Open Problems

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Outline

The Standard Model

A renormalizable quantum field theory of two interactions: strong and electro-weak. Complete theory of everything known except gravity.

So what is the problem?

1. Does it really account for almost everything?
2. Does it give a good account of “bound states”?
3. Is the theory “natural” or “fine-tuned”?

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Natural units

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In cosmology, also realize that $[G] = 1$. So gravitational physics has no dimensions. Natural length scale of any mass is its Schwartzchild radius: $R = 2M$. For the sun $M_s = 47.9 \times 10^{-15}$ pc! But measurement shows $R_s = 0.22 \times 10^{-9}$ pc. Is it natural to have $R_s/M_s \simeq 0.5 \times 10^6$?

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Answer: quantum theory. New constant $\hbar$ implies that there is a dimensionless quantity which one can build out of the electron's charge: $\alpha = e^2/(4\pi\hbar)$. And $\omega \simeq \alpha^2/a$.

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In a 1-loop computation in QED we find the running coupling

$$\alpha(Q^2) = \frac{1}{1 - \beta_0 \log(Q^2/\Lambda^2_{QED})}, \qquad \beta_0 = \frac{1}{3\pi^2}$$

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Using the 1-loop formula, given $\alpha = 1/137$ for $Q = \alpha^2 m_e$ we find

$$\frac{m_e}{\Lambda_{QED}} \simeq \frac{\mathrm{e}^{-204\pi^2}}{19 \times 10^3} \ll 1$$

So the real question: is the electron mass natural?

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Technical naturalness

Dimensionless ratios of fundamental quantities are expected to be of order 1, except when there is an increased symmetry if they vanish. **'t Hooft 1980**

The wrong naturalness problem of the SM

In the standard model the Higgs part of the Lagrangian

$$\mathcal{L}_H = -\frac{1}{2}\mu^2 H^2 + \frac{1}{2}(\partial_\mu H)^2 + \lambda H^4.$$

As a result, the 1-loop contribution to the Higgs self energy is

$$\lambda \int_0^{\Lambda_{EW}} \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - M^2} \propto \lambda \Lambda_{EW}^2.$$

Quadratically divergent with Λ_{EW}.

Susskind 1979

Wrong, since SM is renormalizable, and there are no divergences.
Real problem must be clear after formulating this properly.

The hierarchy problem

SM fitted to physical data gives $M_H = 125$ GeV.

But SM is renormalizable, so all loop integrals can be taken without an UV cutoff. However, gravity becomes a strong force at energies of $M_P \simeq 10^{19}$ GeV. At these scales graviton loops must be taken into account.

There is a hierarchy of experimentally known scales

$$\frac{M_H}{M_P} \simeq 10^{-17}.$$

What is this due to? Why is the scale of gravity so different?

Note the related question:

$$\frac{\Lambda_{QCD}}{M_P} \simeq 10^{-19}.$$

Why?

Non-(quantum gravity) solutions?

No theory of quantum gravity: are there extra dimensions anywhere in the range of length scales between 1 TeV^{-1} and $1/M_P$? If yes, then the hierarchy problem is reduced. No idea!

Can one replace this by a non-gravity theory? If yes, and this theory exists at scales of 1–10 TeV, then it could produce particles which can be seen at the LHC or ILC. Nothing seen yet, but can hope!

It can induce higher-dimensional operators to be added to the Lagrangian of the standard model: effective field theory. Can these be seen? Not yet, but one can work hard on them.

Outline

1. What has been achieved?
2. Naturalness and hierarchy problems
3. **The desert**
4. Unexpected discoveries
5. Summary

Doldrums at the LHC

If there is new physics at the scale $\Lambda_{EW} = 4\pi M_H \simeq 1.6$ TeV, then at LHC one should see the effects of new operators

$$(H^\dagger D_\mu H)^2, \qquad (H^\dagger D_\mu H)(\overline{Q}\gamma_\mu Q), \qquad (H^\dagger D_\mu H)(\overline{L}\gamma_\mu L), \cdots$$

These need to be added into the Lagrangian of the SM with coupling constants of the form c/Λ_{EW}^2.

The natural values $c = \mathcal{O}(1)$ have been ruled out to over 5 sigma for almost all dimension-6 terms.

One can turn the question around and ask if $c = \mathcal{O}(1)$, then how large must Λ_{EW} be? For many of these operators, $\Lambda_{EW} > 10$ TeV with current LHC data.

Little or no hierarchy

The hierarchy in the SM

$$\frac{\Lambda_{QCD}}{M_H} \simeq 0.003 \ll 1$$

is not bothersome. Slightly different logarithmic running of couplings in SM can give small hierarchies of this kind.

Pushing Λ_{EW} to 100 TeV will cause a tension of sorts, but may still be solved by logarithmic running of couplings in the theory BSM. Small hierarchies of this kind are solvable problems.

However, a small hierarchy of this kind still leaves a big hierarchy problem:

$$\frac{\Lambda_{EW}}{M_P} \simeq 10^{-14}.$$

No ideas on how to solve these.

Beyond small coupling theories

Other complicated theories exist where large or small dimensionless numbers are regularly found. Example: Navier-Stokes equations. Reynolds numbers can range from 0.1 to 10^9 for familiar systems.

Not a problem because the equations are not linear. Not solved in perturbation theory. Numerical solutions are regularly used.

Within the SM the sector of QCD has strongly non-perturbative physics and shows a hierarchy of scales—

1. an exponential spectrum of hadrons (which has been explored only up to about 3 GeV)
2. asymptotic scales for total cross sections are 10^4 TeV,
3. ...

Large anomalous dimensions

Typically anomalous dimensions of composite operators computed in perturbation theory:

$$[\psi] = \frac{3}{2} \qquad \text{so} \qquad [\psi^n] = \frac{n}{2} + \mathcal{O}(g^2).$$

However baryon in QCD is a fermion, and EFTs will treat it as a single fermion field. So one has

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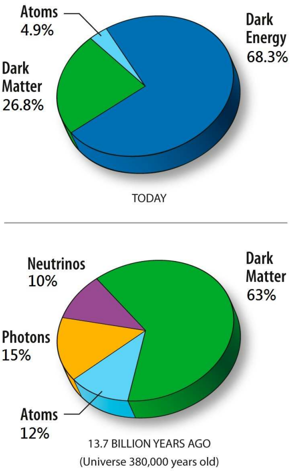
In many modern UV completions of theories BSM, several SM fields appear as composite operators. Their scaling dimensions are not computable in perturbation theory.

Implies that UV completions of SM could have strongly non-perturbative components. These could be used to push Λ_{EW} up to 10^{15} GeV and get rid of the hierarchy problem.

Outline

Dark Energy



Dark energy: observed in the large scale structure of the universe.

$$\frac{\Lambda_C}{M_P} = 3 \times 10^{-122}.$$

Completely unknown physics.

Corresponding length scale:

$$\frac{1}{\Lambda_C} = 2 \times 10^{70}\ \mathrm{pc}$$

No other experimental handle on such scales.

Dark Matter

Dark matter: observed at length scales of galaxies. Galactic mass $M_g \simeq 10^{12} M_s$ and radius $R_g = 30 Kpc$. This means that

$$\frac{R_g}{M_g} \simeq 0.5 \times 10^6$$

Different from all known scales, but perhaps within reach.

1. If dark matter interacted strongly enough to clump together, then there would have been some clumps closer to the center of the galaxy than the sun. The absence of lensing events due to such clumps then implies that dark matter interacts weakly.
2. Since density of atoms inferred from CMB anisotropies is consistent with that from models of nucleo-synthesis, this implies that CDM is not baryonic.

Dark matter around the sun

The standard values for the local density and velocity of dark matter are

$$\rho_\chi = 0.3\mathrm{GeV/cm}^3 = 2.3\ \mathrm{eV}^4$$
$$\overline{v}_\chi = 270\mathrm{Km/s} = 0.9 \times 10^{-3}.$$

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If $M_{DM} = 10$ TeV, then typical electroweak collisions would give cross sections of about 10^{-12} barns or 10^{-40} cm^2. LHC bounds are much lower; dark matter interacts weaker than EW.

Why not $\sigma_{N\chi} = 0$?

If the only interaction of normal matter and dark matter was gravitational, then dark matter produced in the early universe would not annihilate. The relic density, ρ_χ then would be just the primordial density, ρ_χ^0, diluted by Hubble expansion.

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The primordial density would be governed by the rate of production of χ by gravitational processes: for example, graviton fusion to give χs. This process would naturally generate $\rho_\chi^0 \simeq M_P^{-2}$ around the inflationary epoch. Even assuming 60 e-fold dilution since then, the relic density of χ would overclose the universe by a factor of nearly 10^{100}. So dark matter must have been in chemical equilibrium with normal matter at early times, implying $\sigma_{N\chi} > 0$.

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Summary

1. The naturalness problem of SM transmutes to the hierarchy problem due to renormalizability. The hierarchy problem may be changed by new discoveries at any scale between TeV and M_P.
2. The gauge hierarchy problem is a mismatch of Λ_{EW} and M_P. Possible discoveries at LHC (or ILC) would still leave a hierarchy problem from 100 TeV to M_P.
3. There are models which push the scale to 10^{15} GeV or so, thus solving the hierarchy problem. But these models require non-perturbative physics.
4. Non-collider discoveries (dark energy, dark matter) open up new windows for the study of matter and energy.