QGP bubble dynamics in the presence of magnetic field

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Emergent Topics in relativistic Hydrodynamics,
Chirality, Vorticity and Magnetic field
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Puri

S. Namitha, Lakshmi J. Naik and V. Sreekanth,
QGP BUBBLE DYNAMICS IN THE PRESENCE OF MAGNETIC FIELD
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Introduction

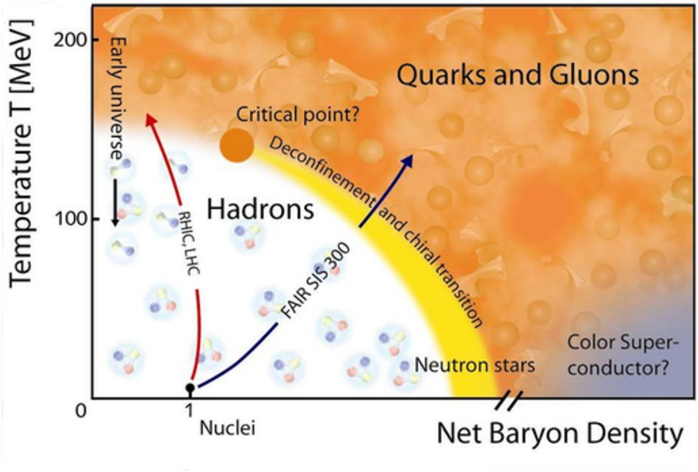


Figure: QCD phase diagram

Introduction

- Nuclear collisions to be performed at FAIR and NICA are expected to create QGP at low temperature and high baryon densities
- In such collisions, the quark-hadron conversion is a first order phase transition - nucleation theory can be employed
- Hadronic matter requires some time to convert to QGP
- Compressed hadronic matter $\longrightarrow$ QGP bubbles

Hadron $\longrightarrow$ QGP phase transition

- The transition involves formation of QGP bubbles in hadronic medium followed by collapse of Hadron gas (HG) bubbles in QGP.

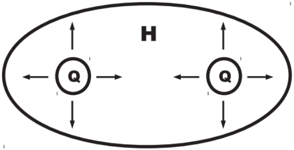


Figure: Expansion of QGP bubbles.

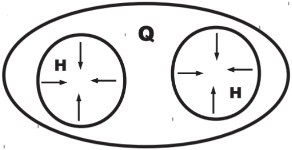
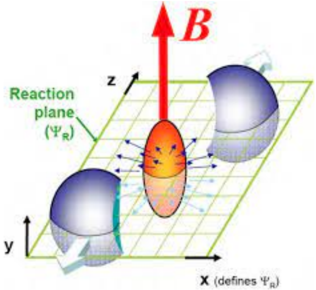
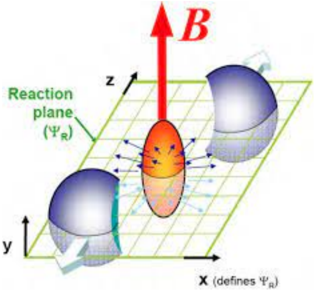


Figure: Collapse of HG bubbles.

Figs. from D. A. Fogaca et al., PRC 93, 055204 (2016)



- Matter created in heavy ion collisions is subjected to large magnetic field produced by the spectators ($\sim eB = 10^{19}$ G)



- Matter created in heavy ion collisions is subjected to large magnetic field produced by the spectators ($\sim eB = 10^{19}$ G)
- AIM: Effect of magnetic field on expansion or collapse of QGP/HG bubbles in a relativistic treatment.

Rayleigh-Plesset Equation

- Collapse and expansion of bubbles in an external medium can be modelled using the Rayleigh-Plesset (RP) Equation
- Collapse of an empty bubble in liquid : Rayleigh equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_i - p_\infty}{\rho}$$

  p_∞ : far-field pressure [Lord Rayleigh, Philos. Mag. 34, 94 (1917)]
- Non-relativistic, incompressible version of RP equation : [M. S. Plesset, (1949)]

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(p_i - p_0 - \frac{2\sigma}{R} - \frac{4\eta\dot{R}}{R}\right)$$

  p_0 - static pressure outside the bubble wall
  p_i - constant pressure inside the bubble
  σ, η - surface tension & shear viscosity

- Relativistic version of the RP equation (RRP) was done by Elze *et al*
- Equation was extended to the relativistic regime at finite temperature by D. A. Fogaca *et al.*
- Lagrangian for a spherically symmetric system

$$\mathcal{L} = -4\pi \int_0^\infty dr\, r^2 \epsilon(n); \quad \text{where } n = \rho/\gamma.$$

- Contributions of internal and external matter :

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{in} + \mathcal{L}_{ex} \\ &= -4\pi \int_0^R dr\, r^2 \epsilon_{in}(n_{in}) - 4\pi \Big\{ \lim_{R_\infty \longrightarrow \infty} \int_R^{R_\infty} dr\, r^2 \epsilon_{ex}(n_{ex}) \Big\} \end{aligned}$$

Elze et al., J. Phys. G 25, 1935 (1999) D. A. Fogaca et al., PRC 93, 055204 (2016)

Relativistic RP Equation [contd.]

- Number density profile for each phase :

$$\begin{aligned} \rho_{in} &= \frac{3N_{in}}{4\pi R^3}, \quad 0 < r < R \\ \rho_{ex} &= \frac{3N_{ex}}{4\pi(R_\infty^3 - R^3)}, \; R < r < R_\infty \end{aligned}$$

- Now the velocity profiles can be obtained as

$$\begin{aligned} v_{in} &= \frac{r}{R}\dot{R}, \; 0 < r < R \\ v_{ex} &= \frac{(R_\infty^3 - r^3)}{(R_\infty^3 - R^3)} \frac{R^2}{r^2}\dot{R}, \; R < r < R_\infty \end{aligned}$$

Relativistic RP Equation [contd.]

- Temperature is introduced with the thermodynamic relation :

$$d\epsilon = T\,ds + \mu\,dn; \quad \mu = \frac{\partial \epsilon}{\partial n}$$

- Euler-Lagrange equation gives the relativistic RP equation :

$$\boxed{\frac{d}{dt}\left[(I_1 + I_2)R^3\dot{R}\right] + (I_1 - 2I_2)R^2\dot{R}^2 = (P^*_{in} - P^*_{ex})R^2}$$

$$I_1 = \int_0^1 dx\, x^4(\epsilon_{in} + P^*_{in})\gamma^2_{in} \qquad P^*_{in} = P_{in} - \frac{\partial P}{\partial T}\,T\Big|_{in}$$

$$I_2 = \int_1^\infty \frac{dx}{x^2}\,(\epsilon_{ex} + P^*_{ex})\,\gamma^2_{ex} \qquad P^*_{ex} = P_{ex} - \frac{\partial P}{\partial T}\,T\Big|_{ex}$$

$x = r/R$.

D. A. Fogaca et al., PRC 93, 055204 (2016)

Inclusion of magnetic field

- Matter created in heavy ion collisions is subjected to large magnetic field produced by the spectators ($\sim eB = 10^{19}$ G)
- Approximations : External magnetic field is constant and homogeneous
- Medium surrounding the bubble is in-compressible, irrotational
- Effects due to magnetic friction is included through dissipation function

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{R}}\right) = \frac{\partial \mathcal{L}}{\partial R} - \frac{\partial \mathcal{F}}{\partial \dot{\mathcal{R}}}$$

Inclusion of magnetic field (contd.)

- Current induced in the medium outside the bubble :

$$\mathbf{j} = \sigma[(\mathbf{v} \times \mathbf{B}) + \mathbf{E}] = \sigma[(\mathbf{v} \times \mathbf{B}) - \nabla\psi]$$

$[\nabla \times E = -\frac{\partial B}{\partial t} = 0]$

- Magnetic force due to the current

$$\begin{aligned} \mathbf{f} = \mathbf{j} \times \mathbf{B} &= \sigma[(\mathbf{v} \times \mathbf{B}) \times \mathbf{B}] \\ &= -\sigma[\mathbf{v}\mathbf{B}^2 - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})] \end{aligned}$$

B.O. Kerbikov et. al PRD101,094004 (2020)

- Energy dissipation due to magnetic friction :

$$\mathcal{Q} = -\mathbf{f} \cdot \mathbf{v} = 4\pi\sigma \mathbf{B}^2 \int_R^{R_\infty} v_{ex}^2 \, r^2 \, dr = 4\pi\sigma \mathbf{B}^2 R^3 \dot{R}^2 = 2\mathcal{F}$$

- Using the dissipation function obtained above, relativistic RP with magnetic field is obtained as

$$\boxed{\frac{d}{dt}\left[(I_1 + I_2)R^3 \dot{R}\right] + (I_1 - 2I_2)R^2 \dot{R}^2 = (P_{in}^* - P_{ex}^*)R^2 + \sigma B^2 R^3 \dot{R}}$$

Inclusion of magnetic field (contd.)

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- RRP equation is solved employing the lattice QCD EoS for QGP phase and the Ideal HRG for HG matter.

Lattice QCD EoS

$$P(T,\mu) = T^4 \int_0^T dT' \frac{e^{-h_1/\tau' - h_2/\tau^2}}{T'} \left[h_0 + \frac{f_0[\tanh(f_1\tau' + f_2) + 1]}{1 + g_1\tau' + g_2\tau'^2} \right] + \frac{\chi_2}{2}\mu^2 T^2$$

$$\epsilon(T,\mu) = 3P(T,\mu) + \frac{\mu^2}{2} T^3 \frac{d\chi_2}{dT} + T^4 e^{-h_1/\tau - h_2/\tau^2} \left[h_0 + \frac{f_0[\tanh(f_1\tau' + f_2) + 1]}{1 + g_1\tau' + g_2\tau'^2} \right]$$

where,

$$\chi_2(T) = e^{-h_3/\tau - h_4/\tau^2} f_3[\tanh(f_4\tau + f_5) + 1]$$

$\tau = T/T_c$ with $T_c = 200$ MeV and $\mu = 350$ MeV.

[S. Borsanyi et al., PLB 730, 99 (2014)]

Hadron Resonance Gas Model

Pressure and energy density of Ideal HRG :

$$
\begin{aligned}
P(T,\mu) &= \sum_i p_i(T,\mu_i) \\
&= \sum_i \frac{d_i}{6\pi^2}\int dm\, f_i(m)\int_0^\infty \frac{dk k^4}{\sqrt{k^2+m^2}} \\
&\quad \times \left[\exp\left(\frac{\sqrt{k^2+m^2}-\mu_i}{T}\right)+\eta_i\right]^{-1} \\
\epsilon(T,\mu) &= \sum_i \epsilon_i(T,\mu_i) \\
&= \sum_i \frac{d_i}{2\pi^2}\int dm\, f_i(m)\int_0^\infty k^2 dk\sqrt{k^2+m^2} \\
&\quad \times \left[\exp\left(\frac{\sqrt{k^2+m^2}-\mu_i}{T}\right)+\eta_i\right]^{-1}
\end{aligned}
$$

(1)

Hadron Resonance Gas Model

- Summation over all strange and non-strange hadrons listed in Particle Data Tables (mesons upto $f_2(2340)$ and baryons upto $N(2600)$) [K. A. Olive et al, Chin. Phys. C 38, 090001 (2014)]
- $f_i(m) = \delta(m - m_i)$, where m_i is the mass of stable hadrons

RRP + B

- RRP equation is solved numerically employing the lattice QCD EoS for QGP phase and the Ideal HRG for HG matter

$$\frac{d}{dt}\left[(I_1 + I_2)R^3\dot{R}\right] + (I_1 - 2I_2)R^2\dot{R}^2 = (P^*_{in} - P^*_{ex})R^2 + \sigma B^2 R^3 \dot{R}$$

QGP bubble expansion

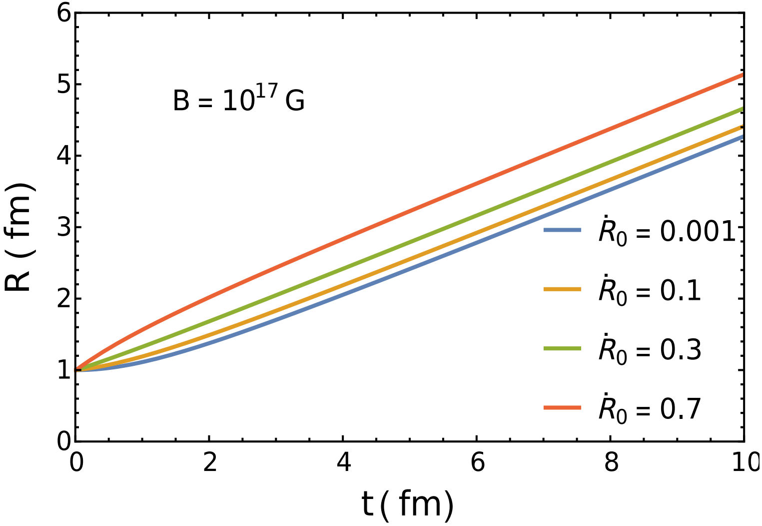

Figure: Expansion of QGP bubble in hadronic medium for $T_{in} = 190$ MeV and $T_{ex} = 180$ MeV.

QGP bubble expansion

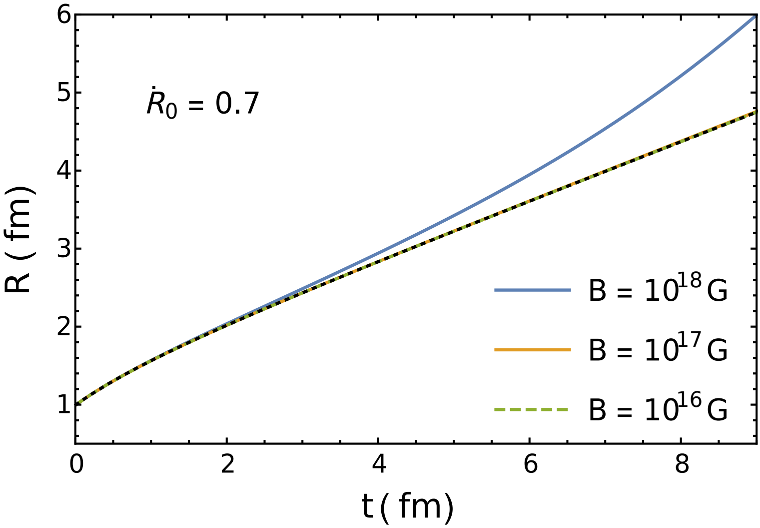

Figure: Expansion of QGP bubble in hadronic medium for $T_{in} = 190$ MeV and $T_{ex} = 180$ MeV.

HG bubble collapse

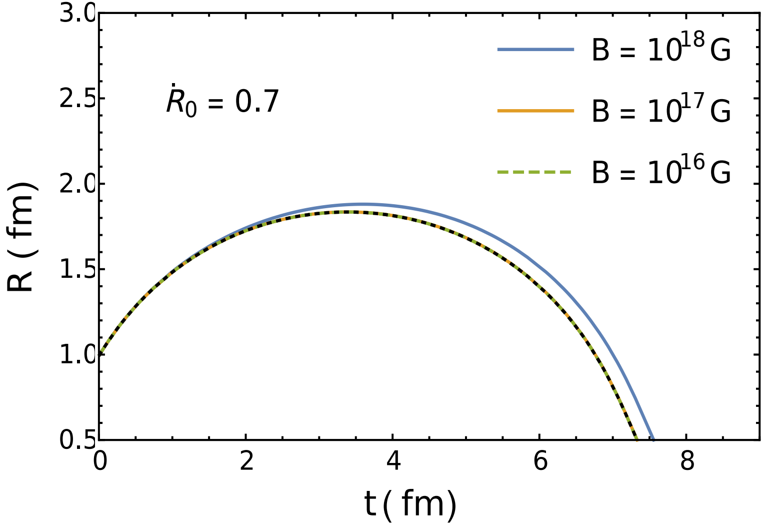

Figure: Collapse of hadron gas bubbles in QGP medium for $T_{in} = 180$ MeV and $T_{ex} = 190$ MeV.

HG bubble collapse

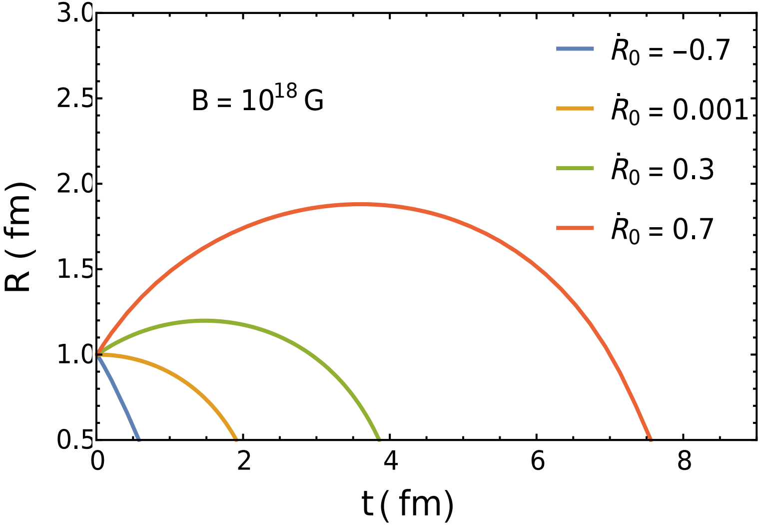

Figure: Collapse of hadron gas bubbles in QGP medium for $T_{in} = 180$ MeV and $T_{ex} = 190$ MeV.

Conclusion

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- High value of magnetic field ($B = 10^{18}$ G) has influence on the dynamics of bubbles
- Initial magnetic field increases the expansion rate of QGP bubbles
- Magnetic field at the initial stage of collision increases the collapse time of HG bubbles
- For large radial velocities, HG bubbles live longer

THANK YOU