Quantized Electrical conduction in BNS magnetized plasma

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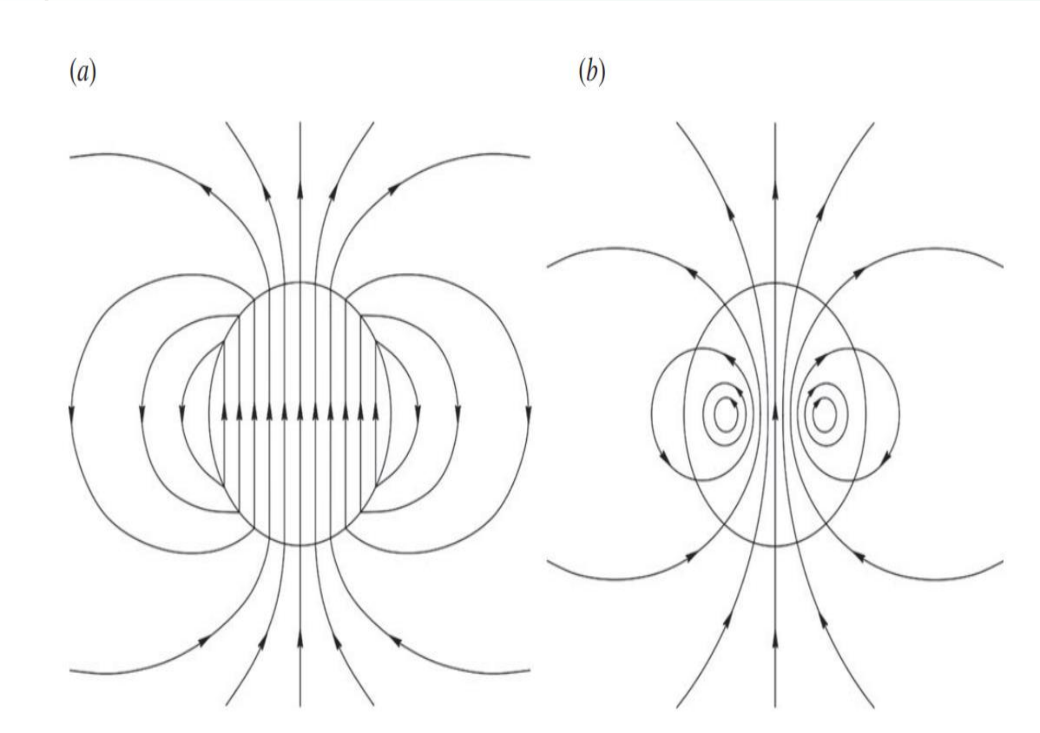
Outline

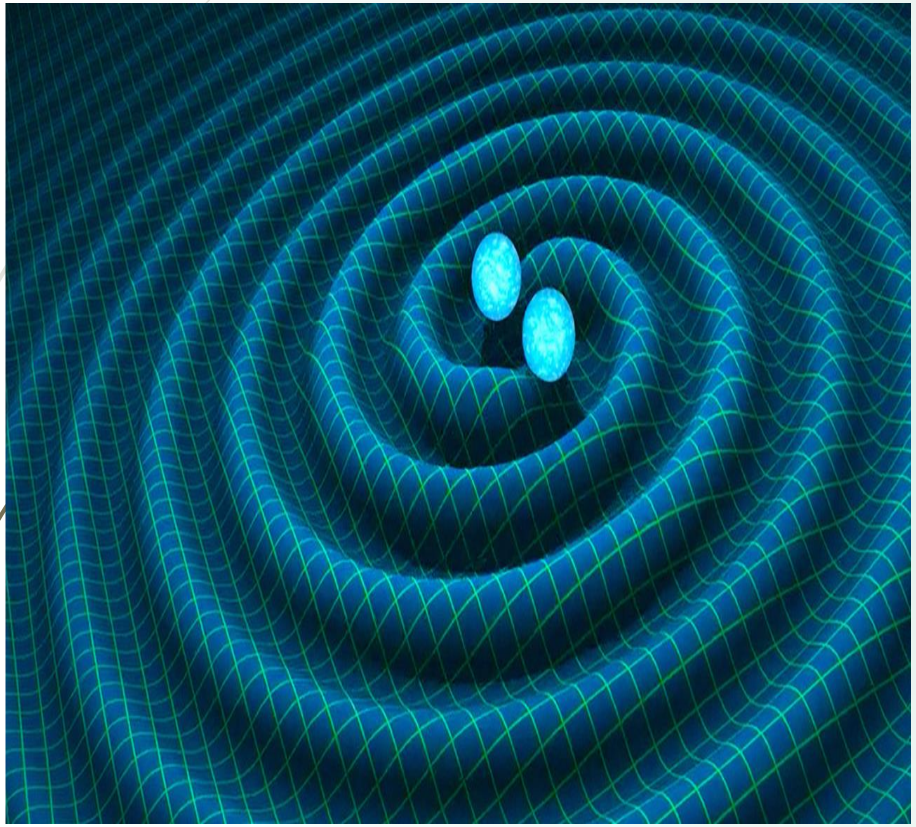
- Introduction
- Motivation: Electrical Conduction in BNS magnetized plasma
- Formalism: Electrical Conductivity
- Physical Condition: Density-Temperature-Magnetic field domain
- Results
- Summary

Electromagnetic Plasma: Neutron Star



- In a plasma slight imbalance in the numbers of particles of opposite sign ions and electrons produce intense electric field.
- Slight anisotropy in motion of particles produce strong magnetic fields.
- Freezing of Magnetic lines in plasma large induction.
- In presence of electric field electrons move opposite to the direction of the field.
- Scattering between electrons-ions equilibrate the distribution functions to local equilibrium.

Introduction: BNS Merger



- LIGO and VIRGO detector ⇒ gravitational wave after the collision of two neutron stars ⇒ unfold the mystery of Neutron star.
- Binary neutron star evolves in time start emitting gravitational wave ⇒ frequency of gravitational wave continues to increase until they merge to form a neutron star or a black hole.
- Natural abundances of Extreme physical conditions ⇒ Conditions are more extreme than anything one observes on the earth. Nuclear material in a neutron star merger⇒ **Density ~ 4nsat ⇒Temperature 5 to 50 MeV⇒high-amplitude density oscillations with f ~ 1 kHz**

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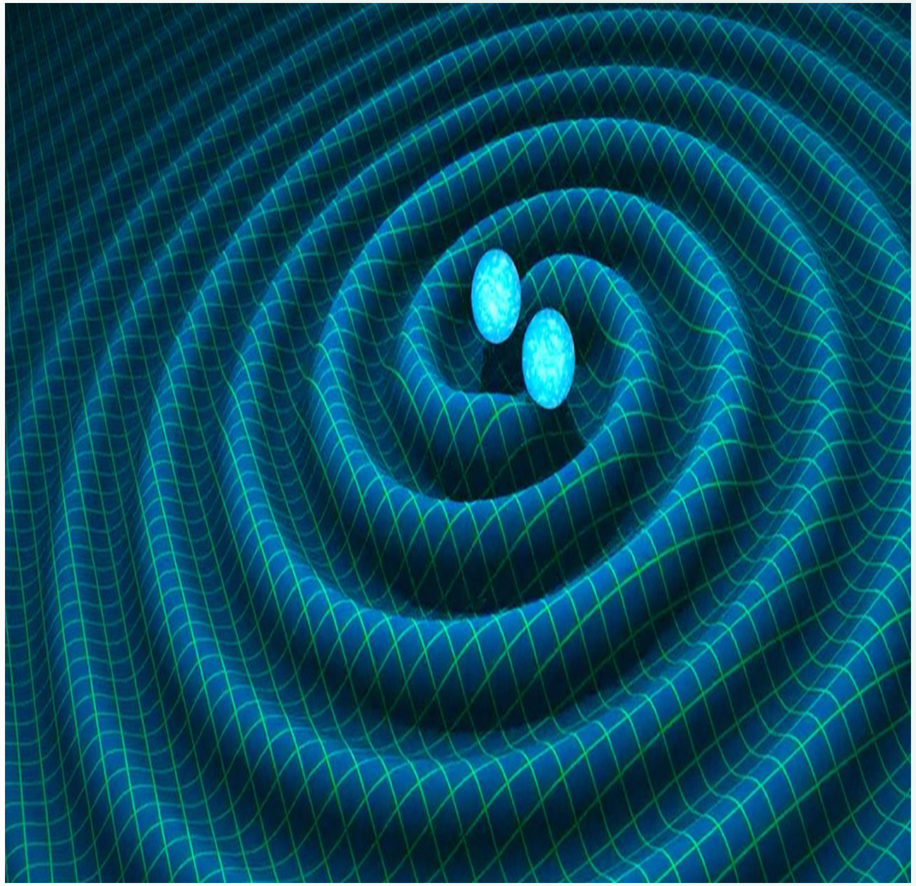
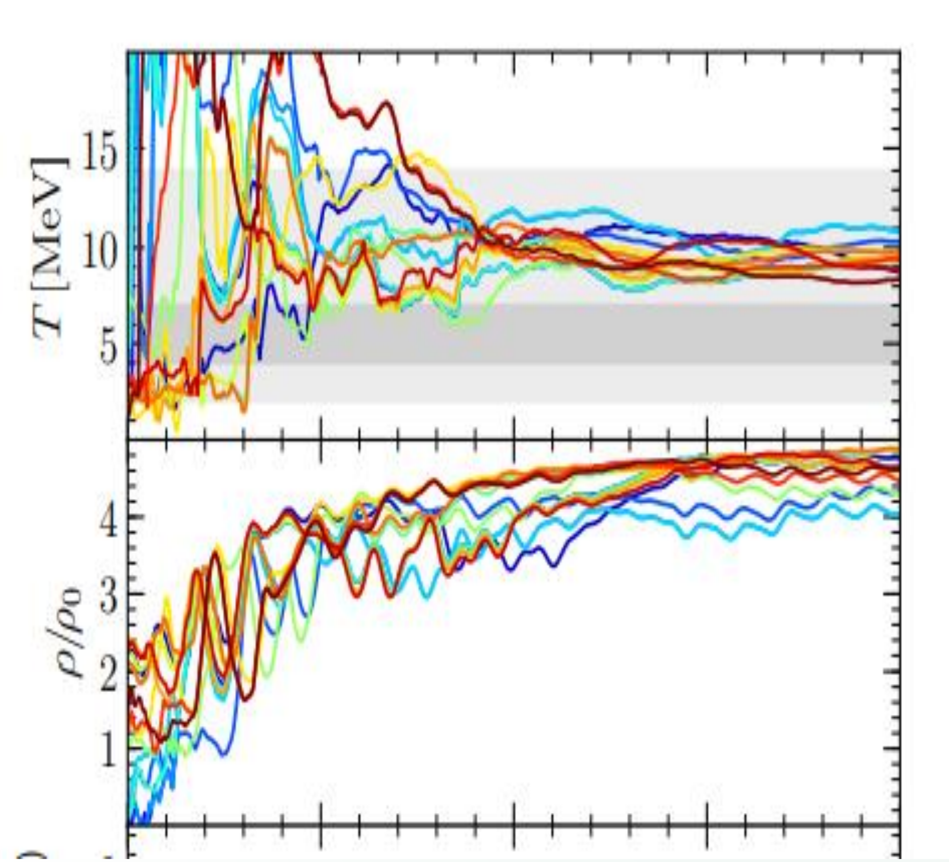


- In the event post merging, a remnant neutron star is created and if the remnant possess a mass beyond Tolman–Oppenheimer–Volkoff (TOV) limiting mass, the merged object survives for 1-100 milliseconds and collapses thereafter.
- The description of neutron star mergers requires the knowledge of General Relativistic Magneto Hydro-dynamics. Most of these numerical simulations account for ideal Magneto Hydro-dynamics.

Motivation



- Maxwell's Eqns:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \vec{v} \times \vec{B} + \frac{c^2}{4\pi\sigma} \nabla^2 \mathrm{B}$$

- Magnetic field decay time scale

$$t_d = \frac{4\pi\,\sigma}{\Lambda^2}$$

- Numerical estimation considers conductivity to be essentially infinite, hence, large magnetic field decay time scale ⇒ larger than survival time period of the post merged object.

Physical Condition

- Simplest possible constituents of post-merger object of electron-ion plasma with fully ionized ions and free mobile electrons in the low density (up to 10^{12}g cm^{-3}), high magnetic field (up to 10^{17}G), high temperature (T ~ 15 MeV).
- Number Density

$$n_e = \frac{m\omega_B}{(2\pi)^2} \int_{-\infty}^{\infty} dp_z \sum_{n,s} f(\epsilon_p).$$

- Electronic Dispersion relation

$$\epsilon_p = \sqrt{p_z^2 + m^2 + 2n\omega_B m}.$$

- Electrons being light particles become relativistic at high density, when, relativistic parameter (x_r)<1 and temperature ~ 5.93X10^9K.

$$x_r = p_F/m \sim 1.008(\frac{\rho_6 Z}{A})^{1/3}$$

$$T_r \sim 5.930\times 10^9 \text{ K}.$$

Quantizing conditions

$$T_{ce} \approx 1.343 \times 10^8 \; B_{12} \text{ K},$$

$$\rho_B = 7.045 \times 10^3 \frac{A}{Z} \, (B_{12})^{3/2} \text{ g cm}^{-3}$$

- The field B is strongly quantizing if $\rho < \rho_B$ and $T \ll T_{ce}$.
- A magnetic field is called weakly quantizing if many Landau levels are occupied for $T \ll T_B$.
- B is nonquantizing if $T \gg T_B$.

Longitudinal Quantized Electrical Conductivity

- The conductivity tensor has a simple form when the magnetic field is along the z direction

$$\sigma = \begin{pmatrix} \sigma_\perp & \sigma_H & 0 \\ \sigma_H & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix}$$

- Electrical conductivity from kinetic theory

$$j = \sigma E \qquad j = \frac{em\omega_B}{4\pi^2}\int_{mc^2}^{\infty} \Phi d\epsilon_p,$$

- The distribution function of electrons has two parts in presence of electromagnetic field

$$f_{n,p_z,s} = f_0(\epsilon_p) + \Phi_{n,p_z,s},$$

Boltzmann equation in electromagnetic plasma

- Boltzmann equation in presence of magnetic field

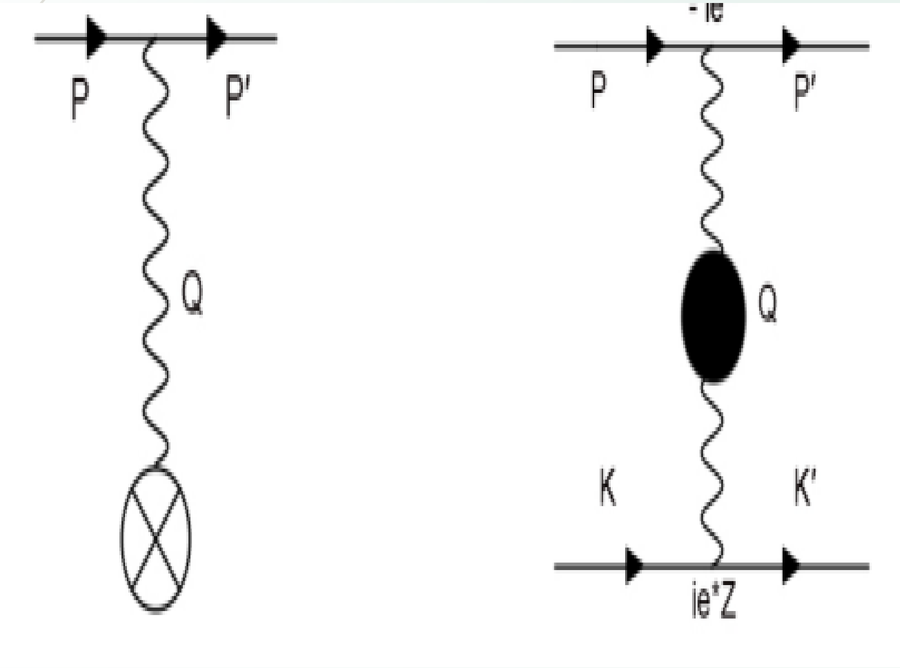
$$\frac{\partial f_{np_zs}}{\partial t} + v_z \frac{\partial f_{np_zs}}{\partial z} - \dot{\mathbf{p}}.\frac{\partial f_{np_zs}}{\partial p_z} = \mathcal{C}[f].$$

- In the Lorentz force, the magnetic field contribution vanishes
- $$(e(v_p \times B).v_p \frac{\partial f_{np_zs}}{\partial \epsilon_{n,p_z,s}} = 0)$$

- The collision integral in relaxation time approximation

$$\mathcal{C}[f] = \left.\frac{\partial f_{n,p_z,s}}{\partial t}\right|_{coll} = \sum_f I_{fi}\left(f_{n,p_z,s\to n',p'_z,s'}\right),$$

Electron-ion scattering rate



- Electron momentum $p = (\epsilon_p, p_z, p_\perp)$ exchanges a virtual photon with an in medium ion of momentum k. The electron emerges with momentum $p' = (\epsilon_{p'}, p'_z, p'_\perp)$ and ion with momentum k'

$$I_{fi} = \frac{1}{2\epsilon_p}\int \frac{d^3p'}{(2\pi)^3 2\epsilon_{p'}} \int \frac{d^3k}{(2\pi)^3 2\epsilon_k} \int \frac{d^3k'}{(2\pi)^3 2\epsilon_{k'}} \left[f_{n,p_z,s}\, g_k \left(1 - f_{n',p'_z,s'}\right) - f_{n',p'_z,s'}\, g_{k'} \left(1 - f_{n,p_z,s}\right)\right] (2\pi)^4 \delta^4\left(p + k - p' - k'\right) |\mathcal{M}_{fi}|^2 \quad (8$$

Electron-ion scattering matrix amplitude

- M_{fi} is the electron-ion scattering matrix

$$M_{fi} = -\Delta_L J_0 J_0' + \Delta_T J_t J_t' = -M_L + M_T,$$

- Effective longitudinal and transverse propagators are given by

$$\Delta_L = \frac{1}{q^2 - \Pi_L},$$
$$\Delta_T = \frac{1}{q^2 - \Pi_T}.$$

- Debye mass :

$$m_D^2 = \left(\frac{e}{\pi}\right)^2 \left(\frac{eB}{2}\right).$$

Solution of Boltzmann equation

- Off-equilibrium distribution function

$$\Psi = \frac{E^2 - 1}{2Q_2}$$

$$Q_2 = \int_0^\infty e^{-u} \times \left[\frac{2}{3\left(u + \frac{\zeta}{3}\right)(u + \zeta)} - \frac{v_k^2}{6u\left(u + \frac{\zeta}{3}\right)} \right] du$$

- $E = \epsilon_p / m,$ $u = \frac{1}{2m\omega_B}(q_x^2 + q_y^2)$ $\zeta = m_D^2 / 2m\omega_B.$

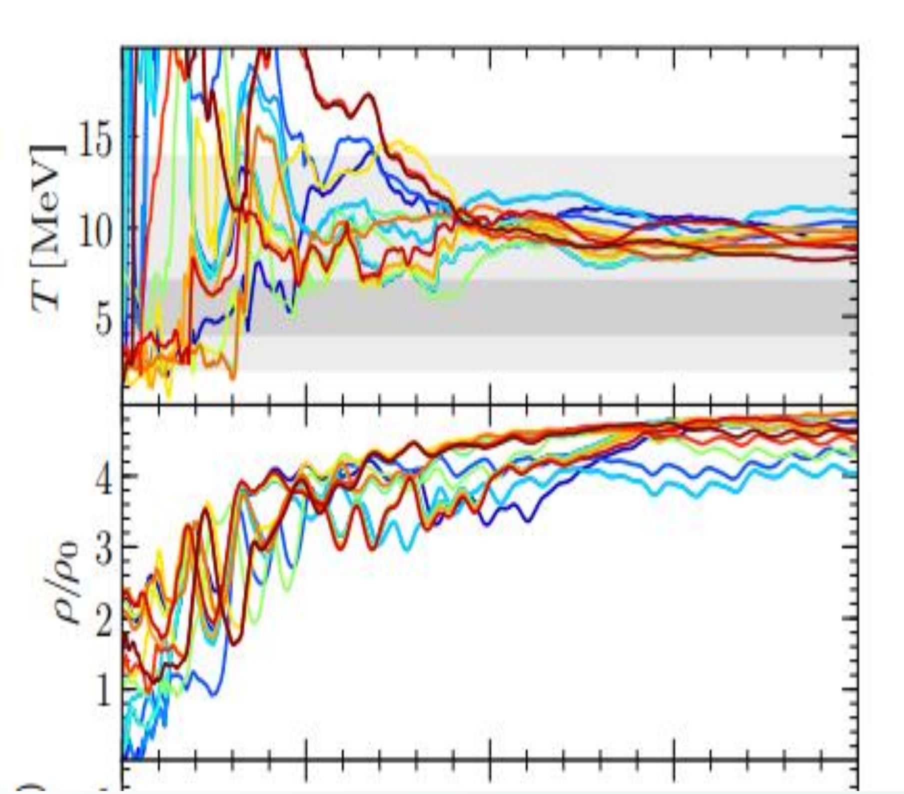
Electrical Conductivity

- Electrical Conductivity

$$\sigma = \frac{\delta_0}{\theta}\int_1^\infty \Psi f_0(1 - f_0)dE,$$

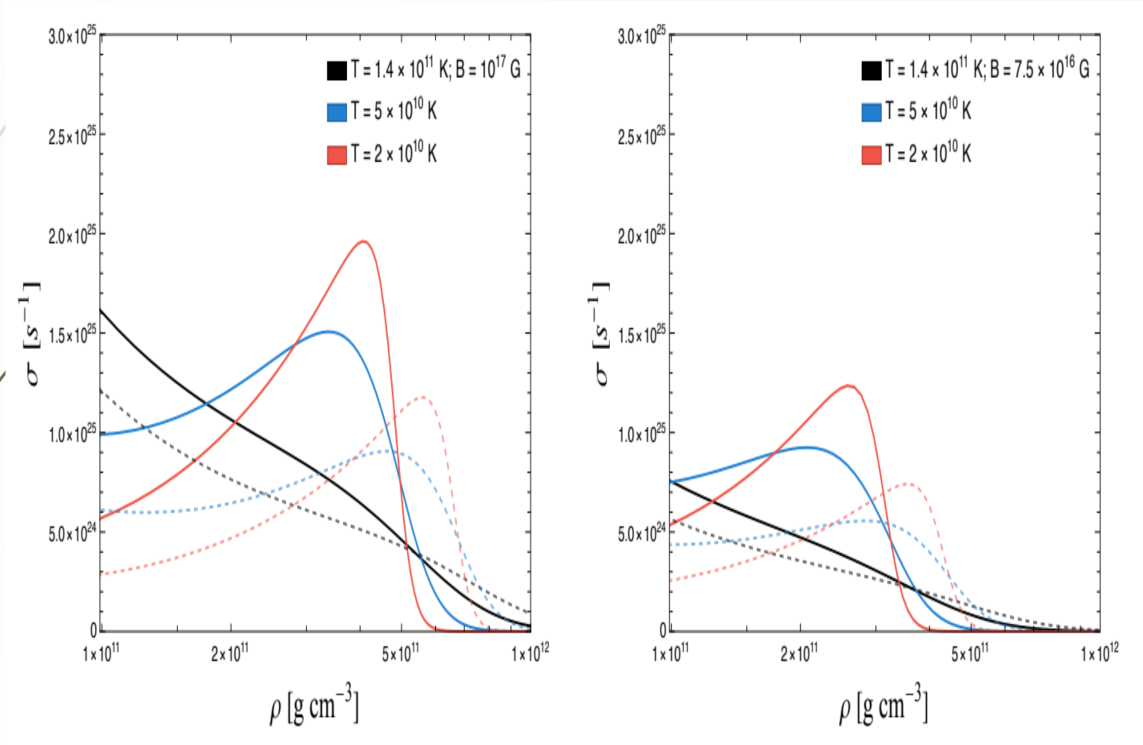
- At low temperature satisfying $(\varepsilon_p - \mu) \sim T$, $\partial f_0/\partial \epsilon_p = \delta(\epsilon_p - \mu)$. σ becomes temperature independent at low temperature.
- v_k reduces the numerator of second term. In the denominator of the same equation $q_\perp^2 < 2m\omega_B$ and $m_D^2 < 2m\omega_B$ lead to $u < 1$, $\zeta < 1$ respectively. Hence, the reduction due to small value of vk gets compensated and interaction rate increases due to frequency dependent dynamical screening.

Density-Temperature-Magnetic Field



- In order to consider electrons to be relativistic, the density and temperatures are chosen as $\rho \gg 10^6$ g cm^{-3} and $T \gg 5 \times 10^9$ K.
- For fixed B, μ increases with ρ and electrons start to populate higher Landau levels. For zeroth Landau level population, the density and temperature should also satisfy $\rho < \rho_B$ and $T \ll T_{ce}$
- Obeying merger condition, we consider $\rho \sim 10^{12}$ g cm^{-3} and $T \gg 2\times10^{10}$ K and $B \sim 10^{16}$ G, 10^{17} G

Variation of σ with density



- The plot for zeroth Landau level population.
- The origin of the hump is due to the fulfillment of the weak degeneracy condition ($|\epsilon_p - \mu| \sim T$) of electron distribution function.
- The nature of the curve resembles differentiated Fermi function at $T \ll \mu$.

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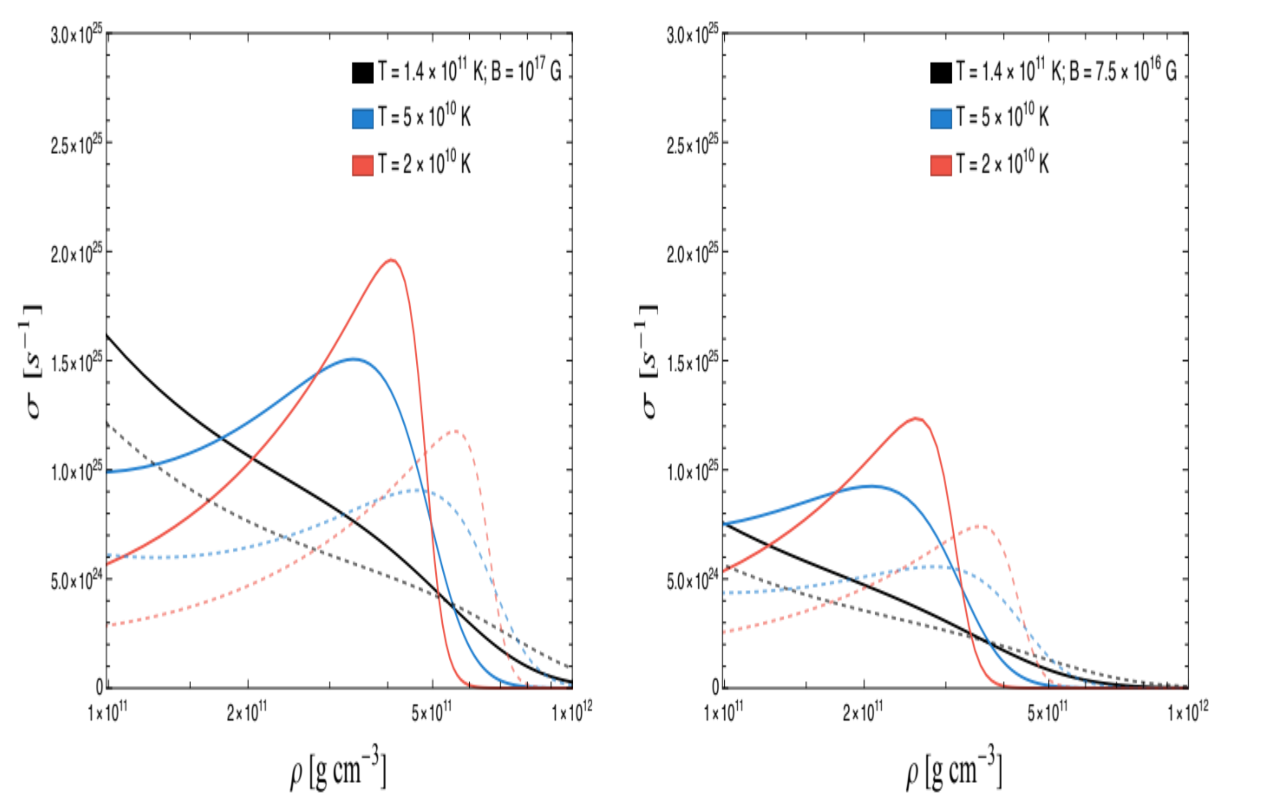
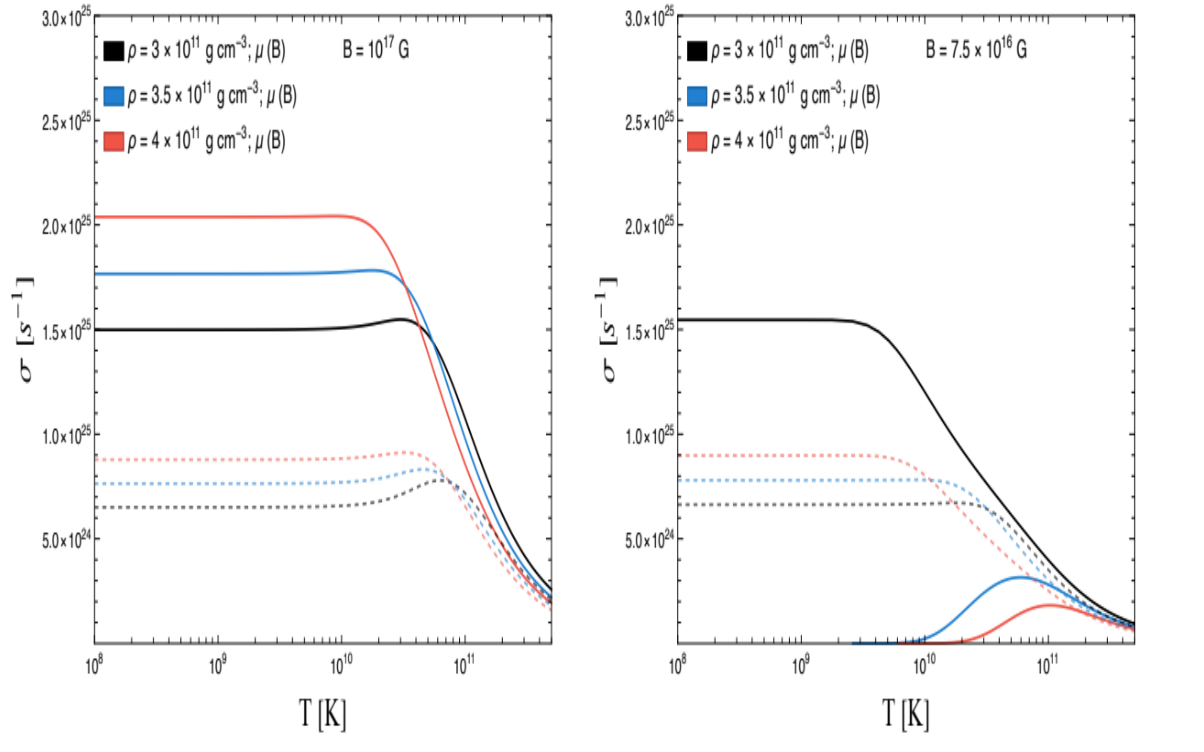


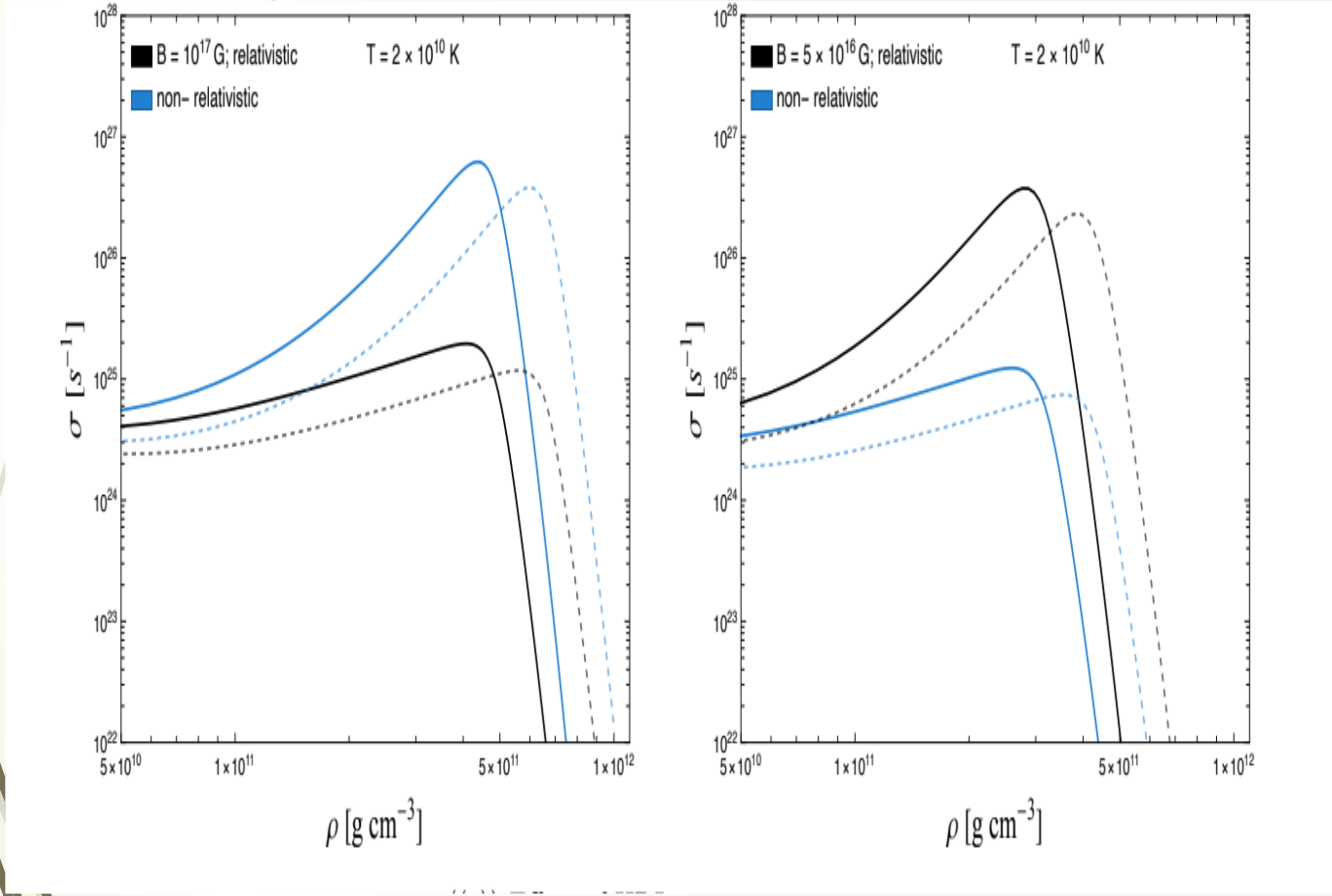
- As the temperature increases, the hump gets flattened since electrons start becoming non-degenerate.
- In each plot of the figure curves are drawn for two different materials Fe (solid lines) and Mo (dotted lines).

Variation of σ with temperature



- $\sigma = (a + b \times T^c)^{-1}$
- At low temperature the effect of T_c is very small, σ is constant.
- At high temperature $\sigma \propto T^{-c}$ and decreases with temperature. Thus, at higher temperatures, the electrons become classical obeying the inverse dependence of temperature.

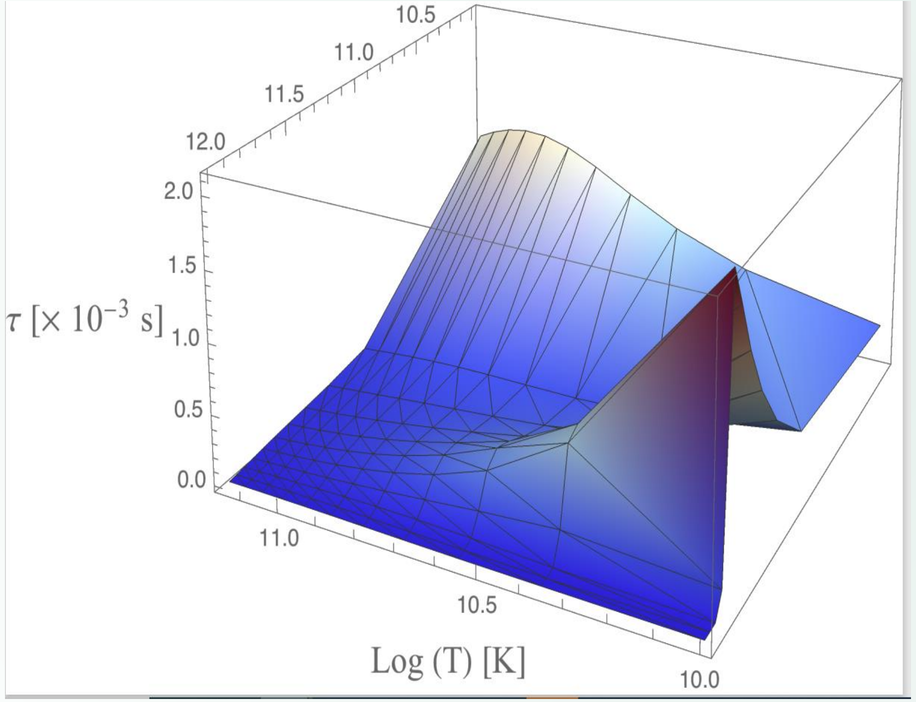
Effect of dynamical screening on conductivity



- The reduction in the transport coefficients arises due to the increase in the interaction rate caused by non-zero frequency in both the transverse component of the propagator

- The off-equilibrium distribution function is inversely proportional to interaction rate and σ is directly proportional to Ψ. Increment in Ψ decreases the coefficient.

Variation of magnetic field decay time scale with density and temperature

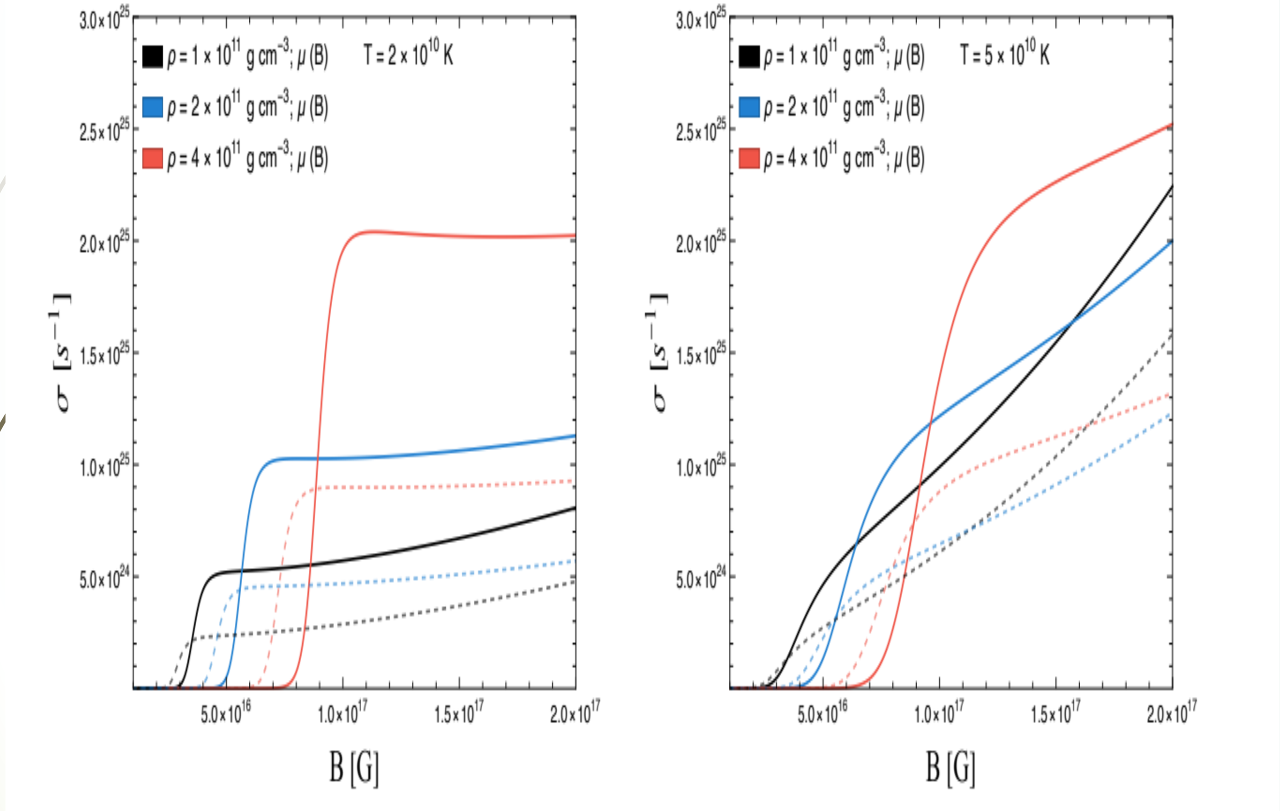


- We find the magnetic field decay timescale is well within the range of survival time period of neutron star merger.

Summary

- The calculation of electrical conductivity, particles are considered slightly out of equilibrium to solve the Boltzmann equation numerically.
- We calculate electron-ion scattering amplitude with screened electromagnetic interaction of magnetically modified spinors
- We have found that the inclusion of the frequency dependent HDL propagator in the relativistic plasma reduces the values of σ in contrast to static screening in the nonrelativistic plasma.
- An estimation of diffusion time scale including frequency dependent screening has found out to be of the same order as the survival time of the merged compact object due to reduction in the value of σ.
- The realistic estimate of timescale can be obtained including all the components of conductivity tensor

THANK YOU



3.0×10^25
2.5×10^25
2.0×10^25
1.5×10^25
1.0×10^25
5.0×10^24
σ⁻ [s⁻¹]
ρ = 1 x 10^11 g cm^-3; μ (B)
ρ = 2 x 10^11 g cm^-3; μ (B)
ρ = 4 x 10^11 g cm^-3; μ (B)
T = 2 x 10^10 K
T = 5 x 10^10 K
5.0×10^16
1.0×10^17
1.5×10^17
2.0×10^17
B [G]