Anisotropic Transport Coefficients of a Magnetized Fluid: a Chapman-Enskog Approach

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Emergent Topics In
Relativistic Hydrodynamics, Chirality, Vorticity and Magnetic Fields

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Motivation

- There is ample evidence of QGP being produced in a heavy ion collision, at RHIC and LHC.

- Based on comparison between hydrodynamic simulation and experimental data, it was found that QGP has a finite but very small value of η/s .

- These calculations were done in the absence of magnetic field, but we know that an intense transient magnetic field is expected to be produced initially.

- Phys. Rev D 100, 114004 (2019) and , Phys. Rev. D 102, 016016 (2020) are some of the works that aims to obtain the Transport coefficients in the presence of magnetic field using Relaxation time approximation.

- Here we aim to obtain the transport coefficients using a modified Chapman-Enskog formalism.

Irreversible Flows

The Thermodynamic flow :-

$$F = Y X$$

$X \longrightarrow$ Thermodynamic force

The Irreversible Pressure tensor :-

$$P^{\mu\nu} = \triangle^{\mu}_{\alpha} \triangle^{\nu}_{\beta} T^{\alpha\beta}$$

$$= \eta^{\mu\nu\kappa\delta} \left(\partial_\kappa U_\delta \right) + \zeta^{\kappa\delta} \left(\partial_\kappa U_\delta \right) \triangle^{\mu\nu}$$

The Heat flow :-

$$I^{\mu}_{q} = \left(U_\nu T^{\nu\sigma} - h N^{\sigma} \right) \triangle^{\mu}_{\sigma}$$

$$= -\lambda^{\mu\nu} \triangle^{\beta}_{\nu} \left(\partial_\beta T - T D U_\beta \right)$$

Boltzmann

Transport Equation

The Covariant form: $p^\mu \partial_\mu f + qF^{\mu\nu} p_\nu \frac{\partial f}{\partial p^\mu} = C(f)$

$$F^{\mu\nu} = -Bb^{\mu\nu}; \qquad b^{\mu\nu} = \epsilon^{\mu\nu\rho\alpha} \frac{B_\rho}{B} U_\alpha$$

No pf particle with p^μ gain — No pf particle with p^μ loss:

$$C(f) = \int d\Gamma_k d\Gamma_{p'} d\Gamma_{k'} \{ f_{p'} f_{k'} (1 \pm f_p)(1 \pm f_k) - f_p f_k (1 \pm f_{p'})(1 \pm f_{k'}) \} W$$

The differential scattering cross-section: $W = \frac{s}{2} \frac{d\sigma}{d\Omega} (2\pi)^6 \delta^4 (p + k - p' - k')$

$$d\Gamma_q = \frac{d^3 q}{(2\pi)^3 q_0}; \quad q_0 = \sqrt{\vec{q}^2 + m^2}$$

Single Particle
Distribution Function

The Single particle distribution function : $f = f(x, p)$

The Particle flow : $N^\mu = \int d\Gamma p^\mu f$

The Energy Momentum tensor : $T^{\mu\nu} = \int d\Gamma p^\mu p^\nu f$

The Hydrodynamic velocity :

Eckart : $U^\mu = \frac{N^\mu}{\sqrt{N_\nu N^\nu}}$

Landau : $U^\mu = \frac{T^{\mu\nu} U_\nu}{\sqrt{U_\rho T^{\rho\sigma} T_{\sigma\tau} U^\tau}}$

The particle density : $n = N^\mu U_\mu$

The Energy density : $ne = U_\mu T^{\mu\nu} U_\mu$

Linearisation of

Boltzmann Equation

The distribution function is expanded as : $f(x,p) = f^0(x,p) + \epsilon f^1(x,p) + \epsilon^2 f^2(x,p) + ...$

The equilibrium distribution function : $f^0 = \left[exp\left(\frac{p^\mu U_\mu - \mu}{T}\right) \mp 1\right]^{-1}$

$$p^\mu \epsilon \partial_\mu \left(f^0 + \epsilon f^1 + \epsilon^2 f^2 + ...\right) + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} \left(f^0 + \epsilon f^1 + \epsilon^2 f^2 + ...\right) = C(f)$$

Equating the coefficients of ϵ in B.E. we get

$$p^\mu \partial_\mu f^0 = -\mathcal{L}(f^1) - qF^{\mu\nu} p_\nu \frac{\partial f^1}{\partial p^\mu}$$

Relaxation time approximation : $\mathcal{L}(f^1) = -\frac{U \cdot p}{\tau_c} f^1$

Series method :

$$\mathcal{L}(f^1) = \int d\Gamma_k d\Gamma_{p'} d\Gamma_{k'} f^0(x,p) f^0(x,k) \left(1 + f^0(x,p')\right) \left(f^0(x,k')\right)$$
$$\left(\phi(x,p) + \phi(x,k) - \phi(x,p') - \phi(x,k')\right) W$$

Conservation Equations

Continuty equation : $\partial_\mu N^\mu = 0;$ $(Dn)^1 = -n\partial_\mu U^\mu$

Equation of Energy conservation : $U_\nu \partial_\mu T^{\mu\nu} = 0\ ;$ $\Rightarrow$ $D(ne)^1 = -hn\partial_\mu U^\mu$

Equation of Momentum conservation : $\triangle_{\mu\nu}\partial_\sigma T^{\nu\sigma} = 0\ ,$ $(DU^\mu)^1 = -\frac{1}{hn}\nabla^\mu P$

Gibb's Duhem Relation : $n^{-1}\partial_\nu P = s\partial_\nu T + T\partial_\nu \left(\frac{\mu}{T}\right)$

Transport equations in terms of

Thermodynamic Forces

Equating the conservation equation we get :

$$DT = (1-\gamma')\,T\partial_\mu U^\mu$$

$$TD\left(\frac{\mu}{T}\right) = \left[(\gamma''-1) - \gamma'''T\right]\partial_\mu U^\mu$$

The hydrodynamic equations contains space derivative of temperature and velocity. In order to make the Boltzmann equation look somewhat like the hydrodynamic equation, the time derivatives will be replaced using these relations.

$$\frac{f^0\left(1+f^0\right)}{T}\left[Q\partial\cdot U + \left(1-\frac{U\cdot p}{h}\right)p^\mu T\nabla_\mu\left(\frac{\mu}{T}\right) - p_\mu p_\nu\langle\partial^\mu U^\nu\rangle\right]$$

$$= -\mathcal{L}\left(f^1\right) - qF^{\mu\nu}p_\nu\frac{\partial}{\partial p_\mu}\left[f^0\left(1+f^0\right)\phi\right]$$

Deviation from equilibrium

The deviation of the distribution function from equilibrium : $f^1 = f^0\left(1+f^0\right)\phi$

$$\phi = \left[\sum_{n=0}^{4} \mathcal{X}_n C_{(n)\mu\nu\alpha\beta}\, p^\mu p^\nu V^{\alpha\beta}\right] + \left[\sum_{n=0}^{2} \mathcal{A}_n C_{(n)}^{\eta\delta}\left(\partial_\eta U_\delta\right) + \mathcal{Y}_n C_{(n)}^{\alpha\beta} p_\alpha \left(T^{-1}\partial_\beta T - DU_\beta\right)\right]$$

No magnetic field : $\phi = \mathcal{X}\triangle_{\mu\nu\alpha\beta}\, p^\mu p^\nu V^{\alpha\beta} + \mathcal{A}\triangle^{\eta\delta}\left(\partial_\eta U_\delta\right) + \mathcal{Y}\triangle^{\alpha\beta} p_\alpha\left(T^{-1}\partial_\beta T - DU_\beta\right)$

$$C_{(0)\mu\nu\alpha\beta} = P^{(0)}_{\langle\mu\nu\rangle\alpha\beta}$$

$$C_{(1)\mu\nu\alpha\beta} = P^{(1)}_{\langle\mu\nu\rangle\alpha\beta} + P^{(-1)}_{\langle\mu\nu\rangle\alpha\beta}; \qquad C_{(2)\mu\nu\alpha\beta} = i\left(P^{(1)}_{\langle\mu\nu\rangle\alpha\beta} + P^{(-1)}_{\langle\mu\nu\rangle\alpha\beta}\right)$$

$$C_{(3)\mu\nu\alpha\beta} = P^{(2)}_{\langle\mu\nu\rangle\alpha\beta} + P^{(-2)}_{\langle\mu\nu\rangle\alpha\beta}; \qquad C_{(4)\mu\nu\alpha\beta} = i\left(P^{(2)}_{\langle\mu\nu\rangle\alpha\beta} + P^{(-2)}_{\langle\mu\nu\rangle\alpha\beta}\right)$$

$$C_{(0)}^{\eta\delta} = P^{0\eta\delta}$$

$$C_{(1)}^{\eta\delta} = P^{1\eta\delta} + P^{-1\eta\delta}\ , \qquad C_{(2)}^{\eta\delta} = i\left(P^{1\eta\delta} - P^{-1\eta\delta}\right)$$

Tensors

$$P^{(m)}_{\langle\mu\nu\rangle\alpha\beta} = P^{(m)}_{\mu\nu\alpha\beta} + P^{(m)}_{\nu\mu\alpha\beta}; \quad P^{(m)}_{\mu\nu\mu'\nu'} = \sum_{m_1=-1}^{1}\sum_{m_2=-1}^{1} P^{(m_1)}_{\mu\mu'} P^{(m_2)}_{\nu\nu'} \delta\left(m, m_1+m_2\right)$$

Hamilton-Cayley equation for the rotational tensor : $H^3 + H = 0$

$$P^m = \prod_{m'\neq m} \frac{H - im'}{im - im'}; \quad m, m' = 0, \pm 1$$

S. Hess; Tensors of Physics. (2015)

$$P^0_{\mu\nu} = b_\mu b_\nu$$

$$P^1_{\mu\nu} = \frac{1}{2}\left(-\triangle^{\mu\nu} - b_\mu b_\nu + i b_{\mu\nu}\right), \qquad P^{-1}_{\mu\nu} = \frac{1}{2}\left(-\triangle^{\mu\nu} - b_\mu b_\nu - i b_{\mu\nu}\right)$$

Properties :

$$P^{(m)}_{\mu\kappa} P^{(m')\kappa}{}_{\mu'} = \delta_{mm'} P^{(m)}_{\mu\mu'} \qquad \left(P^{(m)}_{\mu\nu}\right)^\dagger = P^{(-m)}_{\mu\nu} = P^{(m)}_{\nu\mu}$$

$$\sum_{m=-1}^{1} P^{(m)}_{\mu\nu} = \delta_{\mu\nu}; \quad P^{(m)}_{\mu\mu} = 1 \qquad P^{(m)}_{\mu\nu\alpha\beta} P^{(n)\alpha\beta}{}_{\mu'\nu'} = \delta\left(m,n\right) P^{(m)}_{\mu\mu'\nu\nu'}$$

Transport Coefficients

Shear viscosity :

$$\langle \pi^{\mu\nu} \rangle = \int d\Gamma \ \langle p^{\mu} p^{\nu} \rangle f^{1}$$

$$\eta^{\mu\nu\phi\theta} \triangle^{\lambda\delta}_{\phi\theta} = \sum_{n=0}^{4} \int d\Gamma \langle p^{\mu} p^{\nu} \rangle p^{\alpha} p^{\beta} f^{0} \left(1 + f^{0}\right) \mathcal{X}_{n} C^{\alpha\beta\lambda\delta}_{(n)}$$

Bulk viscosity :

$$\pi \triangle^{\mu\nu} = \int d\Gamma p^{\mu} p^{\nu} f^{1}$$

$$\zeta^{\eta\delta} = \frac{1}{3} \sum_{n=0}^{2} \int d\Gamma \triangle_{\mu\nu} p^{\mu} p^{\nu} f^{0} \left(1 + f^{0}\right) \mathcal{A}_{n} C^{\eta\delta}_{(n)}$$

Thermal Conductivity :

$$I^{\mu}_{q} = \int d\Gamma_{p} \ p^{\sigma} \triangle^{\mu}_{\sigma} \left(p \cdot U - h\right) f^{1}$$

$$\lambda^{\mu\beta} = - \sum_{n=0}^{2} \int d\Gamma_{p} \ p^{\sigma} \triangle^{\mu}_{\sigma} f^{0} \left(1 + A_{0} f^{0}\right) \left(p \cdot U - h\right) \mathcal{Y}_{n} C^{\alpha\beta}_{(n)} p_{\alpha}$$

Expanding the
Coefficients

The coefficients: $\mathcal{X}_n = \sum_{i=0}^{\infty} c_i^{(m)}(B)\, L_i^{\frac{5}{2}}(\tau')\,, \qquad \mathcal{A}_n = \sum_{i=0}^{\infty} a_i^{(n)}(B)\, L_i^{\frac{1}{2}}(\tau')$

Without magnetic field : $\mathcal{X} = \sum_{i=0}^{\infty} c_i L_i^{\frac{5}{2}}(\tau)\,, \qquad \mathcal{A} = \sum_{i=0}^{\infty} a_i L_i^{\frac{1}{2}}(\tau)$

D. Davesne; Phys. Rev C **53,** 3069 (1996)

Depending on how precise we want the coefficients to be we truncate the series accordingly. Depending on the truncation we get different order of the same transport coefficient

$$\eta_{||} = \frac{\rho T^2}{4} \sum_{n=0}^{\infty} c_n^{(0)} \gamma_n^{(0)}$$

$$\eta_{\perp} = \frac{1}{8}\left\{ \sum_{n=0}^{\infty} c_n^{(1)}\gamma_n^{(1)} + \sum_{n=0}^{\infty} c_n^{(2)}\gamma_n^{(2)} \right\} \qquad \eta_{\times} = \frac{1}{8}\left\{ \sum_{n=0}^{\infty} c_n^{(2)}\gamma_n^{(1)} - \sum_{n=0}^{\infty} c_n^{(1)}\gamma_n^{(2)} \right\}$$

$$\eta'_{\perp} = \frac{1}{8}\left\{ \sum_{n=0}^{\infty} c_n^{(3)}\gamma_n^{(3)} + \sum_{n=0}^{\infty} c_n^{(4)}\gamma_n^{(4)} \right\} \qquad \eta'_{\times} = \frac{1}{8}\left\{ \sum_{n=0}^{\infty} c_n^{(4)}\gamma_n^{(3)} - \sum_{n=0}^{\infty} c_n^{(3)}\gamma_n^{(4)} \right\}$$

$$\zeta_{||} = \frac{\rho T}{m} \sum_{i=0}^{\infty} a_j^1 \alpha_j \;, \quad \zeta_{\perp} = \frac{\rho T}{m} \sum_{i=0}^{\infty} a_j^2 \alpha_j`, \quad \zeta_{\times} = \frac{\rho T}{m} \sum_{i=0}^{\infty} a_j^3 \alpha_j$$

Expanding the Coefficients

The coefficient : $$\mathcal{Y}_n\left(B,\tau'\right)=\sum_{m=0}^{\infty} b_m^{(n)}\left(B\right) L_m^{\frac{3}{2}}\left(\tau'\right)$$

Without Magnetic field : $$\mathcal{Y}\left(\tau'\right)=\sum_{m=0}^{\infty} b_m L_m^{\frac{3}{2}}\left(\tau'\right)$$

The expression for the thermal conductivity in terms of the expanded coefficient :

$$\lambda_0=-\frac{\rho T}{m}\sum_{n=0}^{\infty} b_n^{(0)}\beta_n^{(0)}$$

$$\lambda_1=-\frac{\rho T}{2m}\sum_{n=0}^{\infty}\left(b_n^{(1)}\beta_n^{(1)}+b_n^{(2)}\beta_n^{(2)}\right)$$

$$\lambda_2=-\frac{\rho T}{2m}\sum_{n=0}^{\infty}\left(b_n^{(2)}\beta_n^{(1)}-b_n^{(1)}\beta_n^{(2)}\right)$$

Relation
between Coefficients

From the Boltzmann equation we get relation between the coefficients $c_n^{(i)}$.
Using the orthogonality condition of $P^{(m)}_{\langle\mu\nu\rangle\alpha\beta}$, and then integrate it over after multiplying it with $L_j^{\frac{5}{2}}$:

$$\frac{1}{2\rho T}\gamma_j^{(0)} = \sum_{n=0}^{\infty} c_n^{(0)} c_{nj}; \qquad c_{nj} = \frac{1}{m^2T^2}\left[L_n^{\frac{5}{2}}\langle p^\mu p^\nu\rangle, L_j^{\frac{5}{2}}\langle p^\eta p^\delta\rangle\right]_{C_{(0)}}$$

$$\frac{1}{2\rho T}\gamma_j^{(1)} = \sum_{n=0}^{\infty} c_n^{(1)}\left(d_{nj}+\xi_{nj}^{(2)}\right) + \sum_{n=0}^{\infty} c_n^{(2)}\left(e_{nj}-\xi_{nj}^{(1)}\right)$$

$$\frac{1}{2\rho T}\gamma_j^{(2)} = \sum_{n=0}^{\infty} c_n^{(1)}\left(e_{nj}+\xi_{nj}^{(1)}\right) - \sum_{n=0}^{\infty} c_n^{(2)}\left(d_{nj}+\xi_{nj}^{(2)}\right)$$

$$\frac{1}{2\rho T}\gamma_j^{(3)} = \sum_{n=0}^{\infty} c_n^{(3)}\left(l_{nj}+\xi_{nj}^{(4)}\right) + \sum_{n=0}^{\infty} c_n^{(4)}\left(k_{nj}-\xi_{nj}^{(3)}\right)$$

$$\frac{1}{2\rho T}\gamma_j^{(4)} = \sum_{n=0}^{\infty} c_n^{(3)}\left(k_{nj}+\xi_{nj}^{(3)}\right) - \sum_{n=0}^{\infty} c_n^{(4)}\left(l_{nj}+\xi_{nj}^{(4)}\right)$$

Expressions

$$\gamma_j^{(n)} = \frac{1}{\rho T^2}\int d\Gamma_p f^0\left(1+f^0\right)p_\alpha p_\beta p_\eta p_\delta L_j^{\frac{5}{2}}\left(\tau_p\right)C_{(n)}^{\alpha\beta\eta\delta}$$

$$\left[L_m^{\frac{5}{2}}\langle p^\mu p^\nu\rangle, L_j^{\frac{5}{2}}\langle p^\eta p^\delta\rangle\right]_{C_{(l)}} = \frac{m^2}{4\rho^2}\int d\Gamma_p d\Gamma_k d\Gamma_{p'} d\Gamma_{k'}\Big[f_p^0 f_k^0\left(1+f_{p'}^0\right)\left(1+f_{k'}^0\right)$$

$$\left(L_m^{\frac{5}{2}}\left(\tau_p\right)\langle p^\mu p^\nu\rangle + L_m^{\frac{5}{2}}\left(\tau_k\right)\langle k^\mu k^\nu\rangle - L_m^{\frac{5}{2}}\left(\tau_{p'}\right)\langle p'^\mu p'^\nu\rangle - L_m^{\frac{5}{2}}\left(\tau_{k'}\right)\langle k'^\mu k'^\nu\rangle\right)$$

$$\left(L_j^{\frac{5}{2}}\left(\tau_p\right)\langle p^\eta p^\delta\rangle + L_j^{\frac{5}{2}}\left(\tau_k\right)\langle k^\eta k^\delta\rangle - L_j^{\frac{5}{2}}\left(\tau_{p'}\right)\langle p'^\eta p'^\delta\rangle - L_j^{\frac{5}{2}}\left(\tau_{k'}\right)\langle k'^\eta k'^\delta\rangle\right)C_{(l)\mu\nu\eta\delta}p_\mu p_\nu V_{\eta\delta}W\Big]$$

$$d_{nj} = \frac{1}{m^2T^2}\left[L_n^{\frac{5}{2}}, L_j^{\frac{5}{2}}\right]_{C_{(1)}} \qquad e_{nj} = \frac{1}{m^2T^2}\left[L_n^{\frac{5}{2}}, L_j^{\frac{5}{2}}\right]_{C_{(2)}}$$

$$l_{nj} = \frac{1}{m^2T^2}\left[L_n^{\frac{5}{2}}, L_j^{\frac{5}{2}}\right]_{C_{(3)}} \qquad k_{nj} = \frac{1}{m^2T^2}\left[L_n^{\frac{5}{2}}, L_j^{\frac{5}{2}}\right]_{C_{(4)}}$$

Relation between Coefficients

Using the orthogonality condition of $P_{\mu\nu}^{(m)}$ we get relation between $a_n^{(i)}$ the coefficients ,

Then we integrate it over after multiplying it with $L_j^{\frac{1}{2}}$:

$$\sum_{n=0}^{\infty} a_n^0 \left[L_n^{\frac{1}{2}}, L_j^{\frac{1}{2}}\right] = \sum_{n=0}^{\infty} a_n^0 a_{nj} = \frac{m}{\rho}\alpha_j$$

$$\alpha_j = -\frac{m}{\rho T}\int d\Gamma_p f^0 \left(1+f^0\right) Q L_j^{\frac{1}{2}}(\tau)$$

$$\left[a_0, L_j^{\frac{1}{2}}(\tau)\right] = -\frac{m^2}{4\rho^2}\int d\Gamma_p d\Gamma_p d\Gamma_{p'} d\Gamma_{k'} f_p^0 f_k^0 \left(1+f_{p'}^0\right)\left(1+f_{k'}^0\right) W L_j^{\frac{1}{2}}(\tau)$$

$$\left(a_0^p + a_0^k - a_0^{p'} - a_0^{k'}\right)\left(L_0^{\frac{1}{2}}(\tau_p) + L_0^{\frac{1}{2}}(\tau_k) - L_0^{\frac{1}{2}}(\tau_{p'}) - L_0^{\frac{1}{2}}(\tau_{k'})\right)$$

Relation
between Coefficients

From the Boltzmann equation we get relation between the coefficients $b_n^{(i)}$.
Using the orthogonality condition of $P_{\mu\nu}^{(m)}$, and then integrate it over after multiplying it with $L_j^{\frac{3}{2}}$:

$$\frac{1}{\rho}\beta_n^{(0)} = -\sum_{m=1}^{\infty} b_m^{(0)} b_{mn}^{(0)}$$

$$\frac{1}{\rho}\beta_n^{(1)} = -\sum_{m=1}^{\infty} \left[b_m^{(1)} \left(b_{mn}^{(1)} \right) + b_m^{(2)} \left(\theta_{mn}^{(1)} \right) \right]$$

$$0 = -\sum_{m=1}^{\infty} \left[b_m^{(1)} \left(\theta_{mn}^{(1)} \right) - b_m^{(2)} \left(b_{mn}^{(1)} \right) \right]$$

Expressions

$$\beta_n^{(m)} = -\frac{m}{\rho T^2}\int d\Gamma_p f^0\left(1+A_0 f^0\right)(p\cdot U - h)\, p^\mu p^\beta C_{(m)\mu\beta} L_n^{\frac{3}{2}}(\tau')$$

$$\theta_{mn}^{(j)} = \frac{qBm}{\rho T^2}\int d\Gamma_p\ f^0\left(1+A_0 f^0\right) p^\mu p^\beta C_{(j)\mu\beta} L_n^{\frac{3}{2}}(\tau_p)\, L_m^{\frac{3}{2}}(\tau_p)\,.$$

$$\left[L_m^{\frac{3}{2}}(\tau')\, p^\mu, L_n^{\frac{3}{2}}(\tau')\, p^\beta\right]_{C_{(i)}} = mT\ b_{mn}^{(i)}$$

$$\frac{\rho^2}{m^2}\left[L_m^{\frac{3}{2}}(\tau')\, p^\mu, L_n^{\frac{3}{2}}(\tau')\, p^\beta\right]_{C_{(i)}} = \frac{m^2}{4\rho^2}\int d\Gamma_p d\Gamma_k d\Gamma_{p'} d\Gamma_{k'}\Big\{f_p^0 f_p^0\left(1+A_0 f_{p'}^0\right)\left(1+A_0 f_{k'}^0\right)$$
$$\times\left[L_m^{\frac{3}{2}}(\tau_p')\, p^\mu + L_m^{\frac{3}{2}}(\tau_k')\, k^\mu - L_m^{\frac{3}{2}}(\tau_{p'}')\, p'^\mu - L_m^{\frac{3}{2}}(\tau_{k'}')\, k'^\mu\right]$$
$$\times\left[L_n^{\frac{3}{2}}(\tau_p')\, p^\beta + L_n^{\frac{3}{2}}(\tau_k')\, k^\beta - L_n^{\frac{3}{2}}(\tau_{p'}')\, p'^\beta - L_n^{\frac{3}{2}}(\tau_{k'}')\, k'^\beta\right]$$
$$\times C_{(i)\mu\beta} W\Big\}.$$

Transport Coefficients

Now if the expansion of the coefficients are truncated after the first term then :

Shear:

$$[\eta_{||}]_1 = \frac{T}{2}\frac{\gamma_0^{(0)}\left(\gamma_0^{(0)} - \frac{2}{3}\beta_0^{(0)}\right)}{c_{00}}$$

$$[\eta_\perp]_1 = \frac{T}{4}\left[\frac{\left(\gamma_0^{(1)2} - \gamma_0^{(2)2}\right)\left(d_{00} + \xi_{00}^{(2)}\right) + 2\gamma_0^{(1)}\gamma_0^{(2)}\left(e_{00} - \xi_{00}^{(1)}\right)}{\left(d_{00} + \xi_{00}^{(2)}\right)^2 + \left(e_{00} - \xi_{00}^{(1)}\right)^2}\right]$$

$$[\eta_\times]_1 = \frac{T}{4}\left[\frac{\left(\gamma_0^{(1)2} - \gamma_0^{(2)2}\right)\left(e_{00} - \xi_{00}^{(1)}\right) + 2\gamma_0^{(1)}\gamma_0^{(2)}\left(d_{00} + \xi_{00}^{(2)}\right)}{\left(d_{00} + \xi_{00}^{(2)}\right)^2 + \left(e_{00} - \xi_{00}^{(1)}\right)^2}\right]$$

$$[\eta'_\perp]_1 = \frac{T}{4}\left[\frac{\left(\gamma_0^{(3)2} - \gamma_0^{(4)2}\right)\left(l_{00} + 2\xi_{00}^{(4)}\right) + 2\gamma_0^{(3)}\gamma_0^{(4)}\left(k_{00} - 2\xi_{00}^{(3)}\right)}{\left(l_{00} + 2\xi_{00}^{(4)}\right)^2 + \left(k_{00} - 2\xi_{00}^{(3)}\right)^2}\right]$$

$$[\eta'_\times]_1 = \frac{T}{4}\left[\frac{\left(\gamma_0^{(3)2} - \gamma_0^{(4)2}\right)\left(k_{00} - 2\xi_{00}^{(3)}\right) + 2\gamma_0^{(3)}\gamma_0^{(4)}\left(l_{00} + 2\xi_{00}^{(4)}\right)}{\left(l_{00} + 2\xi_{00}^{(4)}\right)^2 + \left(k_{00} - 2\xi_{00}^{(3)}\right)^2}\right]$$

Transport Coefficients

Now if the expansion of the coefficients are truncated after the first term then:

Bulk :

$$[\zeta_{||}]_1 = [\zeta_\perp]_1 = \frac{\rho T}{m} a_0^{2(1)} \alpha_2 = T\frac{\alpha^2}{a_{22}} \ , \qquad [\zeta_\times]_1 = 0$$

$$[\lambda_0]_1 = -\frac{\rho T}{m} b_1^{(0)} \beta_1^{(0)} = \frac{T}{m} \frac{\left(\beta_1^{(0)}\right)^2}{b_{11}^{(0)}}$$

Thermal Conductivity :

$$[\lambda_1]_1 = -\frac{\rho T}{2m}\left(b_1^{(1)}\beta_1^{(1)} + b_1^{(2)}\beta_1^{(2)}\right) = \frac{T}{2m}\frac{\left(\beta_1^{(1)}\right)^2 b_{11}^{(1)}}{\left(b_{11}^{(1)}\right)^2 + \left(\theta_{11}^{(1)}\right)^2}$$

$$[\lambda_2]_1 = -\frac{\rho T}{2m}\left(b_1^{(2)}\beta_1^{(1)} - b_1^{(1)}\beta_1^{(2)}\right) = \frac{T}{2m}\frac{\left(\beta_1^{(1)}\right)^2 \theta_{11}^{(1)}}{\left(b_{11}^{(1)}\right)^2 + \left(\theta_{11}^{(1)}\right)^2}$$

Transport Coefficients

Now if the expansion of the coefficients are truncated after the second term then:

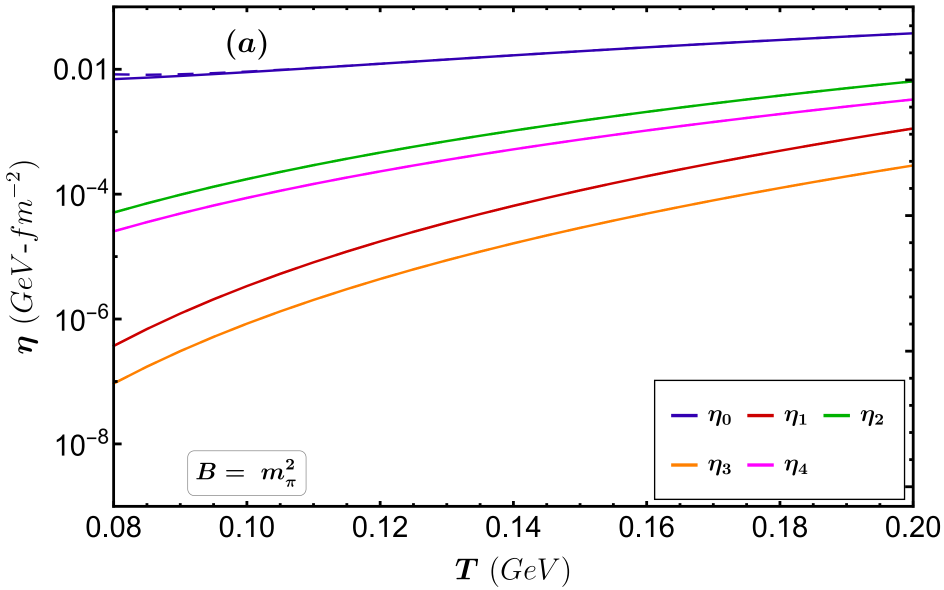
$$[\eta_{||}]_2 = \frac{T}{2}\frac{\left[\left(c_{11}\gamma_1^{(0)} - c_{01}\gamma_0^{(0)}\right)\left(\gamma_0^{(0)} - \frac{2}{3}\beta_0^{(0)}\right) + \left(c_{00}\gamma_0^{(0)} - c_{10}\gamma_1^{(0)}\right)\left(\gamma_0^{(0)} - \frac{2}{3}\beta_0^{(0)}\right)\right]}{(c_{00}c_{11} - c_{01}c_{10})}$$

$$[\zeta_{||}]_2 = [\zeta_\perp]_2 = T\frac{(a_{33}\alpha_2^2 - 2\alpha_2\alpha_3a_{23} + a_{22}\alpha_3^2)}{(a_{22}a_{33} - a_{23}a_{32})}$$

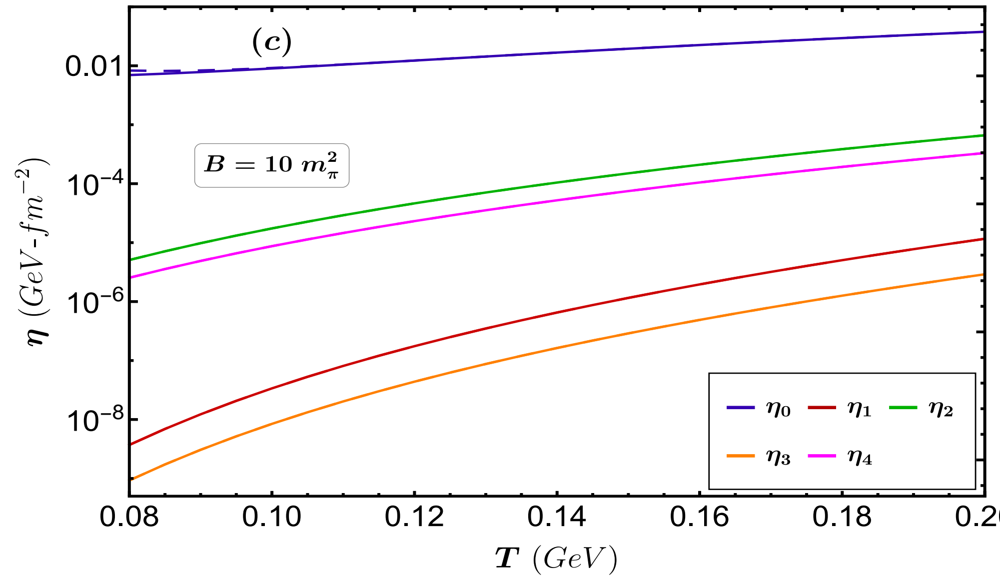
$$[\zeta_\times]_2 = 0$$

Result for Shear

$B = m_\pi^2$



$B = 10m_\pi^2$



U.Gangopadhyaya Et al.;
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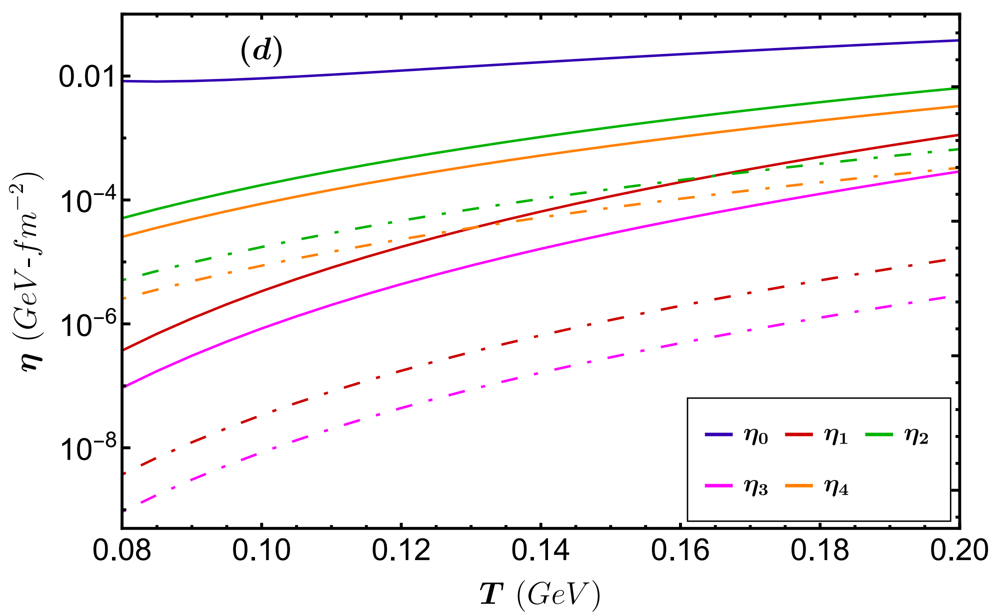
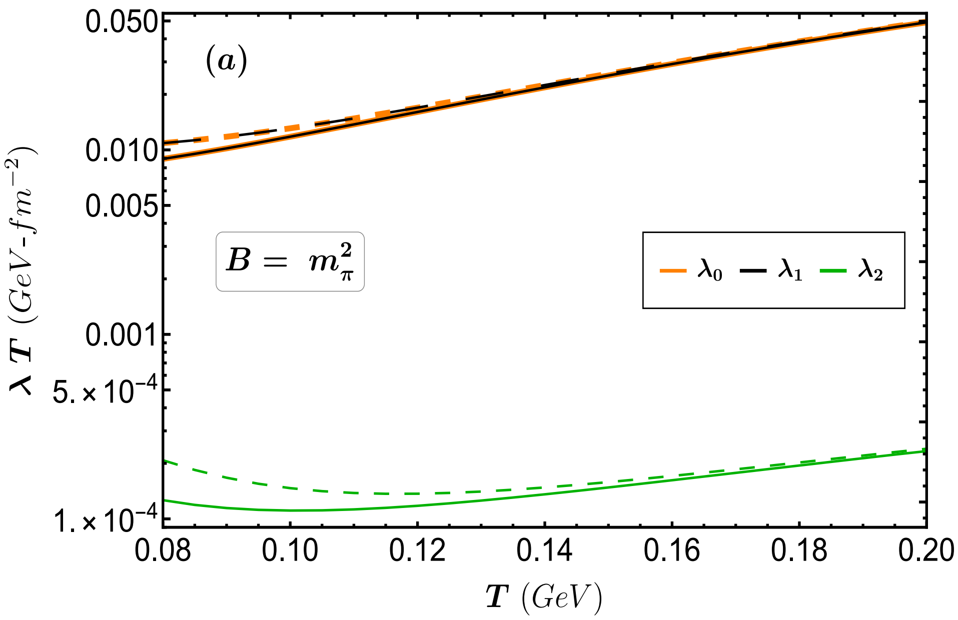


$B = m_\pi^2, \quad 10m_\pi^2$

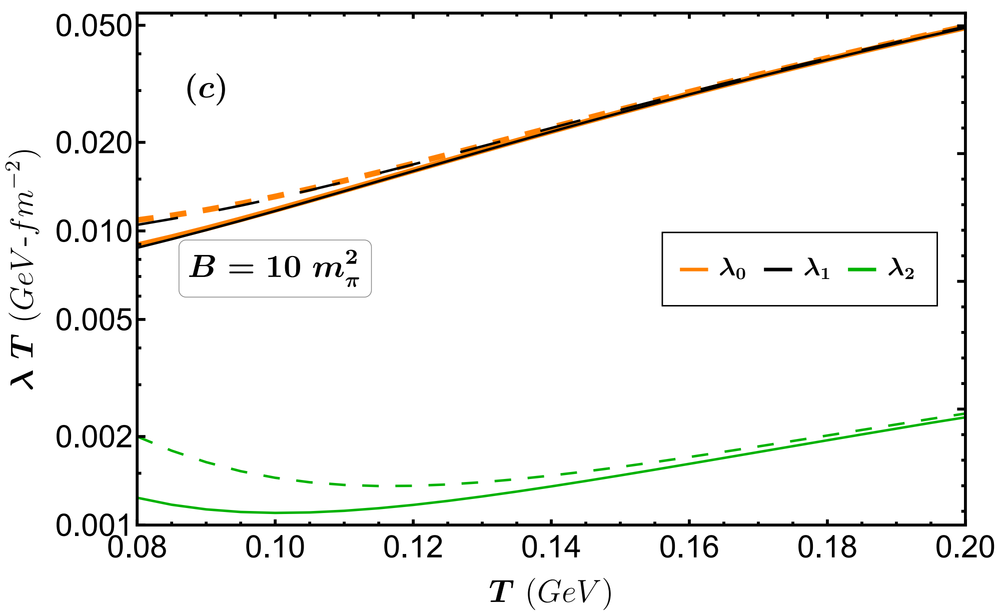
Result for

Thermal conductivity

$B = m_\pi^2$



$B = 10m_\pi^2$



Summary

- Using the Chapman-Enskog method we can get a better result than using the relaxation time approximation.

- There is momentum dependent relaxation time approximation, but it comes with its own problems.

- The Chapman-Enskog method can be modified to include system with magnetic field by making the expansion coefficients a function of magnetic field .

Future work

- Will take in-medium cross-section instead of vaccum cross-section.
- Will introduce effects of magnetic field in the cross-section, and study its effect on the transport properties.
- Will introduce electric field along with magnetic field.

Thank You