Kubo and Kinetic expression of transport coefficients at finite magnetic field

Sabyasachi Ghosh (Physics Dept, IIT Bhilai)

PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

One-loop Kubo estimations of the shear and bulk viscous coefficients for hot and magnetized bosonic and fermionic systems

Snigdha Ghosh and Sabyasachi Ghosh
Phys. Rev. D **103**, 096015 – Published 18 May 2021

Kubo estimation of the electrical conductivity for a hot relativistic fluid in the presence of a magnetic field

Sarthak Satapathy, Snigdha Ghosh, and Sabyasachi Ghosh
Phys. Rev. D **104**, 056030 – Published 28 September 2021

Quantum field theoretical structure of electrical conductivity of cold and dense fermionic matter in the presence of a magnetic field

Sarthak Satapathy, Snigdha Ghosh, and Sabyasachi Ghosh
Phys. Rev. D **106**, 036006 – Published 9 August 2022

Time line of Magnetic field in QGP topic

2004 — SQGP →

2009 — Strong B →

2012 — IMC →

2017 — Vorticity →

2022 — CME test

Estimate of the magnetic field strength in heavy-ion collisions

V. Skokov (Darmstadt, GSI and Frankfurt U., FIAS and Dubna, JINR), A.Yu. Illarionov (Trento U.), V. Toneev (Darmstadt, GSI and Dubna, JINR)

Jul, 2009

8 pages

Published in: *Int.J.Mod.Phys.A* 24 (2009) 5925-5932

QCD quark condensate in external magnetic fields

G.S. Bali (Regensburg U. and Tata Inst.), F. Bruckmann (Regensburg U.), G. Endrodi (Regensburg U.), Z. Fodor (Wuppertal U. and Eotvos U. and Julich, Forschungszentrum), S.D. Katz (Julich, Forschungszentrum) et al.

Phys.Rev.D 86 (2012) 071502 • e-Print: 1206.4205 • *DOI:* 10.1103/PhysRevD.86.071502

STAR Collaboration (Sep 7, 2022)

e-Print: 2209.03467 [nucl-ex]

NJL model at finite B

QCD Thermodynamics at finite B

Transport Coefficients at finite B

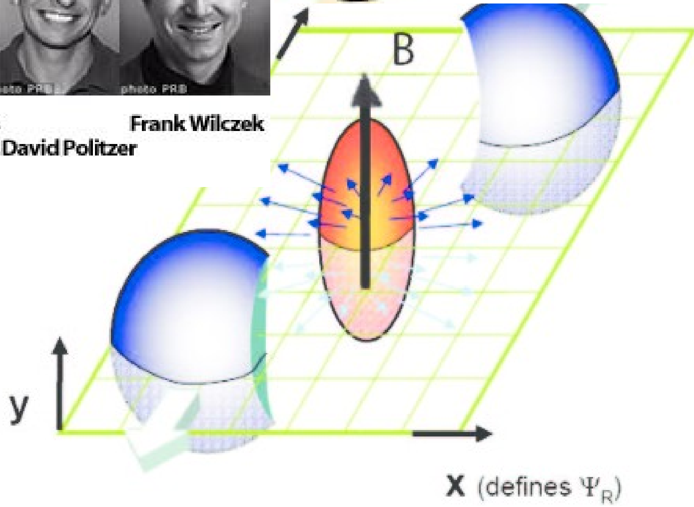
.....& more Institutes

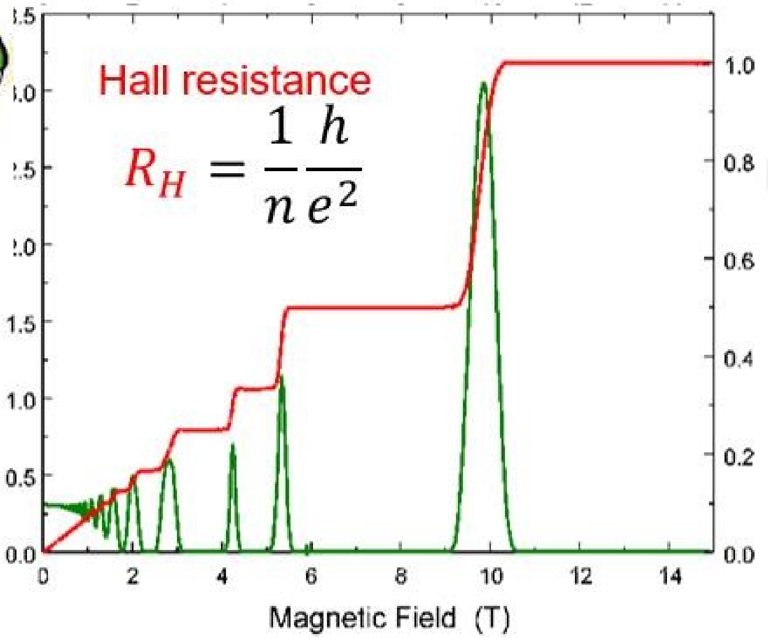
Our Query: Quantum Hall effect in Nuclear Matter?

The Nobel Prize in Physics **1985**

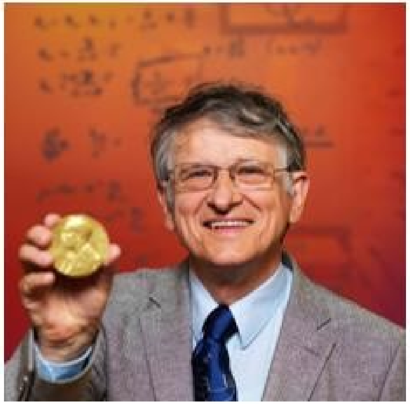
2004 Nobel Prize in Physics

David J. Gross H. David Politzer Frank Wilczek





Quantum Hall effect

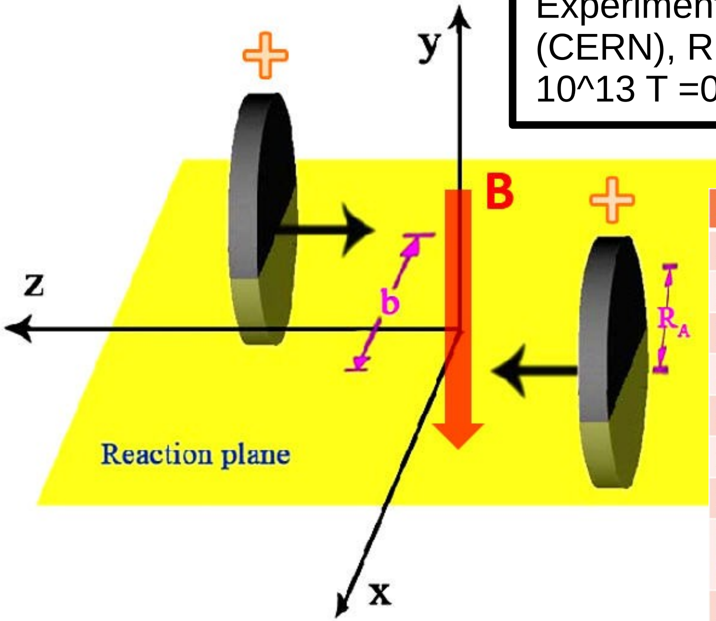


Klaus von Klitzing "for the discovery of the quantized Hall effect"

von Klitzing, et al., PRL 45, 494 (1980);

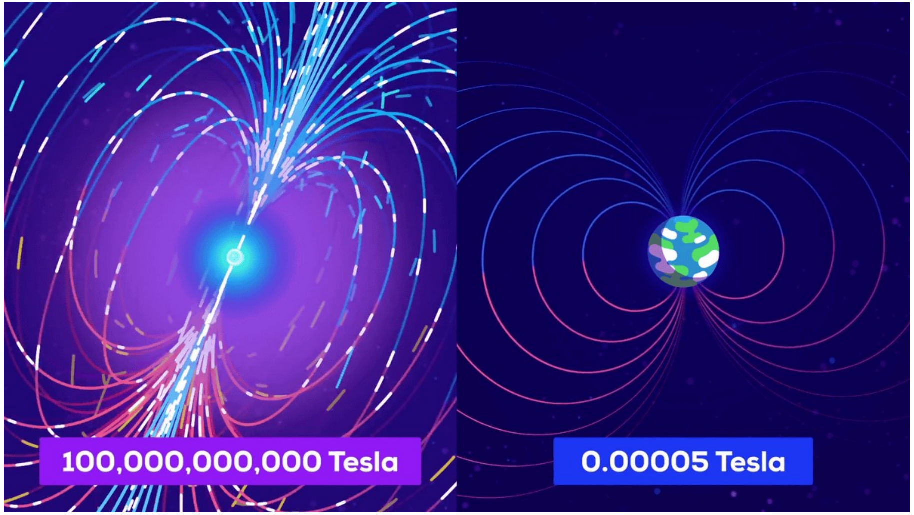
Magnetic Field:

Heavy Ion Collision Experiments : LHC (CERN), RHIC (BNL) : 10^13 T =0.140^2 GeV^2



Neutron Star

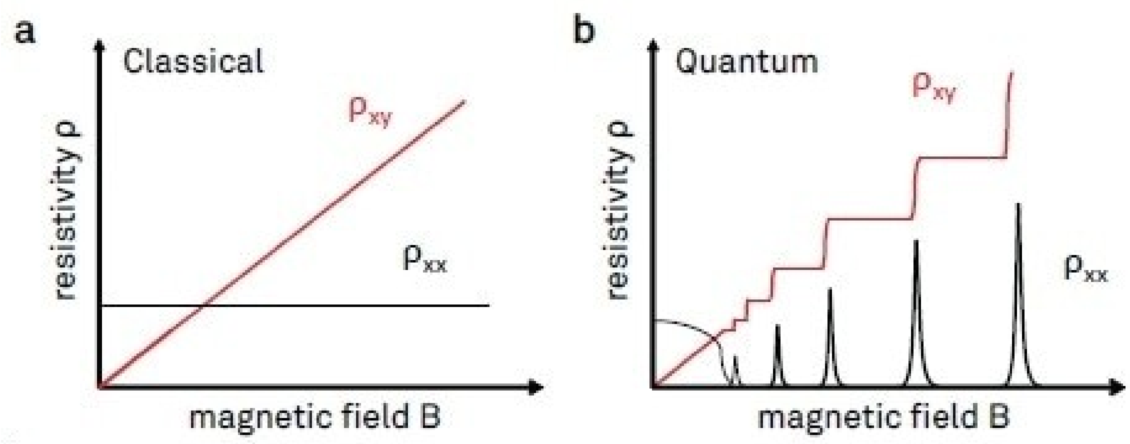
Earth

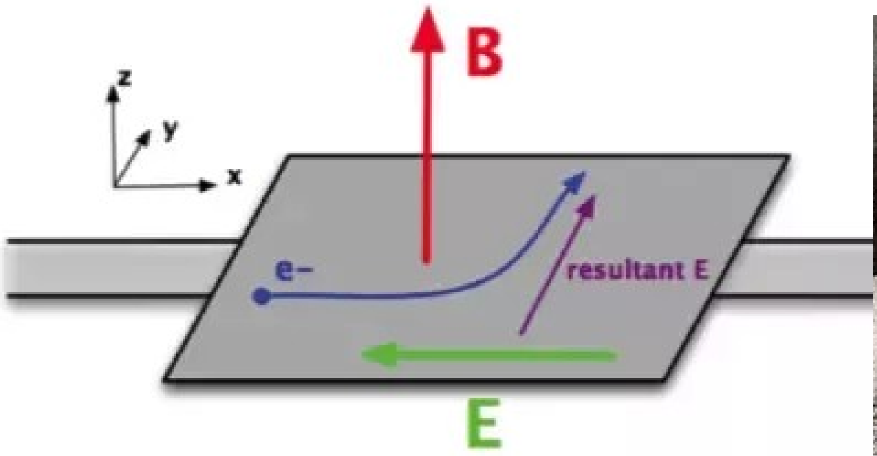


Source	Approximate Magnetic Field
Neuron depolarization (imaged by MEG)	0.5 pT (5 x 10^{-13} T)
Earth's magnetic field	0.5 G (50 μT)
Refrigerator magnet	50 G (5 mT)
Junkyard electromagnet	1 T
Clinical MRI scanners	0.5 - 3.0 T (typical)
Research MRI scanners (human)	7.0 T – 11.7 T
Laboratory NMR spectrometers	6 - 23 T
Largest pulsed field created in lab nondestructively	97 T
Largest pulsed field created in lab (destroying equipment but not the lab)	730 T

Classical and Quantum Hall effect:

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}}$$





(Discovered by Edwin Hall in 1879)

(Discovered by Klaus von Klitzing in 1980)

Cornell University

We gratefully acknowledge support from

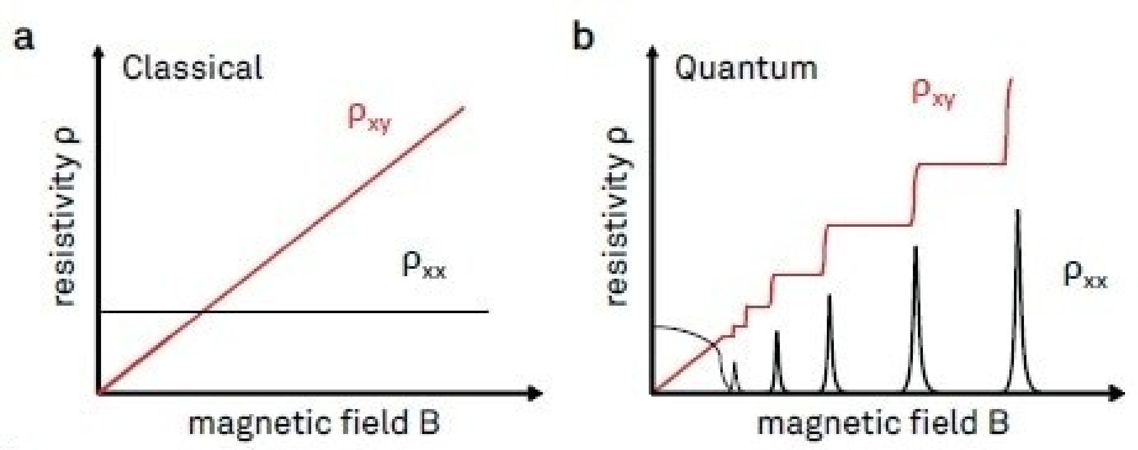
arXiv.org > hep-th > arXiv:1606.06687

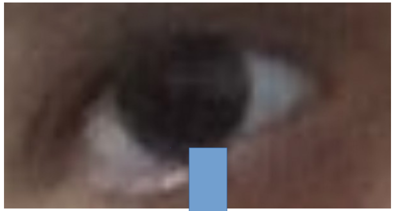
High Energy Physics - Theory

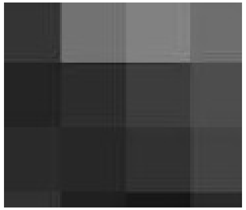
Lectures on the Quantum Hall Effect

David Tong

(Submitted on 21 Jun 2016 (v1), last revised 20 Sep 2016 (this version, v2))







Acknowledgements

These lectures were given in TIFR, Mumbai. I'm grateful to the students, postdocs, faculty and director for their excellent questions and comments which helped me a lot in understanding what I was saying.

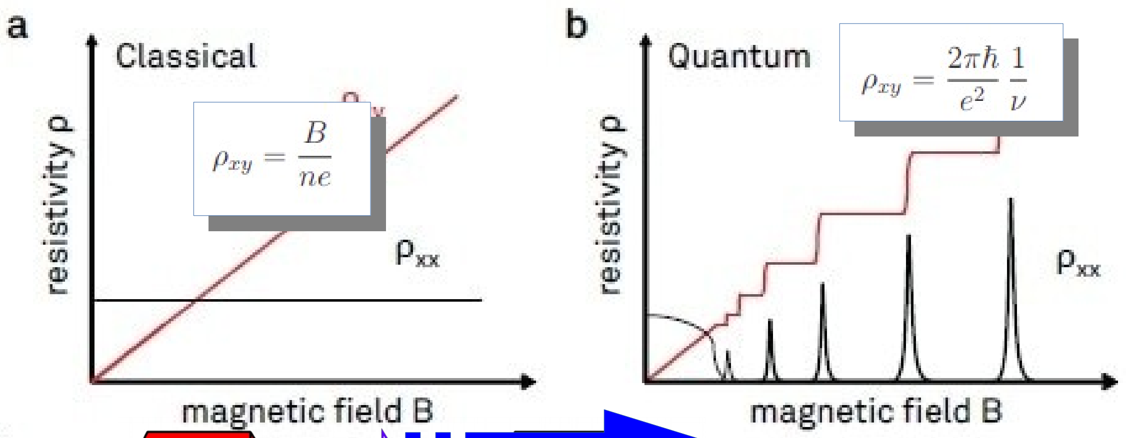
Resistivity Matrix:

The *resistivity* is defined as the inverse of the conduc
both are matrices,

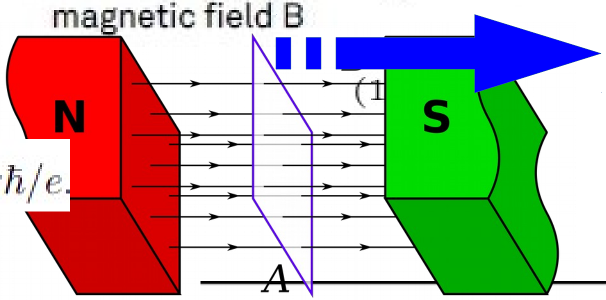
$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$$

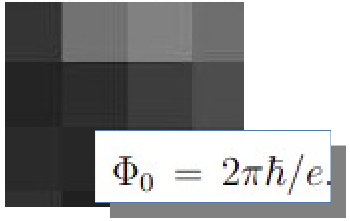
From the Drude model, we have

$$\rho = \frac{1}{\sigma_{DC}} \begin{pmatrix} 1 & \omega_B \tau \\ -\omega_B \tau & 1 \end{pmatrix}$$



$\Phi_0 = 2\pi\hbar/e.$



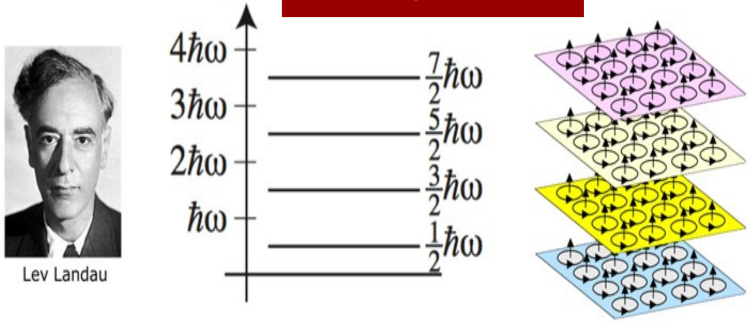


$\Phi_0 = 2\pi\hbar/e.$

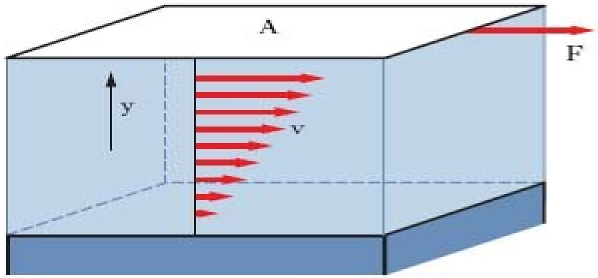
Electrical Conductivity Matrix:

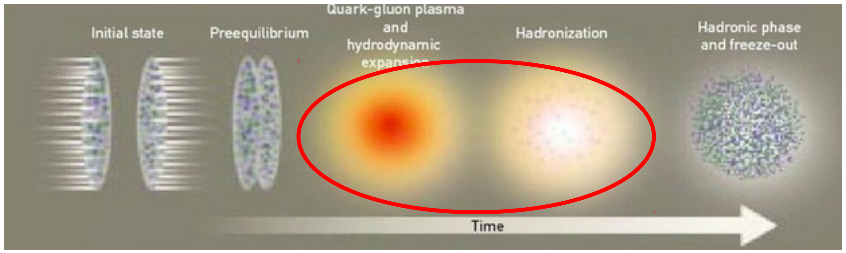
$$\sigma = \frac{\sigma_{DC}}{1 + \omega_B^2 \tau^2} \begin{pmatrix} 1 & -\omega_B \tau \\ \omega_B \tau & 1 \end{pmatrix} \quad \text{with} \quad \sigma_{DC} = \frac{ne^2\tau}{m}$$

Landau Quantization



Kinetic Theory (Relaxation Time Approximation) for B=0





$$\delta f = (A_{ij}U_\eta^{ij} + C_i E^i) f (1 \pm f) \ ,$$

Macro / **Micro**

$$\eta U_\eta^{ij} = T^{ij} = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} \delta f$$

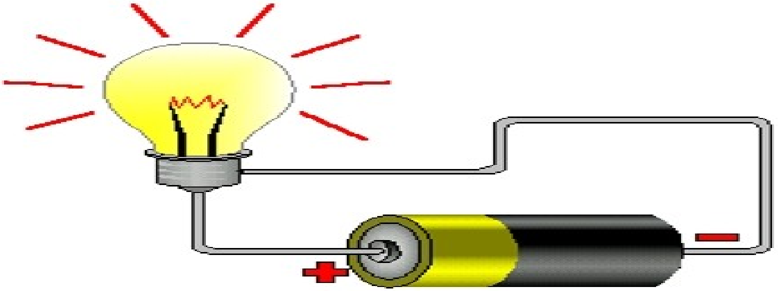
$$\sigma^{ij} E_j = J^i = g_e e \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E} \delta f \ ,$$

$$\frac{p^\mu}{E} \partial_\mu f^\pm + F^\mu \frac{\partial f^\pm}{\partial p^\mu} = -\left(\frac{p^\mu u_\mu}{E}\right) \frac{\delta f^\pm}{\tau_c}$$

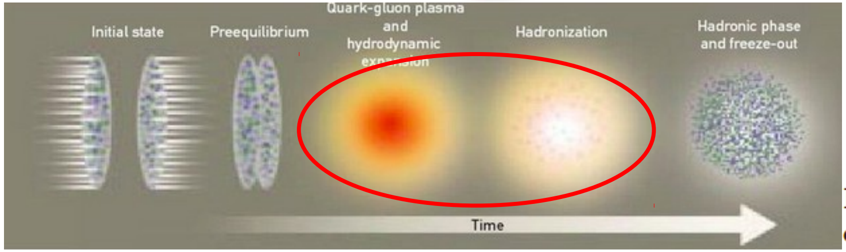
Relativistic Boltzmann Equation

$$\eta_{g,Q} = \frac{g_{g,Q}}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}^2}{\omega}\right)^2 \tau f (1 \pm f)$$

$$\sigma_Q = \frac{e_Q^2 g_e}{3T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}}{\omega}\right)^2 \tau f (1 - f) \ .$$



Kinetic Theory (Relaxation Time Approximation) for finite B



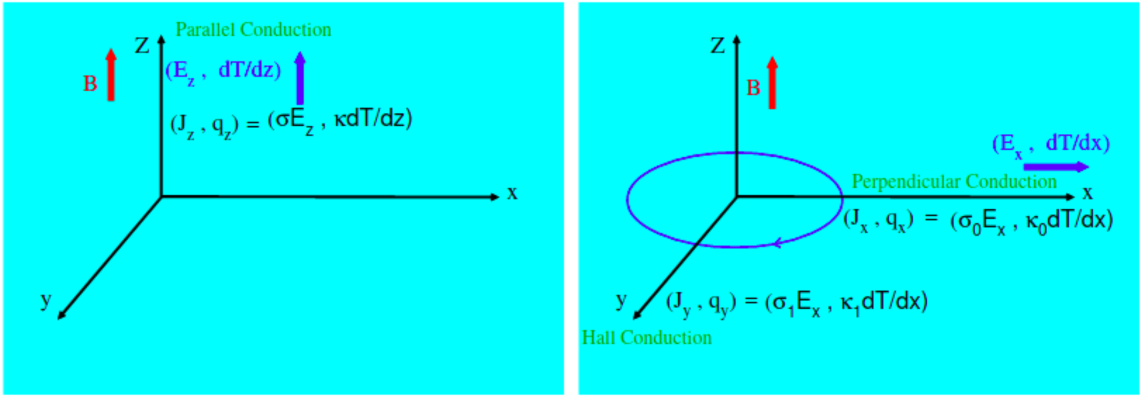


Fig. 5 Schematic diagram of parallel (Left), perpendicular and Hall (Right) components of electrical and thermal conductivity.

Macro | **Micro**

$$\sigma^{ij}E_j = J^i = g_e e\int\frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\delta f\,,$$

$$\sigma_Q = \frac{e_Q^2 g_e}{3T}\int\frac{d^3\vec{p}}{(2\pi)^3}\left(\frac{\vec{p}}{\omega}\right)^2\tau f(1-f)\,.$$

τ_c

$\tau_c/[1+(\tau_c/\tau_B)^2]$

$\tau_c(\tau_c/\tau_B)/[1+(\tau_c/\tau_B)^2]$

Relativistic Boltzmann Equation

$$\frac{p^i}{E}\partial_i f_0^\pm + e_{Q,\bar{Q}}\mathcal{E}^i\frac{\partial f_0^\pm}{\partial p^i} + e_{Q,\bar{Q}}Bb^{ij}v_j\frac{\partial}{\partial p^i}\left(\delta f^\pm\right) = -\frac{\delta f^\pm}{\tau_c}$$

Green-Kubo Relation of Transport coefficients

R. Kubo, J. Phys. Soc. Jpn. 12, 570 (1957).

Energy-momentum tensor & conserved current

Operators

$$\pi^{ij} \equiv T^{ij} - g^{ij}T^k_k/3 ,$$

$$\mathcal{P} \equiv -T^k_k/3 - c_s^2 T^{00} \text{ (for vanishing chemicalpotential } \mu = 0),$$

$$\mathcal{K}^i \equiv T^{0i} - hN^i ,$$

Static Limit

Dynamical structure

Transport Coefficients

Thermal Corelators

Shear Viscosity

$$\eta = \frac{1}{20} \lim_{q_0,\vec{q}\to 0} \frac{A_\eta(q_0,\vec{q})}{q_0} , \quad A_\eta = \int d^4x e^{iq\cdot x} \langle [\pi^{ij}(x), \pi_{ij}(0)] \rangle_\beta ;$$

Bulk Viscosity

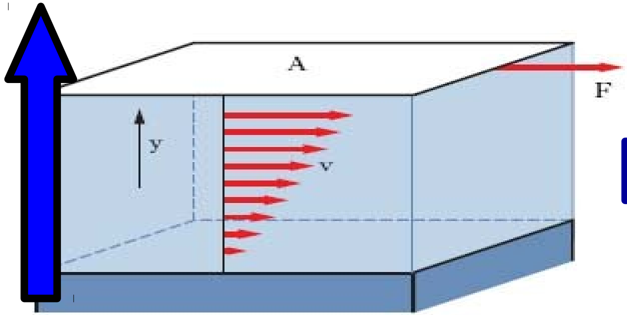
$$\zeta = \frac{1}{2} \lim_{q_0,\vec{q}\to 0} \frac{A_\zeta(q_0,\vec{q})}{q_0} , \quad A_\zeta = \int d^4x e^{iq\cdot x} \langle [\mathcal{P}(x), \mathcal{P}(0)] \rangle_\beta ;$$

Thermal Conductivity

$$\kappa = \frac{\beta}{6} \lim_{q_0,\vec{q}\to 0} \frac{A_\kappa(q_0,\vec{q})}{q_0} , \quad A_\kappa = \int d^4x e^{iq\cdot x} \langle [\mathcal{K}^i(x), \mathcal{K}_i(0)] \rangle_\beta$$

Heat Energy Transport

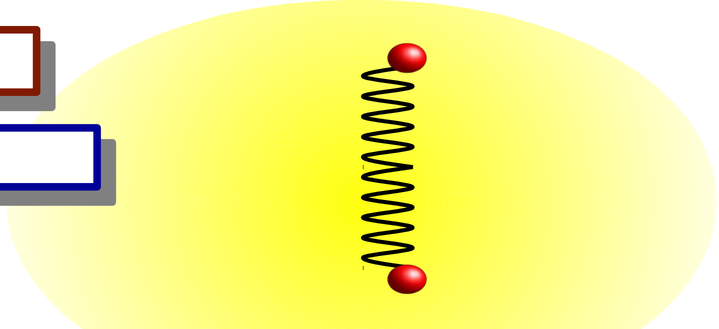
Momentum Transport



Velocity gradient

Temperature gradient

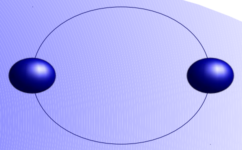
Classical Picture



Picture of QFT at finte temperature

Kubo Formulism: QFT ->TFT -> TFT for finite B

Fermion Self-energy in Medium



$$\langle \mathcal{T}_C J^\mu(x) J^\nu(0) \rangle^B_{11}$$

QFT

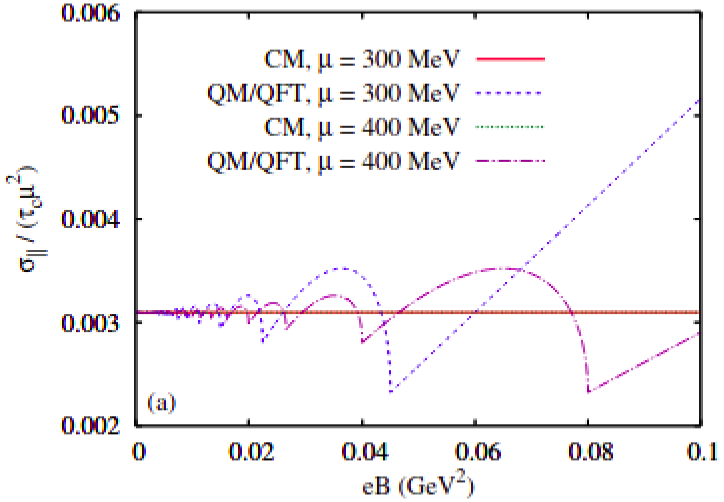
$$\sigma_\parallel = e^2 \left(\frac{eB}{2\pi^2}\right) \frac{1}{\Gamma\mu} \sum_{l=0}^{l_{\rm max}} (2-\delta_l^0) \sqrt{\mu^2 - m_l^2}\, \Theta(\mu - m_l),$$

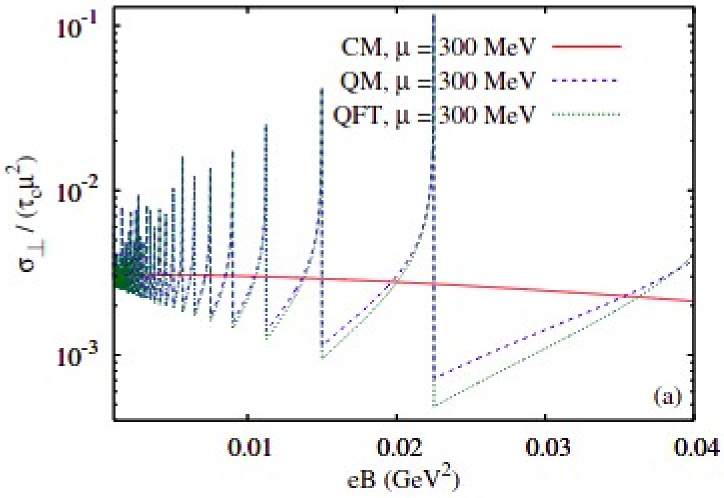
$$\sigma_\perp = e^2 \left(\frac{eB}{2\pi^2}\right) \frac{\Gamma}{\Gamma^2 + (\mu - \sqrt{\mu^2 - 2eB})^2} \frac{1}{\sqrt{\mu^2 - 2eB}} \sum_{l=1}^{l_{\rm max}} \frac{(2l-1)eB}{\sqrt{\mu^2 - m_l^2}} \Theta(\mu - m_l),$$

$$l_{\rm max} = \left\lfloor \frac{\mu^2 - m^2}{2eB} \right\rfloor$$

CM

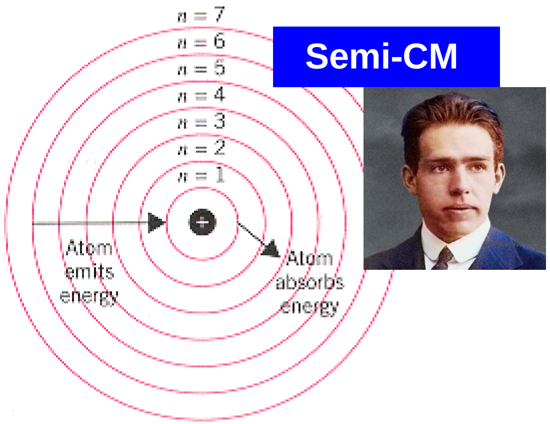
$$\sigma^{\rm RTA}_{\parallel,\perp} = 2e^2 \int \frac{d^3k}{(2\pi)^3} \tau_c^{\parallel,\perp} \frac{\vec{k}^2}{3\omega_k^2} \delta(\omega_k - \mu) = \frac{e^2}{3\pi^2} \frac{(\mu^2 - m^2)^{3/2}}{\mu} \tau_c^{\parallel,\perp}$$



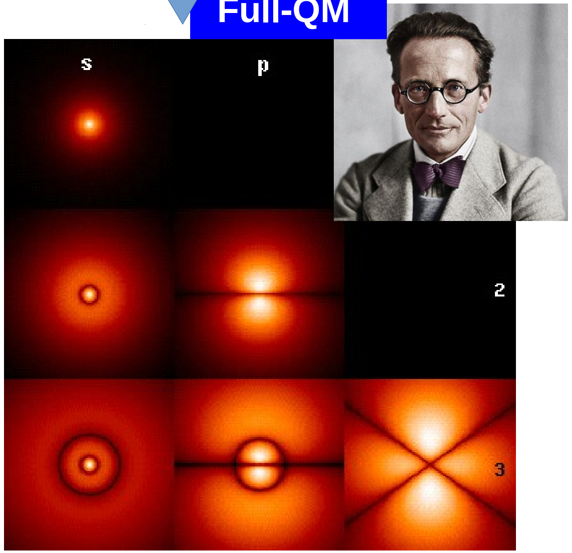


Concluding Remarks and future plans

Semi-CM



Full-QM



Cyclotron time in QFT

$$\tilde{\tau}_c^{\perp} = \frac{\Gamma}{\Gamma^2 + \left(\mu - \sqrt{\mu^2 - 2eB}\right)^2} = \tau_c \Big/ \left(1 + \frac{\tau_c^2}{\tilde{\tau}_B^2}\right),$$

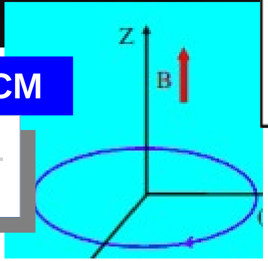
where,

$$\frac{1}{\tilde{\tau}_B} = \left(\mu - \sqrt{\mu^2 - 2eB}\right) = \frac{eB}{\mu}\left\{1 + \frac{eB}{2\mu^2} + \frac{(eB)^2}{2\mu^4} + \frac{5(eB)^3}{8\mu^6} + \cdots\right\} = \frac{1}{\tau_B}\left\{1 + \frac{1}{2\mu\tau_B} + \frac{1}{2(\mu\tau_B)^2} + \frac{5}{8(\mu\tau_B)^3} + \cdots\right\}$$

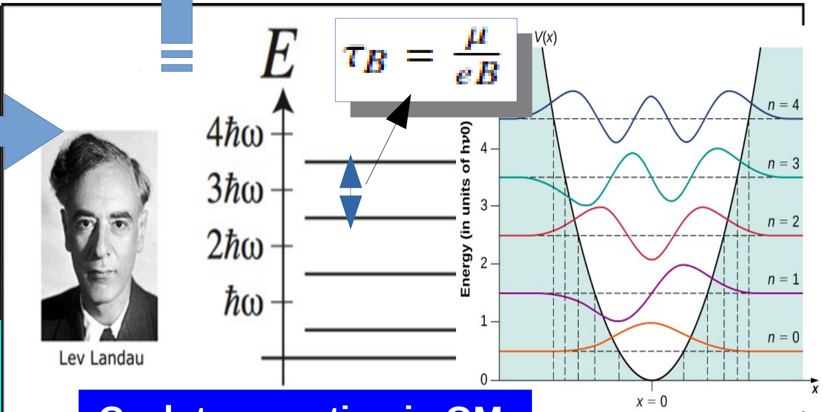
SHM in CM

Cyclotron motion in CM

$$\tau_B = \frac{\mu}{eB}$$



Cyclotron motion in QM



Future plans: impact of this QFT version of cyclotron motion in EM probes