Some invariants of power graphs and combinatorial properties of finite groups

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Abstract

The power graph $\mathcal{G}(G)$ of a group G is a simple and undirected graph with elements of G as the vertex set and distinct vertices u and v are adjacent if $v = u^k$ for some $k \in \mathbb{N}$ or $u = v^l$ for some $l \in \mathbb{N}$. For any integer $n \geq 2$, let C_n denote the cyclic group of order n and let r be the number of distinct prime divisors of n. The minimum degree $\delta(\mathcal{P}(C_n))$ of $\mathcal{P}(C_n)$ is known for $r \in \{1, 2\}$. In this talk, for $r \geq 3$, under certain conditions involving the prime divisors of n, we determine at most $r - 1$ vertices such that $\delta(\mathcal{P}(C_n))$ is equal to the degree of at least one of these vertices. If $r = 3$ or if n is a product of distinct primes, we identify two such vertices without any condition on the prime divisors of n. In the study of graphs constructed from groups, a sought after topic is the extent to which structure of a group is reflected by the corresponding graph. By considering it for power graphs, we characterize (non-cyclic) finite groups of prime exponent and finite elementary abelian 2-groups (of rank at least 2) in terms of connectedness of their power graphs. A nonempty subset of a group is said to be product-free if it contains product of none of its two elements. We review the results on structure and cardinality of maximal and locally maximal product-free sets of finite groups. We conclude each of the aforementioned problems with our future work in that direction.