# Relativistic Spin-magnetohydrodynamics from Kinetic Theory

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**Based On :** PRL 129, 192301 (2022)

**Collaborators :** Wojceich Florkowski, Amaresh Jaiswal, Radoslaw Ryblewski, Avdhesh Kumar

Introduction & Motivation

Relativistic Spin-magnetohydrodynamics :

Summary and Outlook :

# Heavy-ion Collisions :

- Signatures of Quark-Gluon Plasma phase found in collider experiments.

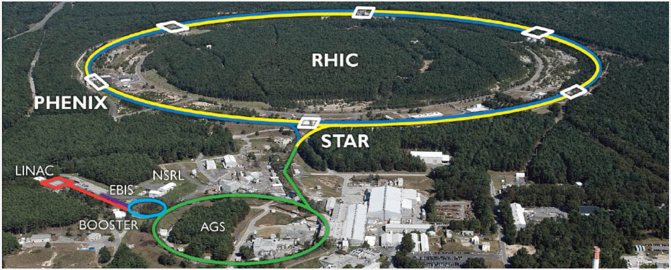



Figure 1: Relativistic Heavy-Ion Collider, BNL. [U.S. Department of Science.]


Figure 2: Large Hadron Collider, CERN. [FORBES, 2016.]

# Features of Non-central Collisions :

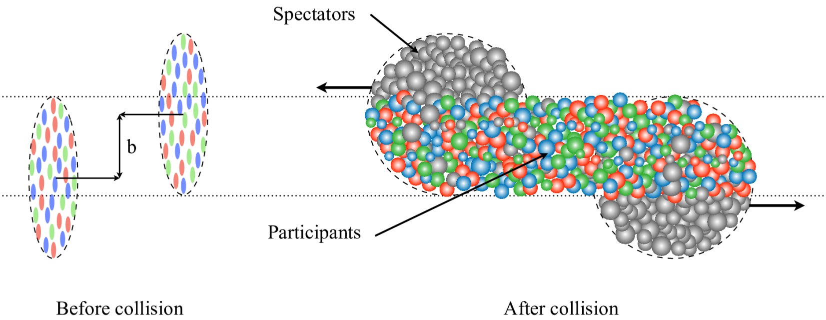



Figure 3: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Properties of the matter produced :
  - Behaves like a fluid (Hydrodynamics applicable).
  - The viscosity ($\eta/s$) is lowest (Dissipative hydrodynamics required).
  - The vorticity is highest (for non-central collisions).

# Features of Non-central Collisions :

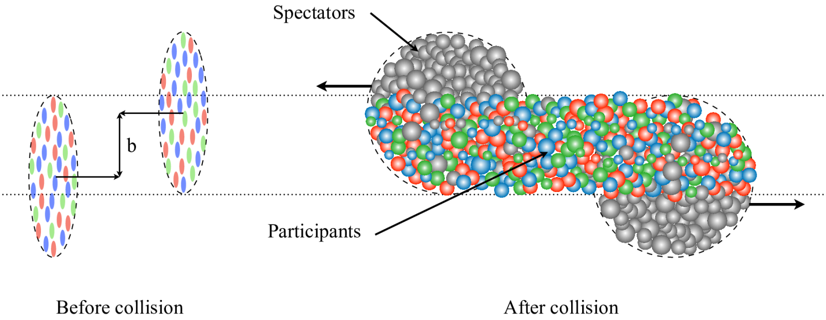



Figure 3: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Special feature of Non-Central Collisions :
  - Large Angular Momentum. [F. Becattini et. al. PRC 77 (2008) 204906]
  - Large Magnetic Field. [A. Bzdak and, V. Skokov, PLB 710 (2012) 171–174]
  - Particle polarization at small $\sqrt{S_{NN}}$. [STAR Collaboration, Nature 548 62-65, 2017]

## Generation of Angular Momentum :

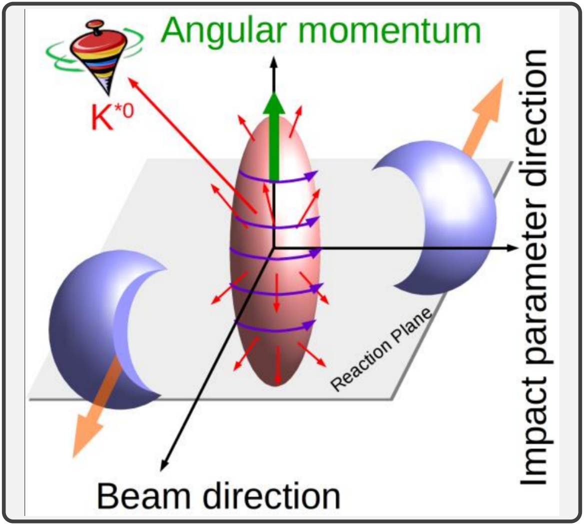



Figure 4: Generation of angular momentum in non-central collisions. [B. Mohanty, ICHEP 2020]

# Generation of Angular Momentum :

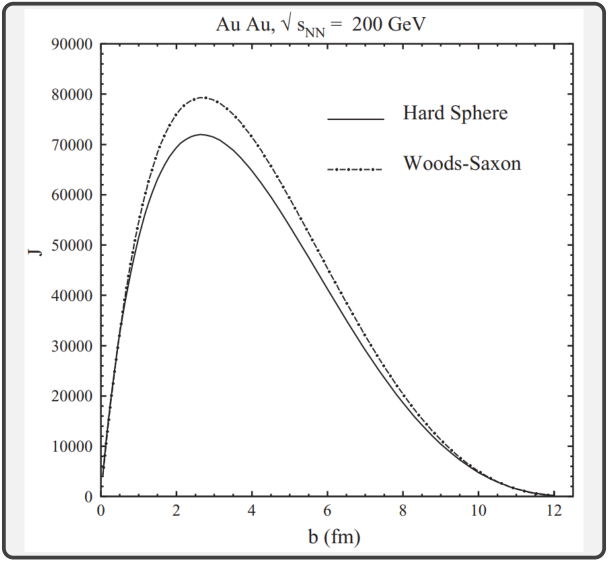



Figure 5: Angular momentum vs impact parameter. [Becattini, Piccinini and, Rizzo, PRC 77 (2008) 204906]

# Generation of Magnetic Field :

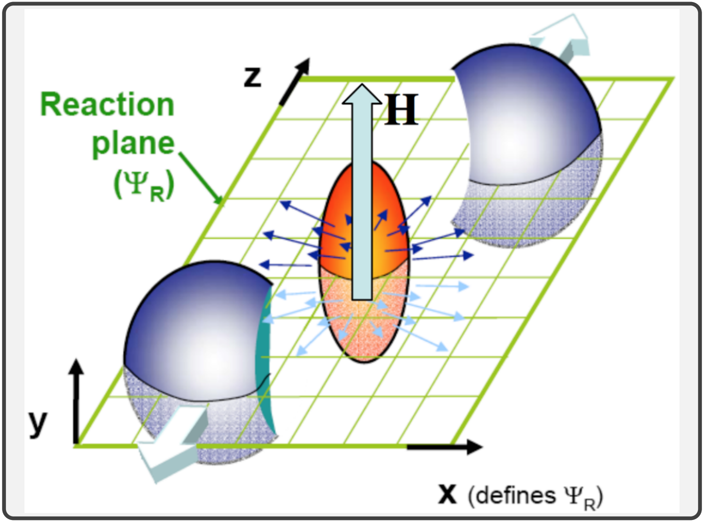



Figure 6: Generation of magnetic field in non-central collisions. [D. E. Kharzeev, PPNP 75 (2014) 133–151]

# Generation of Magnetic Field :

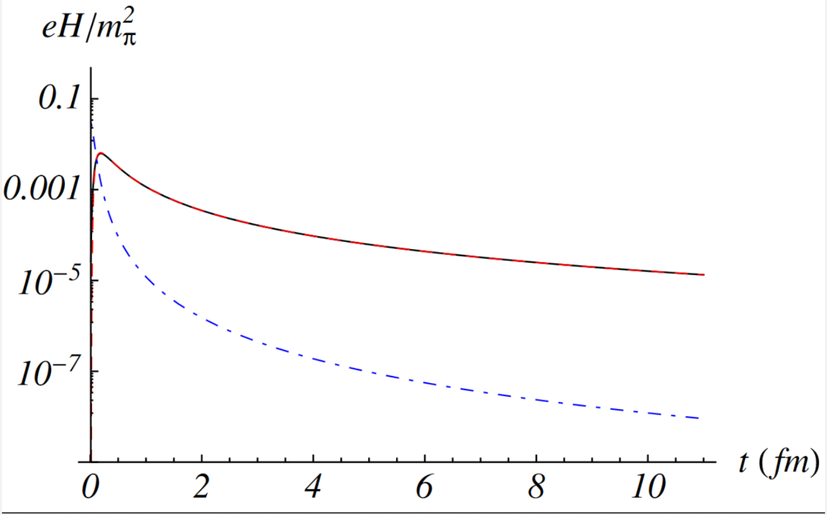



Figure 7: Time evolution of magnetic field. [K. Tuchin, IJMPE 23, No. 1 (2014) 1430001]

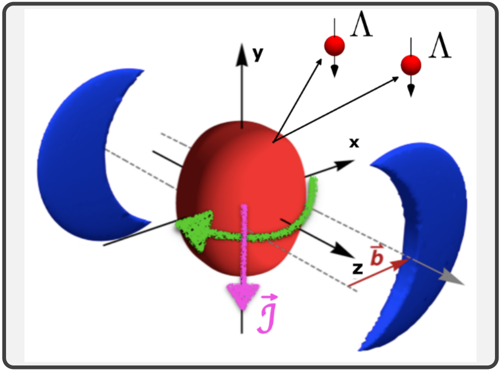



Figure 8: Origin of particle polarization. [W. Florkowski *et al*, PPNP 108 (2019) 103709]

- Large angular momentum → Local vorticities → spin alignment.

  [Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005); Phys. Lett. B 629, 20 (2005)]

# Particle Polarization :

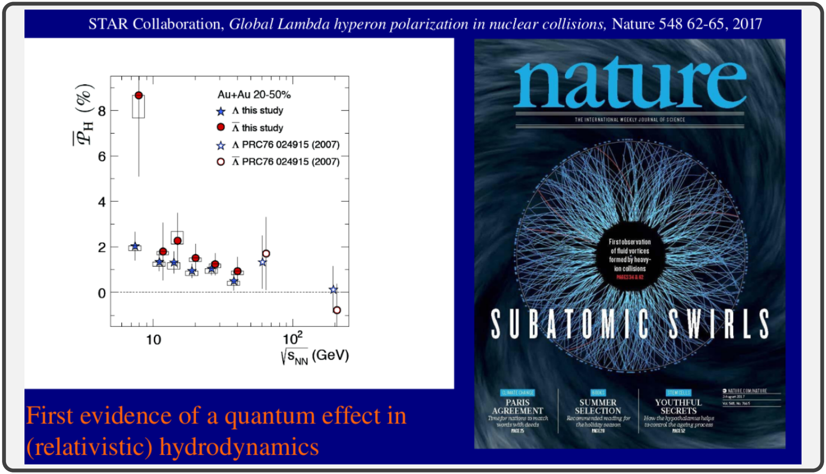



Figure 9: First observation of $\Lambda$-hyperon polarization. [F. Becattini - 'Subatomic Vortices'.]

- STAR collaboration of RHIC provided the first experimental evidence.

  [STAR Collaboration, Nature 548, 62 (2017), Phys. Rev. Lett. 123, 132301 (2019), Phys. Rev. Lett. 126, 162301 (2021)]

- Theoretical models assuming equilibration of spin d.o.f. explains the data.

# Particle Polarization :

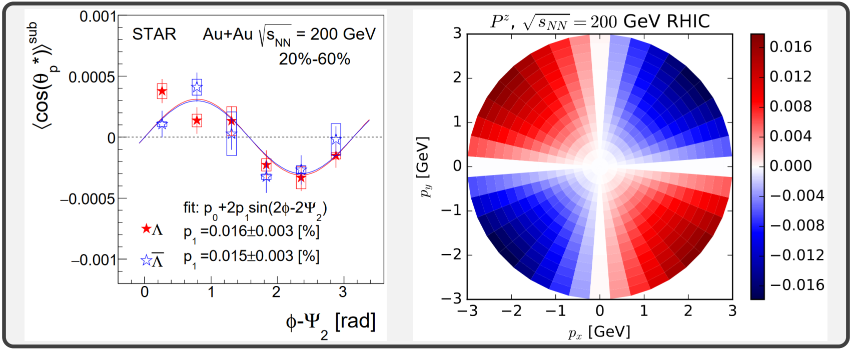



Figure 10: Observation (L) and prediction (R) of longitudinal polarization.
[Left: PRL **123** 132301 (2019); Right: PRL **120** 012302 (2018)]

- Theoretical models assuming equilibration of spin d.o.f. predict the opposite sign.
- Not enough time to thermalize. Dissipative forces at play?

# Einstein-de Haas effect :

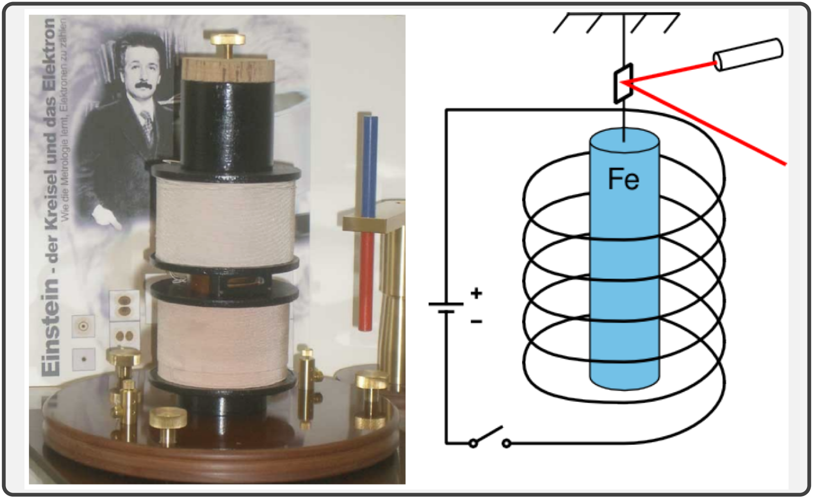
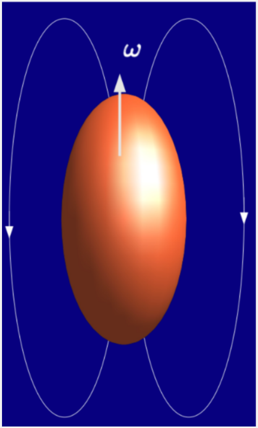
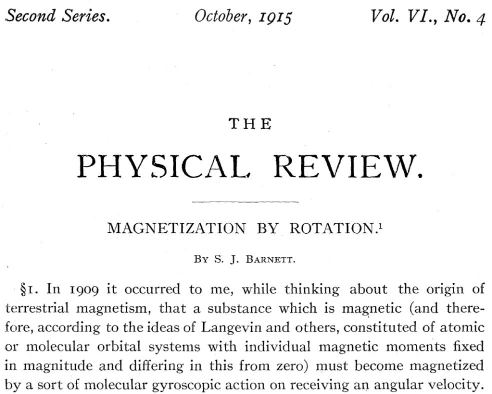



Figure 11: Einstein-de Haas effect. [Amaresh Jaiswal - Excited QCD 2022]

Magnetic field aligns electron spins $\rightarrow$ Matter rotates to conserve angular momentum.

# Barnett effect :





*Second Series.* *October, 1915* *Vol. VI., No. 4*

THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

BY S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

Figure 12: Barnett effect. [Amaresh Jaiswal - Excited QCD 2022]

Non-zero angular momentum → Generation of magnetic field.

# Summary of the Problem :

The main problem we wish to address is :

- To describe a relativistic fluid that is both magnetizable and polarizable.
  - $\rightarrow$ Formulate Dissipative Spin-magnetohydrodynamics.

## Section Outline :

Introduction & Motivation

Relativistic Spin-magnetohydrodynamics :

Summary and Outlook :

# Relativistic Spin-hydrodynamics :

- Inspired by the success of Relativistic Hydrodynamics (RH) in explaining the multitude of properties of QGP evolution, development of a framework of RH with spin was started.

  [F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

  [W. Florkowski *et al*, PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017]

  [D. Montenegro *et al*, PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016]

How to include internal degrees of freedom in a macroscopic theory?

[J. Weyssenhoff, A. Raabe, Acta Phys. Pool. 9 (1947) 7]

# Relativistic Spin-hydrodynamics :

- Origin of spin is purely quantum mechanical.

- Any theory with spin should be built up from Quantum Field Theory (QFT).

- For a hydrodynamic description of a spin-polarizable fluid starting from QFT, it was proved that a spin-polarization tensor ($\omega^{\mu\nu}$) must be introduced.

  [F. Becattini *et al*, PLB 789 (2019) 419-425]

- It has been argued that, at global equilibrium, the spin-polarization tensor should be same as the thermal vorticity.

  [F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

  [N. Weickgenannt *et al*, PRL 127 (2021) 5, 052301]

$$\boxed{\omega^{\mu\nu}|_{\text{geq}} \propto \varpi^{\mu\nu} = \left(\partial^{\mu}\beta^{\nu} - \partial^{\nu}\beta^{\mu}\right)/2}$$

$\beta^{\mu} = u^{\mu}/T$ is the inverse temperature four-vector.

- A theory of ideal spin-hydrodynamics was formulated for fluids in equilibrium.

  [W. Florkowski *et al*, PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017]

  [D. Montenegro *et al*, PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016]

- But, we want description of fluid with non-thermalized spin, where the relation, $\omega^{\mu\nu}|_{\rm geq} \propto \varpi^{\mu\nu}$ may not hold.

- Here, we want to understand, how $\omega^{\mu\nu}$ and hence a out-of-equilibrium system of spin-polarizable particles evolves in presence of magnetic field.

# Covariant Magnetohyrodynamics :

- In the limit of infinite conductivity, field strength tensor is,

$$F^{\mu\nu} \rightarrow B^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\, u_\alpha\, B_\beta$$

- $B^\mu$ is orthogonal to $u^\mu$ and spacelike i.e. $u_\mu B^\mu = 0$ and, $B_\mu B^\mu \leq 0$.

[G. S. Denicol et al. Phys.Rev.D 98 (2018) 7, 076009; A. K. Panda et al., JHEP 03 (2021) 216]

- If the medium if magnetizable, then the Maxwell's equations are given by,

$$\partial_\mu H^{\mu\nu} = J^\nu, \qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0,$$
$$\left(\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\, F_{\alpha\beta}\right)$$

where, $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$ is the induction tensor and $M^{\mu\nu}$ is the magnetization tensor, $J^\nu$ is the charged four-current.

[Balakin, Grav.Cosmol. 13 (2007) 163-177; Hehl and, Obukhov, Phys. Lett. A 311, 277 (2003)]

## Current Sources :

- The charged four-current may have two different origins -

$$J^\mu = J^\mu_{\text{f}} + J^\mu_{\text{ext}}$$

- Charge four-current can be related to particle four-current as, $J^\mu_{\text{f}} = \mathfrak{q} N^\mu_{\text{f}}$.

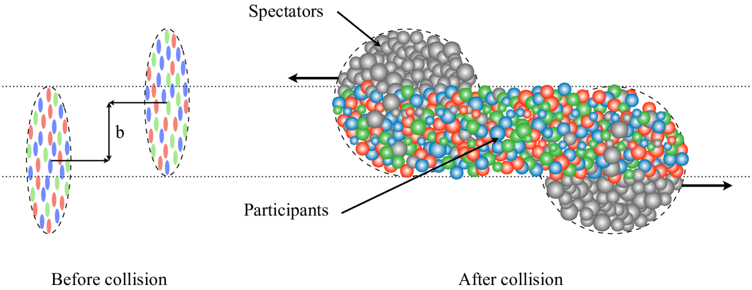



Figure 13: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

## Conserved Currents (Particle Current and Stress-Energy Tensor) :

- The net particle current within the system remains conserved. Hence we have,

$$\partial_\mu N_{\rm f}^\mu = 0$$

- Total stress-energy tensor is, $T^{\mu\nu} = T_{\rm f}^{\mu\nu} + T_{\rm int}^{\mu\nu} + T_{\rm B}^{\mu\nu} + T_{\rm ext}^{\mu\nu}$

- The first three stress-energy tensors are given by,

$$\begin{aligned} T_{\rm f}^{\mu\nu} &= \mathcal{E}\, u^\mu u^\nu - (\mathcal{P} + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu} \\ T_{\rm int}^{\mu\nu} &= -F^\mu{}_\alpha M^{\nu\alpha} \\ T_{\rm B}^{\mu\nu} &= -F^{\mu\alpha} F^\nu{}_\alpha + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \end{aligned}$$

- Due to the external field, stress-energy tensor is not conserved

$$\partial_\nu T^{\mu\nu} = -f_{\rm ext}^\mu\,, \qquad f_{\rm ext}^\mu = F^\mu{}_\alpha J_{\rm ext}^\alpha\,, \qquad \partial_\nu T_{\rm f}^{\mu\nu} = F^\mu{}_\alpha J_{\rm f}^\alpha + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}$$

# Conserved Currents (Angular Momentum Tensor) :

- Similar to stress-energy tensor, the total angular momentum is not conserved in presence of external field and we have,

$$\partial_\lambda J^{\lambda,\mu\nu} = \partial_\lambda L^{\lambda,\mu\nu} + \partial_\lambda S^{\lambda,\mu\nu} = -\tau^{\mu\nu}_{\text{ext}},$$

  where, $\tau^{\mu\nu}_{\text{ext}} = x^\mu f^\nu_{\text{ext}} - x^\nu f^\mu_{\text{ext}}$ is the torque exerted by $J_{\text{ext}}$ on the system.

- However, since $\partial_\lambda L^{\lambda,\mu\nu} = -\tau^{\mu\nu}_{\text{ext}}$, we get a conserved spin angular momentum tensor i.e.

$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

- Conservation laws:

$$\partial_\mu N^\mu = 0, \qquad \partial_\nu T^{\mu\nu}_{\text{f}} = F^\mu{}_\alpha J^\alpha_{\text{f}} + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}, \qquad \partial_\lambda S^{\lambda,\mu\nu} = 0$$

# Boltzmann Equation :

- Phase-space distribution function with spin $\rightarrow f(x,p,s)$.

- In presence of electromagnetic fields, the Boltzmann equation under RTA is,

$$p^\mu \partial_\mu^{(x)} f^\pm + \mathcal{F}^\mu \partial_\mu^{(p)} f^\pm + \mathcal{S}^{\mu\nu} \partial_{\mu\nu}^{(s)} f^\pm = -\frac{(u\cdot p)}{\tau_{\rm R}} \delta f^\pm$$

where, [Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970)]

$$\partial_\mu^{(x)} \equiv \frac{\partial}{\partial x^\mu}, \qquad \partial_\mu^{(p)} \equiv \frac{\partial}{\partial p^\mu}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}},$$

- We can obtain a simplified expression for the equation of motion as,

[Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970), Nora W. et al. PRD 100 (2019) 5, 056018]

$$\mathcal{F}^\alpha = \mathfrak{q}\, F^{\alpha\beta} p_\beta + \frac{m}{2}\left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}$$

where, $m^{\alpha\beta} = \chi\, s^{\alpha\beta}$ is dipole moment tensor, $m$ is the mass of the particle.

- We can use the dipole moment tensor to provide a definition of $M^{\alpha\beta}$ as,

$$M^{\alpha\beta} = m \int \mathrm{dPdS}\, m^{\alpha\beta} \left(f^+ - f^-\right)$$

# Solving Boltzmann Equation :

- Using RTA in Boltzmann equation we can write the 1st order gradient correction as,

$$\delta f^{\pm}_{(1)} = -\mathcal{D} f^{\pm}_{\text{eq}},$$

where,

$$\mathcal{D} = \frac{\tau_R}{(u \cdot p)} \left( p^{\alpha} \frac{\partial}{\partial x^{\alpha}} + \mathcal{F}^{\alpha} \frac{\partial}{\partial p^{\alpha}} \right)$$

[A. K. Panda et al., JHEP 03 (2021) 216]

- For equilibrium distribution function, we use,

$$f^{\pm}_{\text{eq}}(x,p,s) = \left( 1 + \frac{1}{2} \omega_{\alpha\beta}\, s^{\alpha\beta}\, \tilde{f}^{\pm}_{0} \right) f^{\pm}_{0} + \mathcal{O}\left(\omega^2\right)$$

$$\text{with,} \quad f^{\pm}_{0} = \left[ e^{\beta \cdot p \mp \xi} + 1 \right]^{-1} \qquad \text{and,} \qquad \tilde{f}^{\pm}_{0} = 1 - f^{\pm}_{0}$$

[F. Becattini et. al., Annals Phys. 338 (2013); W. Florkowski et. al., Phys.Rev.D 97 (2018)]

# Dissipative Currents in Spin-magnetohydrodynamics:

- The dissipative quantities are defined as,

$$
\begin{aligned}
n^{\mu} &= \Delta^{\mu}_{\alpha} \int dP \int dS\, p^{\alpha} \left(\delta f^{+} - \delta f^{-}\right) \\
\Pi &= -\frac{\Delta_{\alpha\beta}}{3} \int dP \int dS\, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\
\pi^{\mu\nu} &= \Delta^{\mu\nu}_{\alpha\beta} \int dP \int dS\, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\
\delta S^{\lambda,\mu\nu} &= \int dP \int dS\, p^{\lambda} s^{\mu\nu} \left(\delta f^{+} + \delta f^{-}\right)
\end{aligned}
$$

where, $\Delta^{\mu\nu}_{\alpha\beta} = (1/2)(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\beta}\Delta^{\mu}_{\alpha}) - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$ is a traceless symmetric projection operator.

# Dissipative Currents in Spin-magnetohydrodynamics:

- So, the dissipative currents are :

$$X = \tau_{\rm eq}\Big[\beta_{X\Pi}\,\theta + \beta^{\alpha}_{Xn}\left(\nabla_\alpha \xi\right) + \beta^{\alpha}_{Xa}\dot{u}_\alpha + \beta^{\alpha\beta}_{X\pi}\sigma_{\alpha\beta} + \beta^{\alpha\beta}_{X\Omega}\Omega_{\alpha\beta} + \beta^{\alpha\beta}_{XF}\left(\nabla_\alpha B_\beta\right) + \beta^{\alpha\beta\gamma}_{X\Sigma}\left(\nabla_\alpha \omega_{\beta\gamma}\right)\Big],$$

where, $X \equiv n^\mu,\ \Pi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}$.

- Evolution of spin-polarization tensor (obtained from spin-matching) is given by,

$$\dot{\omega}^{\mu\nu} = \mathcal{D}^{\mu\nu}_{\Pi}\,\theta + \mathcal{D}^{\mu\nu\gamma}_{\rm n}\left(\nabla_\gamma \xi\right) + \mathcal{D}^{\mu\nu\gamma}_{\rm a}\dot{u}_\gamma + \mathcal{D}^{\mu\nu\rho\kappa}_{\pi}\sigma_{\rho\kappa} + \mathcal{D}^{\mu\nu\rho\kappa}_{\Omega}\Omega_{\rho\kappa} + \mathcal{D}^{\mu\nu\phi\rho\kappa}_{\Sigma}\left(\nabla_\phi \omega_{\rho\kappa}\right)$$

- Equilibrium magnetization tensor is given by,

$$M^{\mu\nu}_{\rm eq} = a_1(T,\mu)\,\omega^{\mu\nu} + a_2(T,\mu)\,u^{[\mu}u_\gamma\omega^{\nu]\gamma}$$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PRL 129, 192301 (2022)]

# Section Outline :

# Summary and Outlook :

- **Summary :**
  1. All the dissipative currents can depend on multiple hydrodynamic gradients.
  2. Evolution of spin-polarization tensor affected by multiple hydrodynamic gradients.
  3. Magnetomechanical effects exist in a spin-polarizable and magnetizable fluid.

- **Outlook :**
  1. The consequence of "*pure torque*" should be explored.
  2. Formulation of a causal spin-hydrodynamics is required.
  3. Spin-hydrodynamics for spin-1 particles should be explored.

***Thank you.***

## Additional Info :

- Magnetic field at $z = 0$, $b = 7.4$ fm, $\gamma = 100$, $\sigma = 5.8$ MeV.

  [Kirill Tuchin, IJMPE (2014)]

# Kinetic Theory with Spin :

- To import spin in kinetic theory (KT), we start from the Wigner function ($\mathcal{W}_{\alpha\beta}$), that bridges the gap between QFT and KT.

- For spin-1/2 particles we set up kinetic equation of $\mathcal{W}_{\alpha\beta}$ using Dirac equation,

$$\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right] \mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

[Xin-Li Sheng, PhD Thesis (2019), N. Weickgenannt *et al*, PRL 127 (2021) 5, 052301, PRD 100, 056018 (2019).]

- The Wigner function can be decomposed as,

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta}$$

$\mathcal{F} \rightarrow$ scalar component,

$\mathcal{P} \rightarrow$ pseudoscalar component,

$\mathcal{V}_\mu \rightarrow$ vector component,

$\mathcal{A}_\mu \rightarrow$ axial vector component,

$\mathcal{S}_{\mu\nu} \rightarrow$ tensor component.

where, the $\gamma$-matrices are the $4 \times 4$ Dirac $\gamma$-matrices and, $\Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$.

# Kinetic Theory with Spin :

- For spin-hydrodynamics it suffices to consider only $\mathcal{F}$ and $\mathcal{A}_\mu$ components.

  [Xin-Li Sheng, PhD Thesis (2019)]

| | ***Scalar Component*** | ***Axial Component*** |
|---|---|---|
| Kin. Eq. | $k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$ | $k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$ |
| RTA | $C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\Big]$ | $C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\Big]$ |
| Dist. fn. | $\mathcal{F}^\pm(x,k) = 2m\int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k\mp p)$ | $\mathcal{A}^\mu_\pm(x,k) = 2m\int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k\mp p)$ |

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103 (2021) 1, 014030]

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int \mathrm{dP}(\cdots), \quad \int \mathrm{dP} = d^3p/\,(2\pi)^3\,p^0.$

Spin measure $\rightarrow \quad \int_s(\cdots) \rightarrow \int \mathrm{dS}(\cdots), \quad \int \mathrm{dS} = (m/\pi\mathfrak{s})\int d^4s\,\delta(s\cdot s + \mathfrak{s}^2).$