Ph.D. Pre-Synopsis Seminar

Title : Blocking sets of certain line sets in $PG(3,q)$

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Abstract

Let q be a prime power and $PG(n,q)$ be the n-dimensional projective space over a finite field of order q. For a nonempty subset L of the line set of $PG(n,q)$, an *L-blocking* set in $PG(n,q)$ is a subset B of the point set of $PG(n,q)$ such that every line in L contains at least one point of B. Blocking sets in $PG(n,q)$ are combinatorial objects in finite geometry with several applications, and have been the subject of investigation by several researchers with respect to varying sets of lines. An important issue in this context is to determine the minimum size of an L-blocking set and if possible, to describe all L-blocking sets of that minimum cardinality. When L is the set of all lines of $PG(n,q)$, a classical result by Bose and Burton says that: if B is a blocking set in $PG(n,q)$ with respect to all its lines, then $|B| \geq (q^n-1)/(q-1)$, and equality holds if and only if B is a hyperplane of $PG(n,q)$. In view of this result, one may consider L to be a proper subset of the set of all lines of $PG(n,q)$.

Now, let $\mathcal{H}$ be a hyperbolic quadric in $PG(3,q)$, that is, a non-degenerate quadric of Witt index two. Let $\mathbb{E}$ (respectively, $\mathbb{T}$, $\mathbb{S}$) denote the set of all lines of $PG(3,q)$ which are external (respectively, tangent, secant) to $\mathcal{H}$. One can ask the following natural question:

> For $L \in \{\mathbb{T}, \mathbb{E}, \mathbb{S}, \mathbb{T}\cup\mathbb{E}, \mathbb{S}\cup\mathbb{T}, \mathbb{S}\cup\mathbb{E}\}$, what are the minimum size L-blocking sets in $PG(3,q)$?

This question was answered by Biondi et al. [2007, 2009] for all $q \geq 8$ when $L = \mathbb{E}$, and by Sahoo and Sastry [2016] for all even $q \geq 4$ when $L = \mathbb{S}\cup\mathbb{E}$. In this thesis we answer the above question for $L \in \{\mathbb{E}, \mathbb{S}, \mathbb{T}\cup\mathbb{E}, \mathbb{S}\cup\mathbb{T}, \mathbb{S}\cup\mathbb{E}\}$ and for all q. For $L = \mathbb{T}$, we have a complete answer for all even q and for $q = 3$. Along the way, we also discuss the minimum size blocking sets in $PG(2,q)$ of similar line sets defined with respect to an irreducible conic.

References

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