Direct and inverse problems for certain sumsets in additive number theory

Abstract: Additive Number Theory is primarily the study of sumsets of subsets of an additive abelian group. Let G be an additive abelian group. Let $h \geq 2$ be an integer, and let $A_1, A_2, \ldots, A_h$ be nonempty subsets of G. The *Minkowski sumset* or simply the *sumset* of these subsets denoted by $A_1 + A_2 + \cdots + A_h$, is defined by $A_1 + A_2 + \cdots + A_h := \{a_1 + a_2 + \cdots + a_h : a_i \in A_i, i = 1, 2, \ldots, h\}$. Similarly, the *restricted sumset* denoted by $A_1 \hat{+} A_2 \hat{+} \cdots \hat{+} A_h$, is defined by $A_1 \hat{+} A_2 \hat{+} \cdots \hat{+} A_h := \{a_1 + a_2 + \cdots + a_h : a_i \in A_i, i = 1, 2, \ldots, h \text{ and } a_i \neq a_j \text{ for } i \neq j\}$. If $A_i = A$ for $i = 1, 2, \ldots, h$, then the sumset $A_1 + A_2 + \cdots + A_h$ is denoted by hA and the restricted sumset $A_1 \hat{+} A_2 \hat{+} \cdots \hat{+} A_h$ is denoted by $h^\wedge A$.

The problem of finding the minimum size of the sumsets in terms of sizes of their underlying sets is very crucial, which is a direct problem. It is also equally important to find the structure of the underlying sets when the sumsets are very close to it's minimum possible value, which is an inverse problem. For some groups, these two problems have been solved, but for many other groups these are still unsolved. In my Ph.D. thesis, we solved these two problems for some variations and generalizations of the Minkowski sumset and the restricted sumset in the group of integers. By solving the direct and inverse problems for one of those sumsets called *sum of dilates*, in the group of integers, we equivalently solved the direct and inverse problems for the Minkowski sumset in the *Baumslag-Solitar group*. The proposed talk will be a overview of my Ph.D. thesis.