Heavy Quark Polarization: two EFTs

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Emerging Topics in
Hydrodynamics, Chirality, Vorticity and Magnetic Fields
Toshali Sands (February 04, 2023)

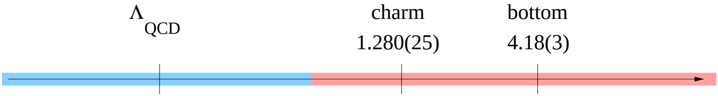
Why

Why are hadrons polarized in HIC?

- Particle spin can only couple to an axial vector
- The only axial vectors available in the fireball are **B** and **J**
- If coupling to **B** then why does the particle remain polarized after initial EM fields decay?
- If coupling to **J** then how does a microscopic element (the particle) feel a macroscopic quantity (value of **J** of the fireball)?

Energy scales and the EFTs

Scale separation at $T = 0$



For physics at scales $Q < \Lambda$, with

$$\Lambda_{QCD} \ll \Lambda \ll M$$

pair creation suppressed by powers of Λ/M. Use effective theory: HQET.

Pauli spinor for heavy quark (HQ) and gluons with $k \ll M$. Effect of all other modes absorbed into low-energy couplings (LECs). Heavy quark velocity $v = p/M$ is conserved. EFT in powers of v; leading terms kinetic.

HQET

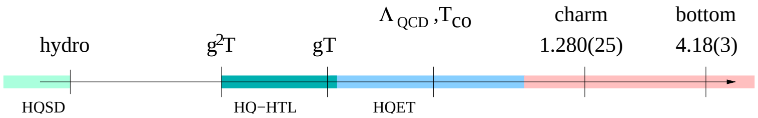
The Lagrangian at order v^0 is

$$L_0 = \overbrace{\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)}^{pure\ \ gauge} + \overbrace{\psi^\dagger \left(iD_0 + \frac{1}{2m}\mathbf{D}^2 \right) \psi}^{Schroedinger}$$

Quark mass term dropped. Notation: $iD_\mu = i\partial_\mu - ieA_\mu$. Use $D_0 = v \cdot D$, with $\mathbf{D} = D - vD_0$ (space and time components in RF of quark). Used Coulomb gauge. No Pauli matrices, so **spin symmetry** at this order. Seen in heavy meson spectra, decays.

Magnetic interactions arise at order v^2. Break spin symmetry: recall fine structure and L-S coupling in atomic physics.

Finite T: rich in scales



No hadrons, but many scales. At LHC, initial $T > T_{co}$. Electric scale gT, magnetic scale $g^2 T$. Realistically $g \simeq 1$, but can control the theory more easily by first setting g small and then taking limit $g \to 1$. Also much longer scale of hydro: no quarks, no gluons, only stress tensor.

Theory at scale T: HQE$\mathbb{T}$ (new). Theory at electric scales previously considered for HQ diffusion. Theory at hydrodynamic scales: heavy quark sono-dynamics (HQSD, new).

Thermal Heavy Quark EFT: HQE$\mathbb{T}$

The HQE$\mathbb{T}$ Lagrangian

Choose Λ, the UV cutoff of the EFT so that $M \gg \Lambda \gg T > T_{co}$. With $\Lambda \simeq 1$ GeV, hierarchy satisfied at LHC for M_b but not M_c. So this theory works for bottom at LHC.

The integration over hard modes can be done in perturbative QCD, with very little difference from the $T = 0$ theory. Carried to order v^2, we have

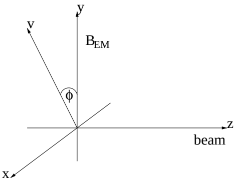
$$L = \frac{1}{2}\overline{\psi}\left[iD_0 - \frac{c_1}{2M}\mathbf{D}^2 + \frac{c_2 g}{4M}\sigma \cdot \mathbf{B}\right]\psi + \cdots$$

with $c_1 = 1 + \mathcal{O}(\alpha_s)$ and $c_2 = 1 + \mathcal{O}(\alpha_s)$, where **B** is the chromomagnetic field and σ its spin tensor in the RF of the HQ.

Weibel Instability

Large **B** at initial times: leads to HQ spin alignment in reaction plane. However, the direction of **B** fluctuates from event to event, so this spin alignment is washed out. Unfortunately no direct evidence for instability.

Heavy quark polarization



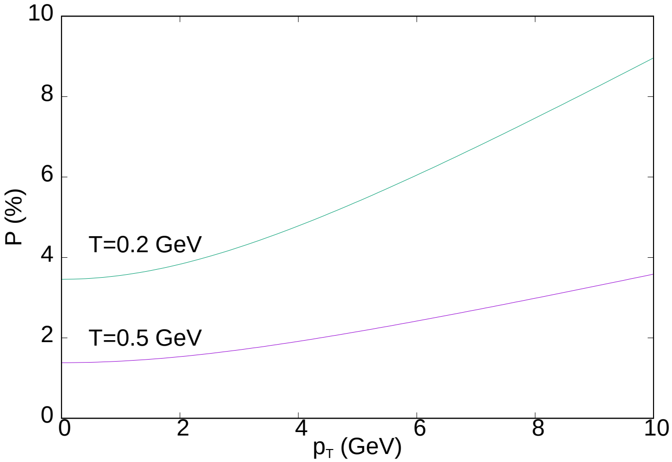
Same construction with EM fields too. In this case $g \to e$ and $\mathbf{B} \to \mathbf{B}_{EM}$. $e\mathbf{B}_{EM} = \zeta T_{co}^2$ with $\zeta = \mathcal{O}(1)$ always out of the reaction plane; $|\mathbf{E}_{EM}| \simeq |\mathbf{B}_{EM}|$. May give observable spin alignment. Boost to RF of HQ first.

When $v = 0$ in the CM of the fireball, then the spin polarization

$$P = \frac{\mathrm{Tr}(\sigma_y \rho)/2}{\mathrm{Tr}\rho} = \frac{c_2 q \zeta T_{co}^2}{8MT},$$

with $q = -1/3$ and $c_2 q \zeta \simeq 1$. So $P \approx 1.7\%$ for $T = 500$ MeV and 4.3% for $T = 200$ MeV. For arbitrary $v = \beta\gamma$, boosting to the RF of HQ and averaging over directions gives $P \simeq (T_{co}^2 m_T)(8M^2 T)$. We have assumed that the HQ is at $y = 0$ so its momentum is p_T and $m_T = \sqrt{M^2 + p_T^2}$.

Initial bottom polarization at LHC



Other quarks

$\sigma \cdot \mathbf{B}$ terms can be guessed for **light quarks** as well. However (1) the coupling is unknown (2) the problem cannot be reduced to a one-particle Hilbert space and (3) the spin of light degrees of freedom changes during thermal evolution of the fireball. This prevents us from making a QCD computation of the polarization of light quarks.

For the polarization of **charm quarks** one could take smaller values of Λ and use HQE$\mathbb{T}$ for charm starting at a later time with $T = 200$ MeV. But this gives a spin polarization only if a strong magnetic field persists until this time.

Does later dynamics change the **bottom quark** spin polarization? HQ-HTL and lattice computations show that gluons may be able to thermalize low-p_T bottom quarks, so it is possible that the magnetic field effect is wiped out by multiple soft interactions.

Heavy Quark Sono-Dynamics

Interactions at the hydrodynamic scale

At momentum scales smaller than g^2T colour is screened, but flavour is not. **Flavour currents are hydrodynamic modes**. Can the heavy and light flavour currents interact? Since the momentum exchange is below g^2T, screening implies that there are no excitations which can couple heavy and light flavours.

However, there is a remnant mode which can couple to HQ at the hydro scale. The breaking of a part of the conformal invariance of the fluid gives rise to a **Goldstone boson** which we know as sound. Since it is a longitudinal pressure wave it has no polarization and is described by a scalar field ϕ.

This EFT cannot be derived from QCD, but we can use symmetry and dimensional arguments, and the fact that ϕ is a Goldstone field, to write the Lagrangian. Its form is valid in all phases, but the couplings depend on the phase.

Kinetic terms and equations of motion

Up to mass dimension 4 we have only mass and kinetic terms. Since ϕ is a Goldstone field, the terms $\phi\overline{\psi}\psi$ is forbidden. We introduce the velocity 4-vector of the fluid u, and the HQ velocity v to write

$$L_4 = \overbrace{\Delta M\overline{\psi}\psi + \frac{1}{2}\overline{\psi}(iD_v)\psi - \frac{c_1}{2M}\overline{\psi}\nabla_v^2\psi}^{heavy\ \ quark} + \overbrace{\frac{1}{2}(D_0\phi)^2 - \frac{c_s^2}{2}\nabla\phi\cdot\nabla\phi}^{sound}$$

The notation is $D_v = v\cdot\partial$ and $\nabla_v = \partial - vD_v$, with $D_0 = u\cdot\partial$ and $\nabla = \partial - uD_0$.

The EoM of sound is $(D_0^2 - c_s^2\nabla^2)\phi = 0$. For bounded ϕ this gives plane waves. However, there is a vector $\mathbf{u}$ in the problem and an axial vector $\mathbf{w} = \nabla\times\mathbf{u}$ of vorticity. So we can look for solutions with $\nabla\phi = \mathcal{M}^2\mathbf{u}$, with the hydrodynamic scale $\mathcal{M}$ of the order of the expansion rate $\nabla\cdot\mathbf{u} \simeq 0.02$ GeV. This gives a solution with $D_0^2\phi = \mathcal{M}^2 c_s^2\nabla\cdot\mathbf{u}$.

Interaction terms: spin-vorticity coupling

Dimension 5 terms are $v \cdot \partial\phi\, \overline{\psi}\psi$ and $\overline{\psi}(v \cdot \sigma \cdot \partial\phi)\psi$. The second is ruled out by CPT, and the first is clearly a contribution to the renormalized mass of the HQ.

There are several dimension 6 terms. One turns out to be a contribution to the renormalized mass of the HQ, and others are corrections to the kinetic terms. The interaction term is

$$L_6 = \frac{c_6}{M}\epsilon^{\mu\nu\lambda\rho}\overline{\psi}\sigma_{\mu\nu}\psi\partial_\lambda\partial_\rho\phi = c_6\frac{\mathcal{M}^2}{M}(v \cdot u)^2\,\overline{\psi}(\sigma \cdot \mathbf{w})\psi.$$

Writing $\sigma \cdot \mathbf{w} = \mathbf{s} \cdot \mathbf{J}$, and taking the angular momentum of the fireball in mid-central collisions to be about $\mathbf{J} \simeq 100$, for $c_6 \approx 1$ and $p_T \simeq 0$, the alignment energy turns out to be about 10 MeV, much smaller than $T \approx 150$ MeV. But due to the factor $\gamma^2 = (v \cdot u)^2$, the alignment energy rapidly exceeds T and leads to complete polarization.

Viscous effects are negligible

If the viscosity is large enough then sound is attenuated. This is a mass term for ϕ, *i.e.*, converts this Goldstone boson to a pseudo-Goldstone boson. The mass is the inverse of the Stokes' attenuation length, *i.e.*, $m_{\rm St} \simeq 1/\ell_{\rm St}$.

Pseudo-Goldstone bosons can interact with terms that are multilinear in the fields. The corresponding couplings are proportional to positive powers of $m_{\rm St}$. The presence of such terms would complicate HQSD.

The Goldstone mass is

$$m_{\rm St} = \frac{2\eta\omega^2}{3\epsilon c_s^3} \simeq \frac{\eta}{S} \times \frac{\mathcal{M}^2}{T} \ll \mathcal{M}$$

where we have assumed $\epsilon \simeq ST$ and $\omega \simeq \mathcal{M}$. So the Goldstone mass would have become important only at times much larger than the actual fireball lifetime.

Experimental outlook

B and D meson polarization

Since b is copiously produced at the LHC, one can study its polarization through the angular distribution of the decay of B^* mesons. However, one can equally well study it in the angular distribution of non-prompt D^* decays.

Due to HQ spin symmetry, the polarization of b quarks is transferred to the decay c quark. Consider the decay $\overline{B}^0 \to D^{*+} e \overline{\nu}_e$. Fully polarized b quarks in $\overline{B}^0$ will lead to fully polarized c quark in the D^{*+}. The polarization of the D^{*+} can be easily computed from particle masses and the Isgur-Wise function. Perfect fit for physics program of **LHCb**.

When either of these measurements is performed it would be possible to fix the low-energy coupling c_6 in HQSD. Consistency check by doing both B^* and D^* polarizations.