Quantum fluctuations of energy in subsystems of a hot relativistic gas

Wojciech Florkowski

Institute of Theoretical Physics, Jagiellonian University, Krakow, Poland

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Motivation

- To model the bulk evolution of the strongly interacting matter produced in relativistic heavy-ion collisions, relativistic viscous hydrodynamics has become the basic theoretical tool. [1], [2].
- Concepts used in hydrodynamics: energy density and pressure, both are defined locally and, formally, the fluid cell has zero size.
- Interestingly, hydro models which are successful in explaining the experimental data can be used to conclude about the energy density attained in the collision processes.
- Is the energy density a well defined concept for a fluid cell of arbitrary size?
- Does quantum fluctuation play any role?
- Noether's theorem does not give a unique choice for the energy-monetum tensor $\rightarrow$ pseudo-gauge choices.
- Possible pseudo-gauge dependence?

[1] C. Gale, S. Jeon, B. Schenke, Int.J.Mod.Phys.A 28 (2013) 1340011

[2] S. Jeon, U. Heinz, Int.J.Mod.Phys.E 24 (2015) 10, 1530010.

Scale dependence of quantum fluctuation

- For a real scalar field the canonical energy-momentum tensor is :

$$\hat{T}^{00} = \pi\dot{\phi} - \mathcal{L} \equiv \mathcal{H}. \qquad (1)$$

In natural units, $\hbar = c = 1$, $[\hat{T}^{00}] = [\mathcal{L}] = [M^d] = [\mathcal{H}]$.

- We define a smeared operator,

$$A(a) = (a\sqrt{\pi})^{1-d} \int d^{d-1}\mathbf{x}\, \mathcal{H}(0,\mathbf{x}) e^{-\mathbf{x}^2/a^2}. \qquad (2)$$

- Variance of the operator $A(a)$,

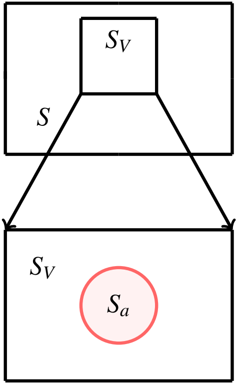
$$\mathrm{var}\, A = \langle A^2 \rangle - \langle A \rangle^2 \implies [\mathrm{var}\, A] = [A]^2 = [\mathcal{H}]^2 = [M]^{2d}. \qquad (3)$$

- If we set,

$$\mathrm{var}\, A(a) \sim a^{\beta} \implies \beta = -2d \qquad (4)$$

- Therefore the fluctuations of the energy density grow rapidly at small distances [3].

[3] Quantum Field Theory: Lectures of Sidney Coleman



- S is closed/isolated system described by microcanonical ensemble.
- S_V is a sub system of the closed system in equilibrium, described by the canonical ensemble. Fluctuation in energy in S_V (large volume limit):

$$\sigma_H^2 = \frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2} = \frac{T^2 C_V}{V\varepsilon^2} \to 0. \quad (5)$$

  H is the Hamiltonian, T is temperature, ε is energy density, C_V is specific heat.
- S_a is a subsystem of S_V which is described by the "Gaussian box": $(a\sqrt{\pi})^3 \exp(-\mathbf{x^2}/a^2)$.

Quantum scalar field

- We describe our system by a quantum scalar field in thermal equilibrium[4].

$$\phi(t,\boldsymbol{x}) = \int \frac{d^3k}{\sqrt{(2\pi)^3\, 2\omega_{\boldsymbol{k}}}} \left(a_{\boldsymbol{k}} e^{-ik\cdot x} + a^\dagger_{\boldsymbol{k}} e^{ik\cdot x} \right); \quad [a_{\boldsymbol{k}}, a^\dagger_{\boldsymbol{k}'}] = \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \qquad (6)$$

  Single particle energy: $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$, $a^\mu b_\mu = a \cdot b = a^0 b^0 - \boldsymbol{a}\cdot\boldsymbol{b}$.
- Hamiltonian density:

$$\mathcal{H} = \frac{1}{2}\left(\dot{\phi}^2 + (\boldsymbol{\nabla}\phi)^2 + m^2\phi^2 \right). \qquad (7)$$

- We define an operator $\mathcal{H}_a$ for a *finite* subsystem S_a placed at the origin of the coordinate system,

$$\mathcal{H}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\, \mathcal{H}(x)\, \exp\left(-\frac{\boldsymbol{x}^2}{a^2} \right). \qquad (8)$$

- Our objective:

$$\sigma^2(a,m,T) = \langle \mathcal{H}_a \mathcal{H}_a \rangle - \langle \mathcal{H}_a \rangle^2, \quad \sigma_n(a,m,T) = \frac{\left(\langle \mathcal{H}_a \mathcal{H}_a \rangle - \langle \mathcal{H}_a \rangle^2\right)^{1/2}}{\langle \mathcal{H}_a \rangle}. \qquad (9)$$

[4] S. Coleman, Lectures of Sidney Coleman on Quantum Field Theory. WSP, Hackensack, 12, 2018.

- Normal ordering: Composite QFT operators → Some "Normal ordering" prescription required.

$$\mathcal{H}_a \to :\mathcal{H}_a: \qquad (10)$$

$$\mathcal{H}_a\mathcal{H}_a \to :\mathcal{H}_a::\mathcal{H}_a: \qquad (11)$$

- To perform thermal averaging, it is sufficient to know the thermal expectation values of the products of two and four creation and/or annihilation operators [5,6,7]

$$\langle a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}'} \rangle = \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}), \qquad (12)$$

$$\langle a^\dagger_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}'} a_{\boldsymbol{p}} a_{\boldsymbol{p}'} \rangle = \left(\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}') + \delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p})\right) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'})$$

The Bose–Einstein distribution function, $f(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta\,\omega_{\boldsymbol{k}}]-1)$.

- Any other combinations of two and four creation and/or annihilation operators can be obtained through the commutation relation between $a_{\boldsymbol{k}}$ and $a^\dagger_{\boldsymbol{k}}$.

[5] C. Cohen-Tannoudji, B. Diu, F. Laloë, and S. R. Hemley, Quantum mechanics: Vol. 3, Wiley, New York, 1977.

[6] T. Evans and D. A. Steer, Nucl. Phys. B 474 (1996) 481.

[7] C. Itzykson and J. Zuber, Quantum Field Theory. International Series In Pure and Applied Physics. McGraw-Hill, New York, 1980.

- The thermal expectation value of the operator $\mathcal{H}_a$ is

$$\langle :\mathcal{H}_a: \rangle = \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}}\, f\left(\omega_{\boldsymbol{k}}\right) \equiv \varepsilon(T) \quad \text{well known result} \tag{13}$$

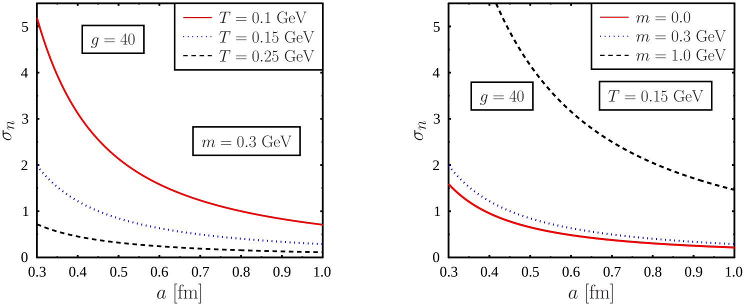
- Important new result: Fluctuation,

$$\begin{aligned}\sigma^2(a,m,T) &= \int dK\, dK' f(\omega_{\boldsymbol{k}})(1+f(\omega_{\boldsymbol{k}'})) \\ &\times \left[(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' + m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} + (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' - m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right].\end{aligned} \tag{14}$$

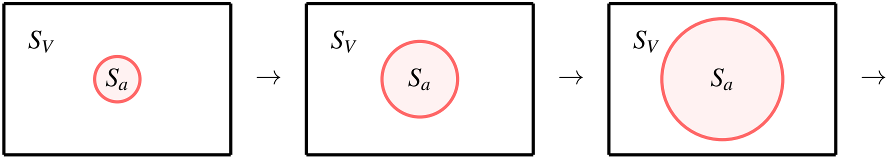
here $dK = d^3k/((2\pi)^3 2\omega_{\boldsymbol{k}})$.

- All the vacuum energy term coming from the composite operator may not be removed by "normal ordering".
- $\langle :\mathcal{H}_a: \rangle$ is independent of the scale a, but the fluctuation $\sigma^2(a,m,T)$ depends on the scale.
- Degeneracy factor: $\varepsilon \to g\varepsilon, \quad \sigma^2 \to g\sigma^2$.

Temperature and mass dependence of σ_n



Thermodynamic limit



- Gaussian representation of the three dimensional Dirac delta function

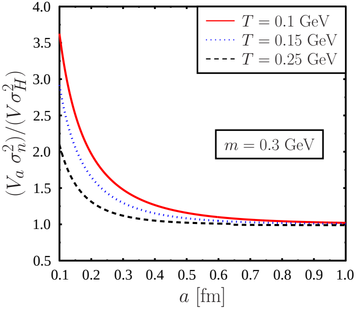
$$\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}) = \lim_{a\to\infty} \frac{a^3}{(2\pi)^{3/2}} e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{p})^2}. \tag{15}$$

- One obtains,

$$V_a \sigma_n^2 = \frac{T^2 c_V}{\varepsilon^2} = V \frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2} \equiv V\sigma_H^2, \tag{16}$$

- $V_a = a^3(2\pi)^{3/2}$ can be considered as the volume of the subsystem S_a — a nontrivial factor of $(2\pi)^{3/2}$ is an artifact of using the “Gaussian” box.
- $V\sigma_H^2$ can be identified as the normalized energy fluctuation in the system S_V.

Approach to the thermodynamic limit



- Variation of $V_a\sigma_n^2/V\sigma_H^2$ with the size of the subsystem S_a in the case where particles have a non vanishing mass and they obey Bose-Einstein statistics.
- One expects that in the thermodynamic limit $V_a\sigma_n^2/V\sigma_H^2$ should approach unity.
- Quantum fluctuations agree with the thermodynamic ones already for $a > 1$ fm.
- Quantum fluctuations become very important at the scale of 0.1 fm.

System of fermions

- We describe our system by a spin-1/2 field in thermal equilibrium. The field operator has the standard form [8]

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}} \Big(U_r(\boldsymbol{k}) a_r(\boldsymbol{k}) e^{-ik\cdot x} + V_r(\boldsymbol{k}) b_r^\dagger(\boldsymbol{k}) e^{ik\cdot x} \Big), \qquad (17)$$

- The canonical anti-commutation relations,

$$\{a_r(\boldsymbol{k}), a_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \qquad (18)$$

$$\{b_r(\boldsymbol{k}), b_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \qquad (19)$$

- Normalization of Dirac spinors,

$$\bar{U}_r(\boldsymbol{k}) U_s(\boldsymbol{k}) = 2m\delta_{rs} \qquad (20)$$

$$\bar{V}_r(\boldsymbol{k}) V_s(\boldsymbol{k}) = -2m\delta_{rs} \qquad (21)$$

[8] L. Tinti and W. Florkowski, arXiv:2007.04029, a chapter in "Strongly Interacting Matter Under Rotation", edited by F. Becattini, J. Liao and M. Lisa

- To perform thermal averaging:

$$\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}), \tag{22}$$

$$\begin{aligned} &\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle \\ &= (2\pi)^6 \Big(\delta_{rs'} \delta_{r's} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\, \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}) \\ &- \delta_{rr'} \delta_{ss'} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\, \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}') \Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}). \end{aligned} \tag{23}$$

  Here $f(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta\,(\omega_{\boldsymbol{k}} - \mu)] + 1)$ is the Fermi–Dirac distribution function for particles.
- Anti particle operators also satisfies similar relation.
- For antiparticles, the Fermi–Dirac distribution function differs by the sign of the chemical potential μ, i.e. $\bar{f}(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta\,(\omega_{\boldsymbol{k}} + \mu)] + 1)$
- We consider a case with zero baryon chemical potential.

- Contrary to the real scalar field, the canonical energy momentum tensor operator is not symmetric,

$$\hat{T}^{\mu\nu}_{\rm Can} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - g^{\mu\nu}\mathcal{L}_{\rm D} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi, \quad \overleftrightarrow{\partial}^{\mu} \equiv \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu} \tag{24}$$

Here $\mathcal{L}_{\rm D}$ denotes the Lagrangian density of a spin-1/2 field, which can be expressed as

$$\mathcal{L}_{\rm D} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi - m\bar{\psi}\psi, \tag{25}$$

- Mathematically, for any original energy-momentum tensor $\hat{T}^{\mu\nu}$ satisfying $\partial_{\mu}\hat{T}^{\mu\nu} = 0$ we can construct a different one by adding the divergence of an antisymmetric object, namely [9]

$$\hat{T}'^{\,\mu\nu} = \hat{T}^{\mu\nu} + \partial_{\lambda}\hat{A}^{\nu\mu\lambda}; \;\; \hat{A}^{\nu\mu\lambda} = -\hat{A}^{\nu\lambda\mu} \tag{26}$$

- By construction, the new tensor is also conserved, i.e., $\partial_{\mu}\hat{T}'^{\,\mu\nu} = 0$.
- For spin $1/2$ field the energy momentum tensor is pseudo-gauge dependent.

[9] E. Speranza and N. Weickgenannt, Spin tensor and pseudo-gauges: from nuclear collisions to gravitational physics, arXiv:2007.00138, Eur. Phys. J. A 57 (2021) 155.

- Belinfante-Rosenfeld framework (BR):

$$\hat{T}^{\mu\nu}_{\rm BR} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - \frac{i}{16}\partial_{\lambda}\left(\bar{\psi}\left\{\gamma^{\lambda}, \left[\gamma^{\mu}, \gamma^{\nu}\right]\right\}\psi\right). \tag{27}$$

- de Groot-van Leeuwen-van Weert framework (GLW)[10]:

$$\begin{aligned}\hat{T}^{\mu\nu}_{\rm GLW} &= -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - g^{\mu\nu}\mathcal{L}_{\rm D} \\ &= \frac{1}{4m}\Big[-\bar{\psi}(\partial^{\mu}\partial^{\nu}\psi) + (\partial^{\mu}\bar{\psi})(\partial^{\nu}\psi) + (\partial^{\nu}\bar{\psi})(\partial^{\mu}\psi) \\ &\qquad - (\partial^{\mu}\partial^{\nu}\bar{\psi})\psi\Big].\end{aligned} \tag{28}$$

- Hilgevoord-Wouthuysen framework (HW)[11]:

$$\begin{aligned}\hat{T}^{\mu\nu}_{\rm HW} = \hat{T}^{\mu\nu}_{\rm Can} &+ \frac{i}{2m}\left(\partial^{\nu}\bar{\psi}\sigma^{\mu\beta}\partial_{\beta}\psi + \partial_{\alpha}\bar{\psi}\sigma^{\alpha\mu}\partial^{\nu}\psi\right) \\ &- \frac{i}{4m}g^{\mu\nu}\partial_{\lambda}\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_{\alpha}\psi\right),\end{aligned} \tag{29}$$

[10] S. R. De Groot, Relativistic Kinetic Theory. Principles and Applications. 1980.

[11] . Hilgevoord and S. Wouthuysen, Nucl. Phys. 40 (1963) 1-12; J. Hilgevoord and E. De Kerf, Physica 31 No.7 (1965) 1002-1016

Energy density is pseudo-gauge independent

- We define an operator, $\hat{T}^{00}_a$:

$$\hat{T}^{00}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\ \hat{T}^{00}(x)\ \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right). \quad (30)$$

- We consider the variance

$$\sigma^2(a,m,T) = \langle : \hat{T}^{00}_a :: \hat{T}^{00}_a : \rangle - \langle : \hat{T}^{00}_a : \rangle^2 \quad (31)$$

and the normalized standard deviation

$$\sigma_n(a,m,T) = \frac{\left(\langle : \hat{T}^{00}_a :: \hat{T}^{00}_a : \rangle - \langle : \hat{T}^{00}_a : \rangle^2\right)^{1/2}}{\langle : \hat{T}^{00}_a : \rangle}. \quad (32)$$

- Mean/thermal averaged $\hat{T}^{00}_a$:

$$\langle : \hat{T}^{00}_{\mathrm{Can},a} : \rangle = 4 \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}} f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\mathrm{Can}}(T) \quad (33)$$

$$= \langle : \hat{T}^{00}_{\mathrm{BR},a} : \rangle = \langle : \hat{T}^{00}_{\mathrm{GLW},a} : \rangle = \langle : \hat{T}^{00}_{\mathrm{HW},a} : \rangle \quad (34)$$

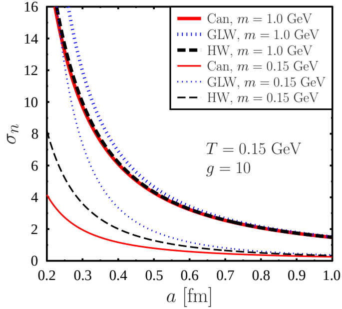
Energy density fluctuation– pseudo-gauge dependent

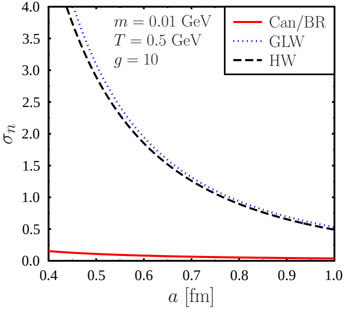
- Contrary to energy density the energy density fluctuation is pseudo-gauge dependent, e.g.
- For the Canonical framework:

$$\sigma^2_{\rm Can}(a,m,T) = 2\int dK\, dK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))$$
$$\times \Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'+m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2}$$
$$-(\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\Big], \tag{35}$$

- For the de Groot-van Leeuwen-van Weert framework:

$$\sigma^2_{\rm GLW}(a,m,T) = \frac{1}{2m^2}\int dK\, dK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))$$
$$\times \Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2}$$
$$-(\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'-m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\Big]. \tag{36}$$





- A comparison of the normalized standard deviation of fluctuations obtained for three different pseudo-gauges (Can=BR, GLW, HW).
- For $a < 0.5$ fm, we observe that the results obtained with various pseudo-gauges differ, with differences growing as a decreases.
- Irrespective of the choice of pseudo-gauges with growing system size the normalized standard deviation of fluctuations (σ_n) decreases.

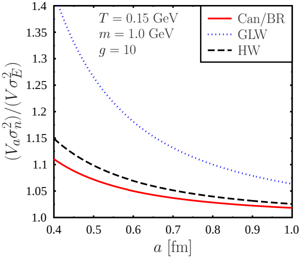
What about thermodynamic limit??

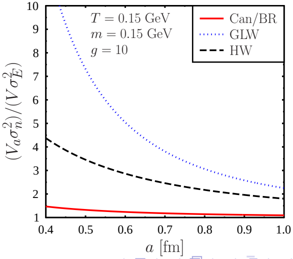
- Using the Gaussian representation of the Dirac delta function it can be shown that, in the large a limit,

$$\sigma^2_{\rm Can} = \frac{4\,g}{(2\pi)^{3/2}a^3}\int \frac{d^3k}{(2\pi)^3}\,\omega^2_{\boldsymbol{k}}\,f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}})) = \sigma^2_{\rm BR} = \sigma^2_{\rm GLW} = \sigma^2_{\rm HW}. \quad (37)$$

- In the large a limit we find,

$$V_a\sigma^2_n = \frac{T^2 c_V}{\varepsilon^2} = V\frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2} \equiv V\sigma^2_E; \quad V_a = a^3(2\pi)^{3/2}. \quad (38)$$





Conclusions

- We have derived the formula characterizing the quantum fluctuation of energy in subsystems of a hot relativistic gas.
- It agrees with the expression for thermodynamic fluctuations, if the size of the subsystem is sufficiently large.
- For smaller sizes the effects of quantum fluctuations become relevant and the classical description with well defined energy density makes sense only after coarse graining over sufficiently large scale.
- For fermions quantum fluctuation of energy density does depend on the choice of the pseudo-gauge.
- On the practical side, the results of our calculations can be used to determine a scale of coarse graining for which the choice of the pseudo-gauge becomes irrelevant.
- This may be useful, in particular, in the context of hydrodynamic modeling of high-energy collisions.
- These results might be relevant for small systems.

Pseudo-gauge personal manisfest

- QFT POV: all pseudo-gauges are equivalent – if field equations are satisfied, all equations considered in different pseudo-gauges are fulfilled
  canonical: $\partial_\mu T^{\mu\nu}_{\rm can} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu}_{\rm can} = T^{\nu\mu}_{\rm can} - T^{\mu\nu}_{\rm can},$
  Belinfante: $\partial_\mu T^{\mu\nu}_{\rm Bel} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu}_{\rm Bel} = 0.$
  GLW: $\partial_\mu T^{\mu\nu}_{\rm GLW} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW} = T^{\nu\mu}_{\rm can} - T^{\mu\nu}_{\rm can},$
- Densities of locally conserved currents have no absolute sense (Coleman's lectures), perhaps, if integrated over sufficiently large volumes (this talk).
- $T^{\mu\nu}_{\rm Bel}$ couples to gravity, $S^{\lambda,\mu\nu}_{\rm can} = \varepsilon^{\lambda\mu\nu\rho} A_\rho$ couples to weak interactions (with A being the axial current) .
- Lessons from the proton spin puzzle (discussion by Leader and Lorce): gauge invariance constrains observables but gauge dependent expressions describe certain observables after fixing the gauge.
- Pseudo-gauge dependent expressions describe certain observables after fixing the pseudo-gauge.
- We cannot require that the only reasonable energy-momentum tensor is a symmetric one.

Normal ordering: alternative approach

- For a composite operator we considered the following normal ordering method: $\mathcal{H}_a\mathcal{H}_a \to :\mathcal{H}_a::\mathcal{H}_a:$.
- Therefore we are first normal ordering first then then multiplying to construct the composite operator.
- Alternatively one can also argue about different method of normal ordering:

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Semiclassical gravity theory and quantum fluctuations

Chung-I Kuo and L. H. Ford
Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 9 December 1992)

$$\Delta(x) \equiv \left| \frac{\langle :T_{00}{}^2(x): \rangle - \langle :T_{00}(x): \rangle^2}{\langle :T_{00}{}^2(x): \rangle} \right| .$$

- If we consider such a normal ordering then:

$$\sigma^2(a,m,T) = \langle :\mathcal{H}_a\mathcal{H}_a: \rangle - \langle :\mathcal{H}_a: \rangle^2 = \int dK\, dK' f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'})$$
$$\times \left[(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' + m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} + (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' - m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2} \right]. \quad (39)$$