimark theorem for commutative Banach algebras/ C*-algebras; Functional calculus for C*-algebras; Positive cone in a C*-algebra; Existance of an approximate identity in a C*-algebra; Ideals and Quotients of a C*-algebra; Positive linear functionals on a C*-algebra; GNS construction. Locally convex topologies on the algebras of bounded operators on a Hilbert space, von-Neumann's bicommutant theorem; Kaplansky's density theorem. Ruan's characterization of Operator Spaces (if time permites).

References:

1. R. V. Kadison, J. R. Ringrose, "Fundamentals of the Theory of Operator Algebras Vol. I", Graduate Studies in Mathematics 15, American Mathematical Society, 1997.
2. G. K. Pedersen, "C*–algebras and their Automorphism Groups", London Mathematical Society Monographs 14, Academic Press, 1979.
3. V. S. Sunder, "An Invitation to von Neumann Algebras", Universitext, Springer-Verlag, 1987.
4. M. Takesaki, "Theory of Operator Algebras Vol. I", Springer-Verlag, 2002.

M558: Representations of Linear Lie Groups

Prerequisites: M205, M304, M306

L	*P*	*T*	*C*
3	0	1	4

Outcome: This course is a more basic than M556 and M564 laying foundation for the students who want to take up one of the branches of mainstream mathematics namely, non-abelian harmonic analysis. The prototype of the complications that might arise in the study of non-abelian harmonic analysis, is amply found in the study of Linear Lie groups. Yet, these linear Lie groups are plausible to understand as they are concrete examples of non-compact non-abelian Lie groups.

The course starts from the first principles of representations and goes upto understanding the important examples of 3 different types of groups, viz., compact, nilpotent and solvable groups. It is quite beneficial for the students who want to get into representation theory.

Contents: Introduction to topological group, Haar measure on locally compact group, Representation theory of compact groups, Peter Weyl theorem, Linear Lie groups, Exponential map, Lie algebra, Invariant Differentail operators, Representation of the group and its Lie algebra. Fourier analysis on $SU(2)$ and $SU(3)$. Representation theory of Heisenberg group . Representation of Euclidean motion group.

References:

1. J. E. Humphreys, "Introduction to Lie algebras and representation theory", Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram, U. B. Tiwari, "A first course on representation theory and linear Lie groups", University Press, 2000.
3. Mitsou Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
4. Sundaram Thangavelu, "Harmonic Analysis on the Heisenberg Group", Birkhauser, 1998.
5. Sundaram Thangavelu, "An Introduction to the Uncertainty Principle", Birkhauser, 2003.

M559: Harmonic Analysis on Compact Groups

Prerequisites: M205, M304, M401

L	P	T	C
3	0	1	4

Outcome: Knowledge on representaion on compact lie groups with examples $SU(2)$, $SO(n)$.

Contents: Review of General Theory: Locally compact groups, Computation of Haar measure on $\mathbb{R}$, $\mathbb{T}$, $SU(2)$, $SO(3)$ and some simple matrix groups, Convolution, the Banach algebra $L^1(G)$. Representation Theory: General properties of representations of a locally compact group, Complete reducibility, Basic operations on representations, Irreducible representations. Representations of Compact groups: Unitarilzibality of representations, Matrix coefficients, Schur's orthogonality relations, Finite dimensionality of irreducible representations of compact groups. Various forms of Peter-Weyl theorem, Fourier analysis on Compact groups, Character of a representation. Schur's orthogonality relations among characters. Weyl's Chracter formula, Computing the Unitary dual of $SU(2)$, $SO(3)$; Fourier analysis on $SO(n)$.

References:

1. T. Brocker, T. Dieck, "Representations of Compact Lie Groups", Springer-Verlag, 1985.
2. J. L. Clerc, "Les Représentatios des Groupes Compacts, Analyse Harmonique" (J. L. Clerc et. al., ed.), C.I.M.P.A., 1982.
3. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
4. M. Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
5. E. B. Vinberg, "Linear Representations of Groups", Birkhäuser/Springer, 2010.
6. A. Wawrzyńczyk, "Group Representations and Special Functions", PWN–Polish Scientific Publishers, 1984.

M560: Modular Forms of One Variable

Prerequisites: M202, M205, M207, M308

L	P	T	C
3	0	1	4

Outcome: This course gives an introduction to modular forms over 2 and their congruence subgroups, and their Hecke theory.

Contents: $SL_2(\mathbb{Z})$ and its congruence subgroups, Modular forms for $SL_2(\mathbb{Z})$, Modular forms for congruence subgroups, Modular forms and differential operators, Hecke theory, L-series, Theta functions and transformation formula.

References:

1. J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
2. N. Koblitz, "Introduction to Elliptic Curves and Modular Forms", Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Bruinier, G. van der Geer, G. Harder, D. Zagier, "The 1-2-3 of Modular Forms", Universitext, Springer-Verlag, 2008.
4. F. Diamond, J. Shurman, "A First Course in Modular Forms", Graduate Texts in Mathematics 228, Springer-Verlag, 2005.
5. S. Lang, "Introduction to Modular Forms", Springer-Verlag, 1995.
6. G. Shimura, "Introduction to the Arithmetic Theory of Automorphic Forms", Princeton University Press, 1994.

M561: Elliptic Curves

Prerequisites: M202, M207, M308

L	P	T	C
3	0	1	4

Outcome: This course gives an introduction to elliptic curves and the structure of their rational points.

Contents: Congruent numbers, Elliptic curves, Elliptic curves in Weierstrass form, Addition law, Mordell–Weil Theorem, Points of finite order, Points over finite fields, Hasse-Weil L-function and its functional equation, Complex multiplication.

References:

1. J. H. Silverman, J. Tate, "Rational Points on Elliptic Curves", Undergraduate Texts in Mathematics, Springer-Verlag, 1992.
2. N. Koblitz, "Introduction to Elliptic Curves and Modular Forms", Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Silverman, "The Arithmetic of Elliptic Curves", Graduate Texts in Mathematics 106, Springer, 2009.
4. A. W. Knapp, "Elliptic Curves", Mathematical Notes 40, Princeton University Press, 1992.
5. J. H. Silverman, "Advanced Topics in the Arithmetic of Elliptic Curves", Graduate Texts in Mathematics 151, Springer-Verlag, 1994.

M562: Brownian Motion and Stochastic Calculus

Prerequisites: M472

L	P	T	C
3	0	1	4

Outcome: Students will learn about the theory of Brownian motion and it applications to stochastic differential equations.

Contents: Brownian Motion, Martingale, Stochastic integrals, extension of stochastic integrals, stochastic integrals for martingales, Itô's formula, Application of Itô's formula, stochastic differential equations.

References:

1. H. H. Kuo, "Introduction to Stochastic Integration", Springer, 2006.
2. J M Steele, "Stochastic Calculus and Financial Applications", Springer-Verlag, 2001.
3. F. C. Klebaner, "Introduction to Stochastic Calculus with Applications", Imperial College, 2005.

M563: Differentiable Manifolds and Lie Groups

Prerequisites: M304, M306

L	P	T	C
3	0	1	4

Outcome: This course will introduce the students to the fundamentals of Lie groups and Lie Algebras. After studying this course, students will get a working knowledge of smooth manifolds, but unlike M483, this course will get into an in depth study of Lie Groups. The students will study about (bi)invariant vector fields, integration on Lie Groups, Cartan's Theorem etc. After studying this course, students will be fully equipped to study Abstract Harmonic Analysis on Lie Groups (and the construction of Haar Measure).

Contents: Review of Several variable Calculus: Directional Derivatives, Inverse Function Theorem, Implicit function Theorem, Level sets in $\mathbb{R}^n$, Taylor's theorem, Smooth function with compact support. Manifolds: Differentiable manifold, Partition of Unity, Tangent vectors, Derivative, Lie groups, Immersions and submersions, Submanifolds. Vector Fields: Left invariant vector fields of Lie groups, Lie algebra of a Lie group, Computing the Lie algebra of various classical Lie groups. Flows: Flows of a vector field, Taylor's formula, Complete vector fields. Exponential Map: Exponential map of a Lie group, One parameter subgroups, Frobenius theorem (without proof). Lie Groups and Lie Algebras: Properties of Exponential function, product formula, Cartan's Theorem, Adjoint representation, Uniqueness of differential structure on Lie groups. Homogeneous Spaces: Various examples and Properties. Coverings: Covering spaces, Simply connected Lie groups, Universal covering group of a connected Lie group. Finite dimensional representations of Lie groups and Lie algebras.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. S. Helgason, "Differential Geometry, Lie Groups and Symmetric Spaces", Graduate Studies in Mathematics 34, American Mathematical Society, 2001.
3. S. Kumaresan, "A Course in Differential Geometry and Lie Groups", Texts and Readings in Mathematics 22, Hindustan Book agency, 2002.
4. F. W. Warner, "Foundations of Differentiable Manifolds and Lie Groups", Graduate Texts in Mathematics 94, Springer-Verlag, 1983.

M564: Lie Groups and Lie Algebras - II

Prerequisites: M556

L	P	T	C
3	0	1	4

Outcome: This course is the sequel of M556 - Lie groups and Lie algebras I. It mostly deals with the representation theory of Lie groups. Lie groups that are studied in this course, are compact Lie groups and the group SL(2,C) . Another aspect of this course is to classify all simple Lie algebras through root system. As it is well known, the study of subatomic particles depend on the irreducible representations of certain Lie groups that are contained in $\mathrm{GL}(n, \mathbb{R})$. This course gives a vivid account of mathematics that is needed to understand these representations. To sum up, it is a gateway for the students of mathematics to pursue harmonic analysis of Lie groups.

Contents: General theory of representations, operations on representations, irreducible representations, Schur's lemma, Unitary representations and complete reducibility. Compact Lie groups, Haar measure on compact Lie groups, Schur's Theorem, characters, Peter-Weyl theorem, universal enveloping algebra, Poincare-Birkoff-Witt theorem, Representations of Lie($SL(2,\mathbb{C})$). Abstract root systems, Weyl group, rank 2 root systems, Positive roots, simple roots, weight lattice, root lattice, Weyl chambers, simple reflections, Dynkin diagrams, classification of root systems, Classification of semisimple Lie algebras. Representations of Semisimple Lie algebras, weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, Weyl Character formula, The representation theory of $SU(3)$, Frobenius Reciprocity theorem, Spherical Harmonics.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
6. A. W. Knapp, "Lie Groups: Beyond an introduction", Birkäuser, 2002.
7. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.

M565: Mathematical Foundations for Finance

Prerequisites: M472

L	P	T	C
3	0	1	4

Outcome: Students will learn about the mathematical modeling of simple stock markets and techniques to analyze them.

Contents: Financial market models in finite discrete time, Absence of arbitrage and martingale measures, Valuation and hedging in complete markets, Basic facts about Brownian motion, Stochastic integration, Stochastic calculus: Itô's formula, Girsanov transformation, Itô's representation theorem, Black-Scholes formula

References:

1. J. Jacod, P. Protter, "Probability Essentials", Universitext, Springer-Verlag, 2003.
2. D. Lamberton, B. Lapeyre, "Introduction to Stochastic Calculus Applied to Finance", Chapman-Hall, 2008.
3. H. Föllmer, A. Schied, "Stochastic Finance: An Introduction in Discrete Time", de Gruyter, 2011.

M566: Designs and Codes

Prerequisites: M205, M307

L	P	T	C
3	0	1	4

Outcome: Understanding the technique used for constructing combinatorial designs and its relation with linear codes.

Contents: Incidence structures, affine planes, translation plane, projective planes, conics and ovals, blocking sets. Introduction to Balanced Incomplete Block Designs (BIBD), Symmetric BIBDs, Difference sets, Hadamard matrices and designs, Resolvable BIBDs, Latin squares. Basic concepts of Linear Codes, Hamming codes, Golay codes, Reed-Muller codes, Bounds on the size of codes, Cyclic codes, BCH codes, Reed-Solomon codes.

References:

1. G. Eric Moorhouse, “Incidence Geometry”, 2007 (available online).
2. Douglas R. Stinson, “Combinatorial Designs”, Springer-Verlag, New York, 2004.
3. W. Cary Huffman, V. Pless, “Fundamentals of Error-correcting Codes”, Cambridge Uinversity Press, Cambridge, 2003.

M567: Statistical Inference II

Prerequisites: Statistical Inference I or equivalent courses

L	*P*	*T*	*C*
3	0	1	4

Outcome: Students will be introduced to decision theory and learn about Bayesian estimation and testing. Moreover, students will learn about large sample theory including asymptotic tests, confidence intervals, asymptotic efficiency and optimality of estimators and tests.

Contents: General decision problem, loss and risk function, minimax estimation, minimaxity and admissibility in exponential family. Introduction to Bayesian estimation, Bayes rule as average risk optimality, prior and posterior, conjugate families, generalized Bayes rules. Bayesian intervals and construction of credible sets, Bayesian hypothesis testing. Empirical and nonparametric empirical Bayes analysis, admissibility of Bayes and generalized Bayes rules, discussion on Bayes versus non-Bayes approaches. Large sample theory: review of modes of convergences, Slutsky's theorem, Berry-Essen bound, delta method, CLT for iid and non iid cases, multivariate extensions. Asymptotic level α tests, asymptotic equivalence, comparison of tests: relative efficiency, asymptotic comparison of estimators, efficient estimators and tests, local asymptotic optimality. Bootstrap sampling: estimation and testing.

References:

1. E. L. Lehmann and G. Casella, “Theory of Point Estimation”, 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann, “Elements of Large-Sample Theory”, Springer-Verlag, 1999.
3. E. L. Lehmann and J. P. Romano, “Testing Statistical Hypothesis”, 3rd edition, Springer, 2005.
4. James O Berger, “Statistical Decision Theory and Bayesian Analysis”, 2nd edition, Springer, New York, 1985.