Natural units in High Energy Physics (i)

Fundamental units ~~are~~ have dimension of length (L), mass (M) and time [t].
All other units are derived from these.

Fundamental constants relevant in high energy physics are
i) velocity of light $c = 2.99 \times 10^{10}$ cm/sec. $\simeq 3 \times 10^{8}$ m/s

ii) Plancks constant divided by 2π : $\hbar = 6.6 \times 10^{-22}$ MeV. Sec
In other units, $\hbar = 1.055 \times 10^{-27}$ erg. sec
$= 1.055 \times 10^{-34}$ J.s $= 1.055 \times 10^{-34}$ kg m^2/s

Velocity of light appears in the relation between mass, energy & momentum : $E^2 = p^2c^2 + m^2c^4$

Planck's constant relates the momentum of a particle to de Broglie wavelength $\lambda = 2\pi\hbar/p$

Natural units correspond to the convention of setting
$c = 1, \quad \hbar = 1$

$\hbar . c = 6.58 \times 10^{-22}$ MeV. sec $\times 2.99 \times 10^{10}$ cm/sec $= 1.973 \times 10^{-11}$ MeV. cm.
$\because \hbar c = 1$ in natural units, we get
$1 \text{ fm} = 10^{-13} \text{ cm} = \frac{1}{1.973} \times 10^{-2} \text{ MeV}^{-1} = \frac{1}{197.3} \text{ MeV}^{-1} \simeq 5 \text{ GeV}^{-1}$

or, $1 \text{ fm} = \frac{1}{200 \text{ MeV}} = \hbar c / Mc^2$ where mass unit : $M = 1$ GeV
Again in particle physics, preferred length unit is
1 fermi $= 1$ fm $= 10^{-15}$ m $= 10^{-13}$ cm.
Proton radius ~ 1 fm. $= r_p$
Xsec^n is measured in typical unit of barn $= 10^{-24}$ cm^2
Radius of proton corresponds to cross section $\pi r_p^2 = 31$ mb.
At LHC energies W production xsec^n ~ 1 nb $= 1$ nanobarn
$= 10^{-33}$ cm^2

Natural Units in HEP (2)

Equivalence relations in natural units

$c = 1 \Rightarrow [c] = [L]/[T] = 1 \Rightarrow [L] = [T]$

ie in natural units dimension of length and time are equivalent

$\hbar = 1 \Rightarrow [E][T] = 1 \Rightarrow [E] = [T]^{-1} = [L]^{-1} = [M]$

$\because E^2 = (pc)^2 + (mc^2)^2$, E, pc, mc^2 all have dimension $[E]$

with $c = 1$, $[m] = [E]$ and similarly $[p] = [E]$.

Thus the conversion of SI units to natural units are

$m = m_{NU}/c^2$, $p = p_{NU}/c$, $L_{SI} = L_{NU} \times \hbar c$

$\therefore$ [xsection] $= [L]^2 = [E]^{-2} \rightarrow \sigma_{SI} = \sigma_{NU} \times (\hbar c)^2$

Similarly [time] $= [E]^{-1} \Rightarrow t_{SI} = t_{NU} \times \hbar$

Reaction decay rate : $d/dt \Rightarrow$ [rate] $= [T]^{-1} \Rightarrow R_{SI} = R_{NU}/\hbar$

Note, $c = 3 \times 10^8$ m/s $\Rightarrow GeV^{-1} \sim 6 \times 10^{-25}$ /s

With fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

we choose $\epsilon_0 = 1$ & $\mu_0 = 1$ so as to keep $c = 1/\sqrt{\mu_0 \epsilon_0} = 1$

$\Rightarrow e = \sqrt{4\pi\alpha} \simeq 0.303$ $\rightarrow$ dimensionless!

Conversion factors between SI units & natural units

$1\,GeV = 10^9\,eV = 1.60 \times 10^{-10}\,J$, $c = 3 \times 10^8$ m/s.

$\hbar = 6.58 \times 10^{-25}$ GeV/sec., $\hbar c = 0.197$ GeV. fm.

$(\hbar c)^2 = 389\,GeV^2\,\mu b$ $\because 1\,\mu b = 10^{-30}\,cm^2$

$1\,GeV^{-2} = 389\,\mu b$

Further, ~~$\hbar c = 6.58 \times 10^{-25}$ GeV/s $\times$ 3×10^8 m/s~~