Effect of critical point on Λ-hyperon spin polarization

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based on arXiv:2110.15604

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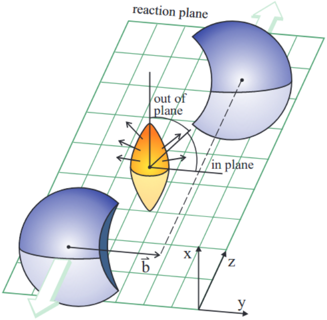
Large OAM in non-central heavy-ion collision

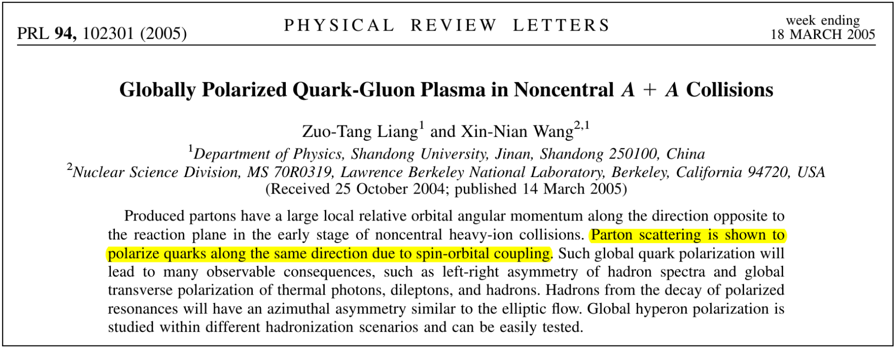


arXiv:0910.4114

- Nuclei carry a large orbital angular momentum (OAM), $L_0 = pb \simeq A\sqrt{s_{NN}}b/2$.
- e.g. for $\sqrt{s_{NN}} = 200$ GeV and $b = 5$ fm, $L_0 \sim 5 \times 10^5$.
- A fraction of L_0 is transferred to QGP fireball.

Spin polarization of hadrons

Parton scattering polarizes quarks along the OAM direction due to spin-orbital coupling in QCD, $P_q \sim -0.3$ at RHIC.



PRL **94,** 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

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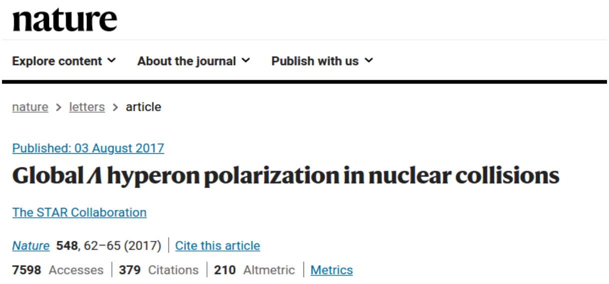
Produced partons have a large local relative orbital angular momentum along the direction opposite to the reaction plane in the early stage of noncentral heavy-ion collisions. Parton scattering is shown to polarize quarks along the same direction due to spin-orbital coupling. Such global quark polarization will lead to many observable consequences, such as left-right asymmetry of hadron spectra and global transverse polarization of thermal photons, dileptons, and hadrons. Hadrons from the decay of polarized resonances will have an azimuthal asymmetry similar to the elliptic flow. Global hyperon polarization is studied within different hadronization scenarios and can be easily tested.

One distinctive signature of an OAM would be the polarization of the emitted hadrons. Considering hadronization via quark recombination, $P_\Lambda = P_s \approx P_q$, for example.

Experimental observation of Λ-polarization



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Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration

Nature **548**, 62–65 (2017) | Cite this article

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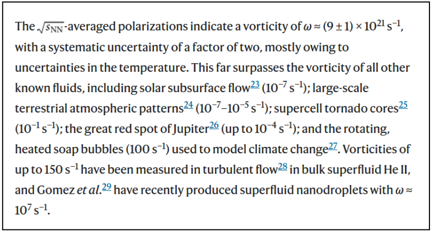
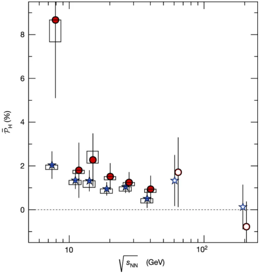
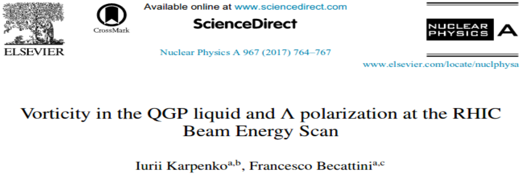


The $\sqrt{s_{NN}}$-averaged polarizations indicate a vorticity of $\omega \approx (9 \pm 1) \times 10^{21}\ s^{-1}$, with a systematic uncertainty of a factor of two, mostly owing to uncertainties in the temperature. This far surpasses the vorticity of all other known fluids, including solar subsurface flow[23] ($10^{-7}\ s^{-1}$); large-scale terrestrial atmospheric patterns[24] (10^{-7}–$10^{-5}\ s^{-1}$); supercell tornado cores[25] ($10^{-1}\ s^{-1}$); the great red spot of Jupiter[26] (up to $10^{-4}\ s^{-1}$); and the rotating, heated soap bubbles ($100\ s^{-1}$) used to model climate change[27]. Vorticities of up to $150\ s^{-1}$ have been measured in turbulent flow[28] in bulk superfluid He II, and Gomez *et al.*[29] have recently produced superfluid nanodroplets with $\omega \approx 10^7\ s^{-1}$.



Hydrodynamic simulation for global polarization



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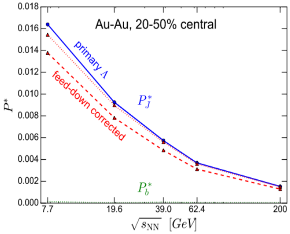
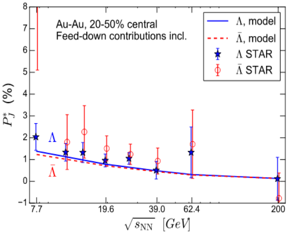
Vorticity in the QGP liquid and Λ polarization at the RHIC
Beam Energy Scan

Iurii Karpenko[a,b], Francesco Becattini[a,c]

Initial condition : UrQMD string/hadron cascade, all components of thermal vorticity tensor are initially non-vanishing. Simulation on a constant energy density hypersurface (0.5 GeV/fm^3).





Cooper-Frye formula for particles with spin

Momentum spectrum of of i^{th} hadron is given by

$$E\frac{dN_i}{d^3p} = \int_\Sigma (d\Sigma.p)\, f_i(x,p) \qquad \rightarrow \qquad \text{Cooper-Frye prescription}$$

Polarization vector for spin-1/2 particles

$$P_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau + \mathcal{O}(\varpi^2)$$

where

$$\varpi^{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \qquad \text{with} \qquad \beta_\rho = \frac{u_\rho}{T}$$

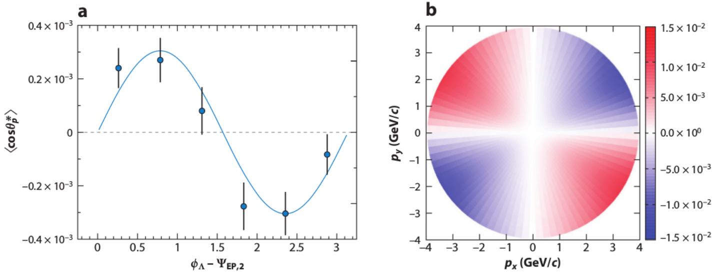
Ann. Phys. 338:32 (2013)

Space-integrated mean polarization vector

$$P_\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, P_\mu(x,p) n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$

Spin sign puzzle

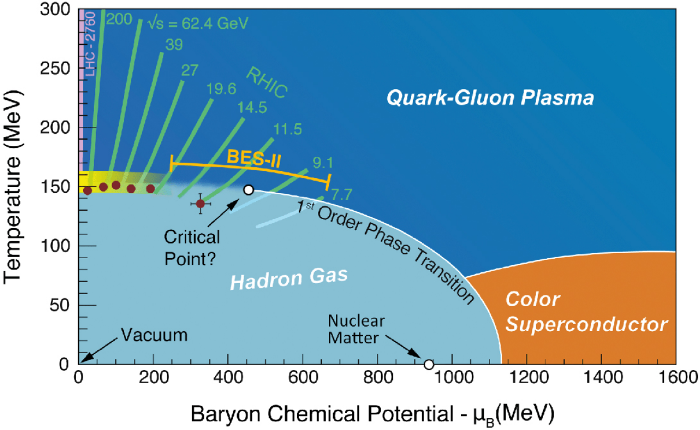
"Hydrodynamic and transport-hybrid calculations predict a negative sign of the longitudinal component of the polarization vector. The magnitude of the effect is significantly larger in the model."



Ann. Rev. Nucl. Part. Sci. 70 (2020) 395

This and several other questions led to the development of relativistic dissipative spin hydrodynamics.

QCD phase diagram ("Conjectured")



CERN courier, February 2021

What is the effect of critical point on spin polarization ?

Critical point

- The correlation length, ξ, diverges at the critical point.

- If the dynamical universality class of the QCD critical point is that of Model H, then

$$\eta \sim \xi^{0.05} \;,\; \zeta \sim \xi^{3} \;,\; \kappa_T \sim \xi$$

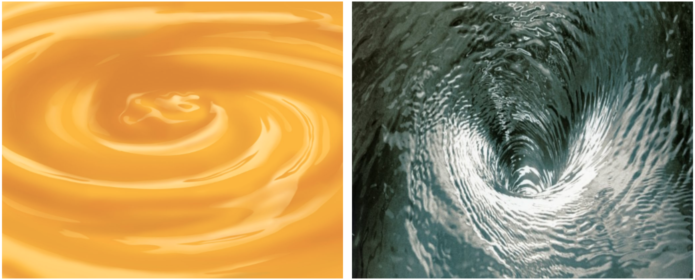
- Similar scaling laws for relaxation times (τ_π, τ_Π etc.)

- Near CP, the fireball will feel enhanced viscosity.

A non-relativistic analogy

For a non-relativistic fluid with constant η and ζ

$$\frac{\partial\vec{\omega}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{\omega}+\theta\vec{\omega}=\left(\vec{\omega}\cdot\vec{\nabla}\right)\vec{v}+\frac{1}{\rho^2}\vec{\nabla}\rho\times\vec{\nabla}p-\frac{1}{\rho^2}\left(\zeta+\frac{1}{3}\eta\right)\vec{\nabla}\rho\times\vec{\nabla}\theta-\frac{\eta}{\rho^2}\vec{\nabla}\rho\times\nabla^2\vec{v}+\frac{\eta}{\rho}\nabla^2\vec{\omega}.$$



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Another motivation !!!

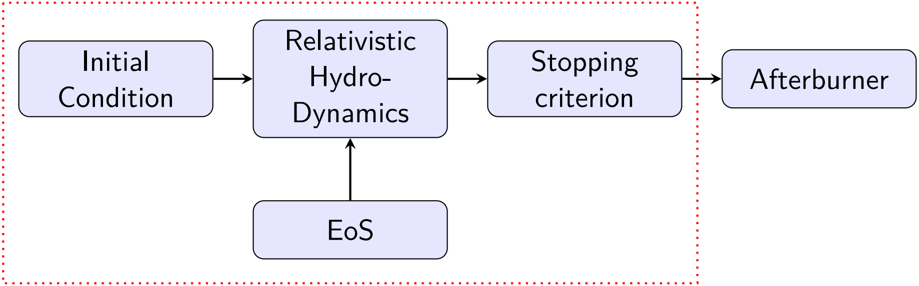
The spin polarization vector in the rest frame of a hyperon at some point in the fluid is given by (Ann. Rev. Nucl. Part. Sci. 70 (2020) 395)

$$\vec{S}^*(x,p) \propto \frac{\gamma}{T^2}\vec{v}\times\nabla T + \frac{1}{T}\left(\vec{\omega} - (\vec{\omega}\cdot\vec{v})\vec{v}\right) + \frac{1}{T}\gamma\vec{A}\times\vec{v}$$

- $\vec{S}^*$ depends on ∇T, $\vec{\omega} = \nabla\times\vec{v}$ and acceleration of fluid cell.
- In nutshell, gradients of temperature and flow-velocity.
- Gradients depend on the expansion dynamics of the system.
- Expansion depends on the EoS.

Modeling the fireball evolution

Flow Chart for simulation



- Initial condition : Shifted Glauber
- EoS : BEST model
- Stopping criterion : constant energy density (CORNELIUS)
- Afterburner : UrQMD

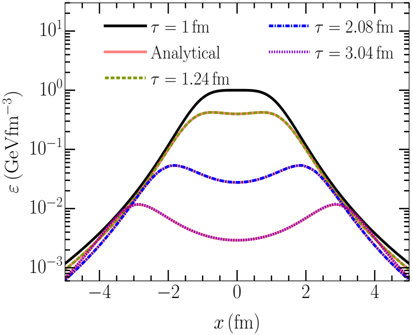
Relativistic Hydrodynamics

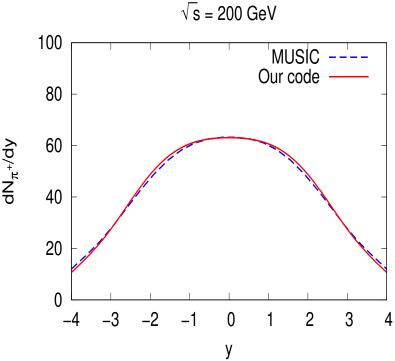
- Ref : **SKS** *& J. Alam, VECC/IR/2018/04 (VECC Internal Report)*
- Hydrodynamic equations

$$D_\mu T^{\mu\nu} = 0$$
$$D_\mu N_B^\mu = 0$$

$$\Delta^{\mu\nu}_{\alpha\beta} u^\gamma D_\gamma \pi^{\alpha\beta} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu} D_\gamma u^\gamma$$
$$u^\gamma D_\gamma \Pi = -\frac{\Pi - \Pi_{NS}}{\tau_\Pi} - \frac{4}{3}\Pi D_\gamma u^\gamma$$

- We develop the code using relativistic HLLE algorithm and test it against known analytical results and with output from publicly available MUSIC and vHLLE codes.

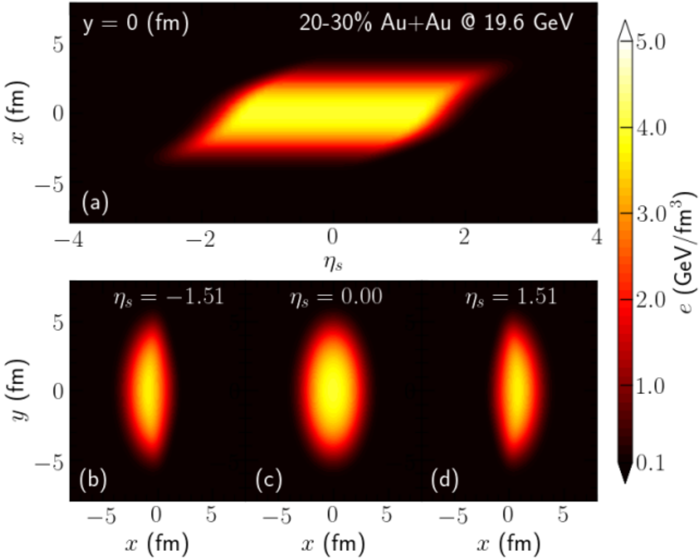




SKS and J. Alam, arXiv:2110.15604

Initial condition

- $S^y \propto [p_\tau \varpi_{\eta x} + p_x \varpi_{\tau\eta} + p_\eta \varpi_{x\tau}]$ where $\varpi_{\mu\nu} = \frac{1}{2}\left[\partial_\nu\left(\frac{u_\mu}{T}\right) - \partial_\mu\left(\frac{u_\nu}{T}\right)\right]$.
- Need an IC with non-zero $\partial_\eta u_x$ or $\partial_x u_\eta$. Simple Glauber model will not work.
- Glauber model for transverse profile along with symmetric rapidity profile for energy density has zero $\varpi_{\eta x}$ at all times.
- We use Glauber model + symmetric rapidity profile + local energy-momentum conservation. C. Shen *et al.* PRC 102 (2020) 014909
- For non-zero initial angular momentum, we use the model of S. Ryu *et al.* PRC 104, 054908 (2021)



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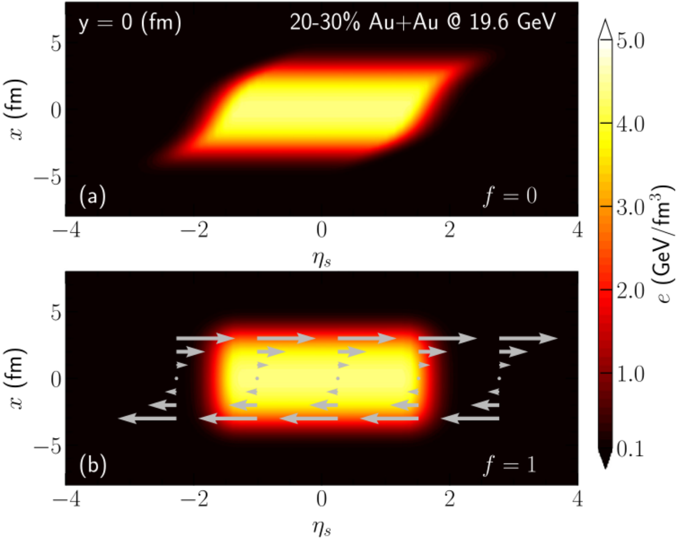
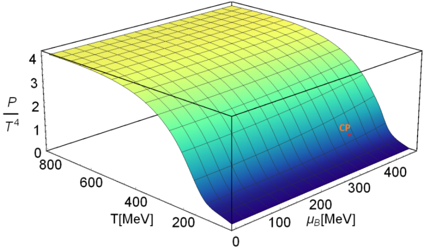
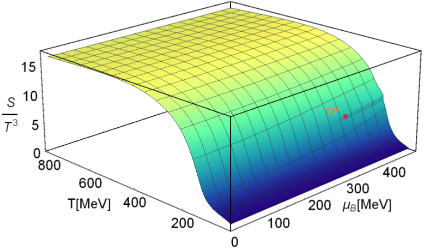


PRC 104, 054908 (2021)

Equation of state & transport coefficients

- EoS model of BEST collaboration PRC 101, 034901 (2020).
- $P_{QCD}(\mu_B, T) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$
- $T^4 c_n(T) \rightarrow T^4 c_n^{\text{Non-Ising}}(T) + T_c^4 c_n^{\text{Ising}}(T)$ or $P_{QCD}(\mu_B, T) = P^{\text{reg}}(\mu_B, T) + P^{\text{crit}}(\mu_B, T)$.
- Obtain P^{crit} by mapping to 3D-Ising model.
- Choose and adjust P^{reg} such that $P_{QCD}(0, T) = P^{\text{LAT}}(T)$.





Near the critical point, the transport coefficients diverge as

$$\zeta \sim \xi^3 \qquad , \qquad \eta \sim \xi^{0.05}$$

The critical behavior of these transport coefficients can be modeled as

$$\zeta = \zeta_0 \left(\frac{\xi}{\xi_0}\right)^3 \qquad , \qquad \eta = \eta_0 \left(\frac{\xi}{\xi_0}\right)^{0.05}$$

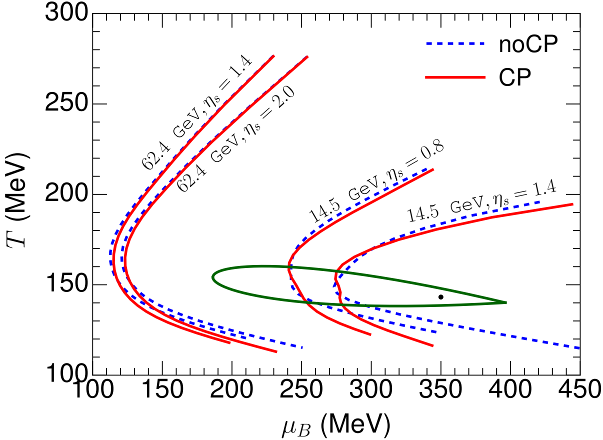
ξ_0 is a parameter for deciding the boundary of the critical region. We choose $\xi_0 = 1.75$ fm (mostly taken as 1 fm). ζ_0 and η_0 taken as

$$\eta_0(\mu_B, T) = 0.08 \left(\frac{\varepsilon + p}{T}\right)$$
$$\zeta_0(\mu_B, T) = 15\eta_0(\mu_B, T) \left(\frac{1}{3} - c_s^2\right)^2$$

Stopping criterion

- Constant energy density, $\varepsilon = 0.3$ GeV/fm^3. Close to transition line.
- The surface is found using the CORNELIUS code.
- The surface is input to the UrQMD.
- The spin polarization analysis is done on this surface

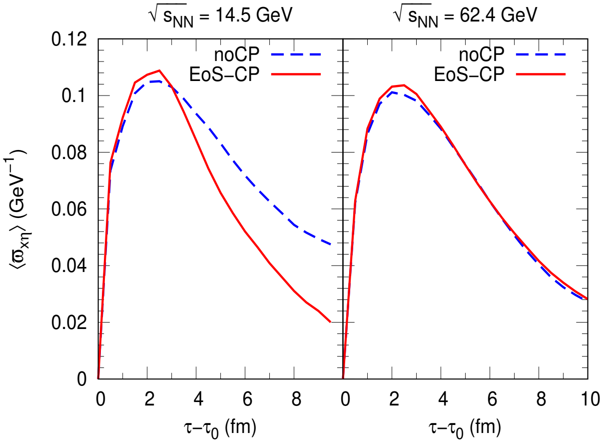
Hydrodynamic trajectories in phase diagram



SKS and J. Alam, arXiv:2205.14469

Case I : Zero initial angular momentum

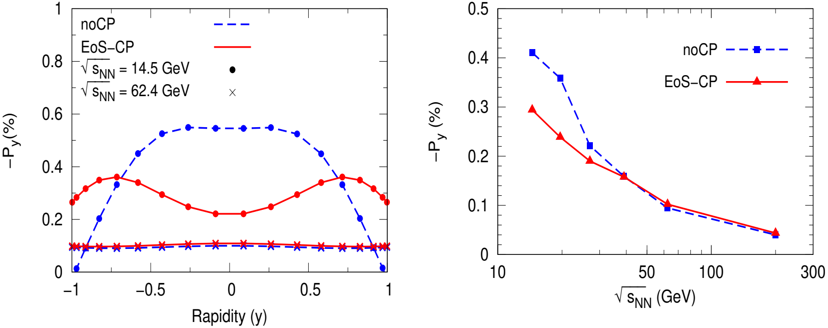
Evolution of thermal vorticity



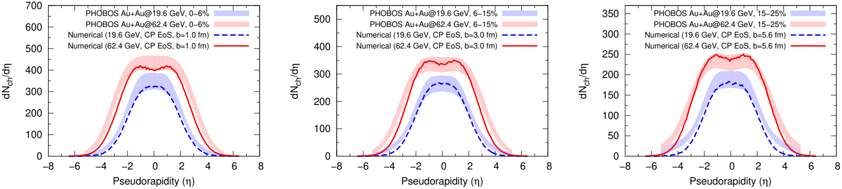
SKS and J. Alam, arXiv:2110.15604

Suppression of spin polarization of Λ-hyperons

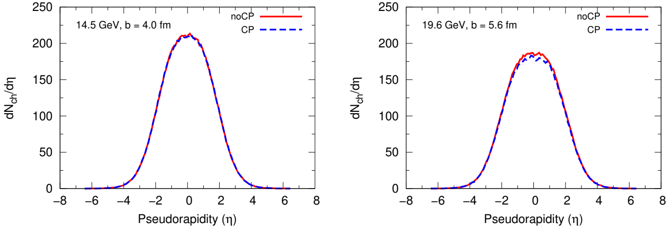
- Polarization calculated on constant energy density hypersurface 0.3 GeV/fm^3. No afterburner.



SKS and J. Alam, arXiv:2110.15604



SKS and J. Alam, arXiv:2110.15604

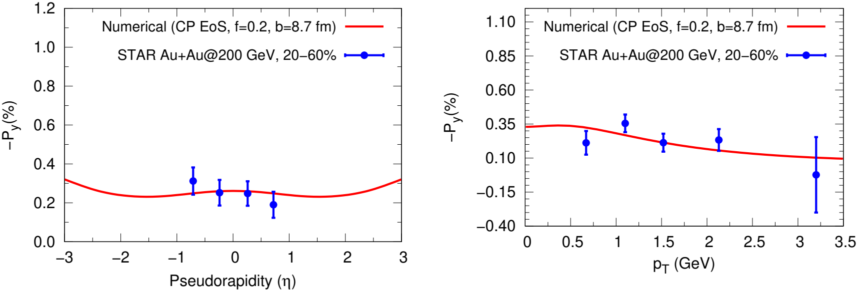


SKS and J. Alam, arXiv:2110.15604

Case II : Non-zero initial angular momentum

Comparison with experimental data

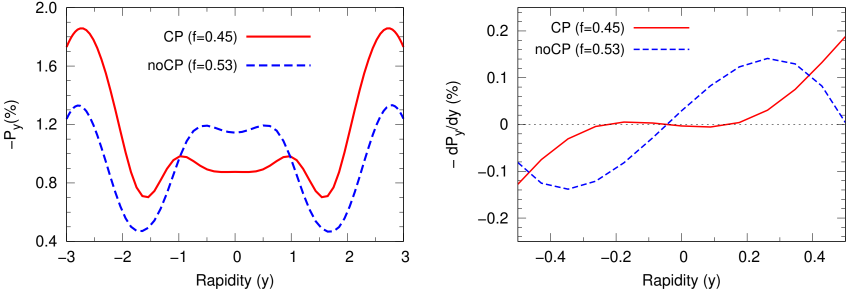
- In S. Ryu *et al.*, PRC 104, 054908 (2021), non-zero initial vorticity is obtained by introducing a parameter *f* that controls the fraction of longitudinal momentum that can be attributed to the flow velocity.



SKS and J. Alam, arXiv:2110.15604

Prediction near critical point

Au+Au collisions at $\sqrt{s_{NN}} = 14.5$ GeV with $b = 5.6$ fm



SKS and J. Alam, arXiv:2110.15604

- We also find that the other bulk observables like elliptic flow, p_T-spectra etc. are not much affected due to the CP. SKS and J. Alam, arXiv:2205.14469.

Summary

- Observables dependent on gradients are more sensitive to the EoS.
- We observe a suppression in thermal vorticity and hence, polarization of Λ-hyperons, as the CP is approached.
- Suppression in the rapidity distribution of spin polarization may be useful for locating CP. Further study needed.

Thank You !!!