SERC School - Flavour physics tutorial

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1. Define isospin in terms of the strong interaction and explain its relationship to SU(2) symmetry.

   Further define the charge conjugation operator $\hat{C}$ and explain what observation led to the conclusion that the π^0 is an eigenstate of $\hat{C}$ with an eigenvalue $\eta_C = +1$.

   Define the G-parity operator $\hat{G}$ and explain how $\pi^{\pm}$ as well as π^0 are G-parity eigenstates.

   First, show that $\hat{G}\,|\pi^0\rangle = -\,|\pi^0\rangle$, then $\hat{G}\,|\pi^{\pm}\rangle = -\,|\pi^{\pm}\rangle$, stating clearly any assumptions you make. *Hint: you will need* $Y^l_m(\theta,\phi) = P_l(\cos\theta)\,e^{im\phi}$ *and ladder operators with the phase convention*

   $$\hat{I}_{\pm}\,|I\ I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\,|I\ I_3 \pm 1\rangle\ .$$

   Given that the eigenvalue of $\hat{G}$ of a light meson is $(-1)^I \eta_C(h^0)$, where $\eta_C(h^0)$ is the eigenvalue of the neutral member of the isospin multiplet, explain why

   $$\frac{\Gamma\left(\omega \to \pi^+\pi^-\right)}{\Gamma\left(\omega \to \pi^+\pi^-\pi^0\right)} = 0.017\ .$$

   Draw a Feynman diagram of $\omega \to \pi^+\pi^-$ decay.

2. The Δ^+ baryon decays strongly into either $\pi^+ n$ and $\pi^0 p$. Use the isospin of the initial and final states to calculate the branching fractions for these two decays.

3. Show that for two pions with zero relative orbital angular momentum, the combination $\pi^+\pi^-$ is an eigenstate of CP with eigenvalue $+1$ and $\pi^+\pi^-\pi^0$ is an eigenstate of CP with eigenvalue -1. [Perkins Problem 3.3]

4. Estimate the relative rates of $D^0 \to K^-\pi^+$, $D^0 \to K^+\pi^-$ and $D^0 \to \pi^+\pi^-$.

   Given that the partial rate $\Gamma(K^+ \to \pi^0 e^+ \nu) = 4 \times 10^6\ \mathrm{s}^{-1}$, calculate the rate for $D^0 \to K^- e^+ \nu$. Hence, estimate the lifetime of the D^0 given the branching fraction for $D^0 \to K^- e^+ \nu$ is 3.55%. [Halzen and Martin 12.21 and 12.22]

5. Show that the charge-lowering weak current (i.e. $e^+ \to W^+ \bar{\nu}_e$) is $J^{\dagger}_{\mu}$, where J_{μ} is the charge-raising weak current (i.e. $e^- \to W^- \nu_e$). Hence explain why imaginary CKM matrix elements can lead to CP violating observables.

6. Explain why the rows (and columns) of the CKM matrix are orthonormal to one another. Show that the orthogonality of the 1st and 3rd columns yields a relation $1 + z_1 + z_2 = 0$ where $z_1 = \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}$ and $z_2 = \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}$. Using the Wolfenstein parameterization of the CKM matrix and the current best values of the parameters $\lambda = 0.225$, $A = 0.811$, $\rho = 0.134$ and $\eta = 0.353$ draw a suitable triangle in the Argand plane representing the orthogonality condition. Given that the measured uncertainty on the ρ is approximately 20% do you think the orthogonality relation is satisfied within the uncertianties?

7. Assuming that CP is conserved in the decays of neutral kaons, show that the neutral kaon states with definite lifetimes are given by

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left\{|K^0\rangle + |\bar{K}^0\rangle\right\} \quad \text{and} \quad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left\{|K^0\rangle - |\bar{K}^0\rangle\right\} ,$$

where $|K^0\rangle$ and $|\bar{K}^0\rangle$ are the eigenstates of the strong interaction.

Explain why the lifetimes of the neutral kaons are very different. Would you expect the lifetimes of the equivalent neutral B mesons to be different?

Mixing of the pure CP even and odd states in $|K_{\rm S}\rangle$ and $|K_{\rm L}\rangle$ can be written as

$$\begin{aligned} |K_{\rm S}^0\rangle &= A\left\{(1+\epsilon)\,|K^0\rangle + (1-\epsilon)\,|\bar{K}^0\rangle\right\} \\ |K_{\rm L}^0\rangle &= A\left\{(1+\epsilon)\,|K^0\rangle - (1-\epsilon)\,|\bar{K}^0\rangle\right\} , \end{aligned}$$

where A is the appropriate normalisation and $\epsilon = |\epsilon|\,e^{i\phi}$. It is a good approximation to assume that ϵ is the only parameter needed to account for CP violation in kaon decays. Estimate $|\epsilon|$ from the data given at the end of the question.

Show that the charge asymmetry δ defined by

$$\delta = \frac{\Gamma\left(K_{\rm L}^0 \to \pi^- e^+ \nu_e\right) - \Gamma\left(K_{\rm L}^0 \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(K_{\rm L}^0 \to \pi^- e^+ \nu_e\right) + \Gamma\left(K_{\rm L}^0 \to \pi^+ e^- \bar{\nu}_e\right)} \approx 2\Re\epsilon ,$$

if one neglects quantities of the order $|\epsilon|^2$. Given that $\phi = 0.785$ rad, calculate δ and compared it to the measured value derived from the data at the end of the question.

From the above unambiguously define positive and negative electric charge.

[Useful data for the decay modes of $K_{\rm S}^0$ and $K_{\rm L}^0$ mesons are given in the table below. In addition, note $\tau_{K_{\rm S}^0} = 0.89 \times 10^{-10}$ s and $\tau_{K_{\rm L}^0} = 5.17 \times 10^{-8}$ s.]

	Branching fractions	
Decay mode	K_S^0	K_L^0
$\pi^+\pi^-$	0.686	0.002
$\pi^- e^+ \nu_e$	—	0.19463
$\pi^+ e^- \bar{\nu}_e$	—	0.19337

8. Explain why the CKM matrix can be described by just four parameters.

   Calculate the maximum momentum in the rest frame of the B meson of the electron coming from (a) the decay $B^0 \to D^- e^+ \nu_e$ and (b) the decay $B^0 \to \pi^- e^+ \nu_e$.

   Hence, explain why counting the number of muons with a momentum greater than 2.3 GeV coming from B decays is one of the most accurate methods to determine $|V_{ub}|$. ($m_B = 5280$ MeV, $m_D = 1870$ MeV, $m_\pi = 140$ MeV and $m_\mu = 106$ MeV.)

9. Draw a 2nd order weak diagram that changes a B^0 mesons into $\overline{B}^0$. Calculate the probability that a B^0 meson is a $\overline{B}^0$ one lifetime (1.5 ps) after it was produced. [The difference between heavy and light eigenstates of the B^0 is 3.34×10^{-10} MeV.]

10. Design flavour experiments to measure (a) time-dependent CP violation in the decay $B_s^0 \to J/\psi\phi$, (b) to search for the rare decay $B^0 \to \nu\bar{\nu}$, and (c) the rare decay $K^+ \to \pi^+\nu\bar{\nu}$. Highlight the key features to produce the samples and to detect the decays.