Perfect fluid dynamics with spin

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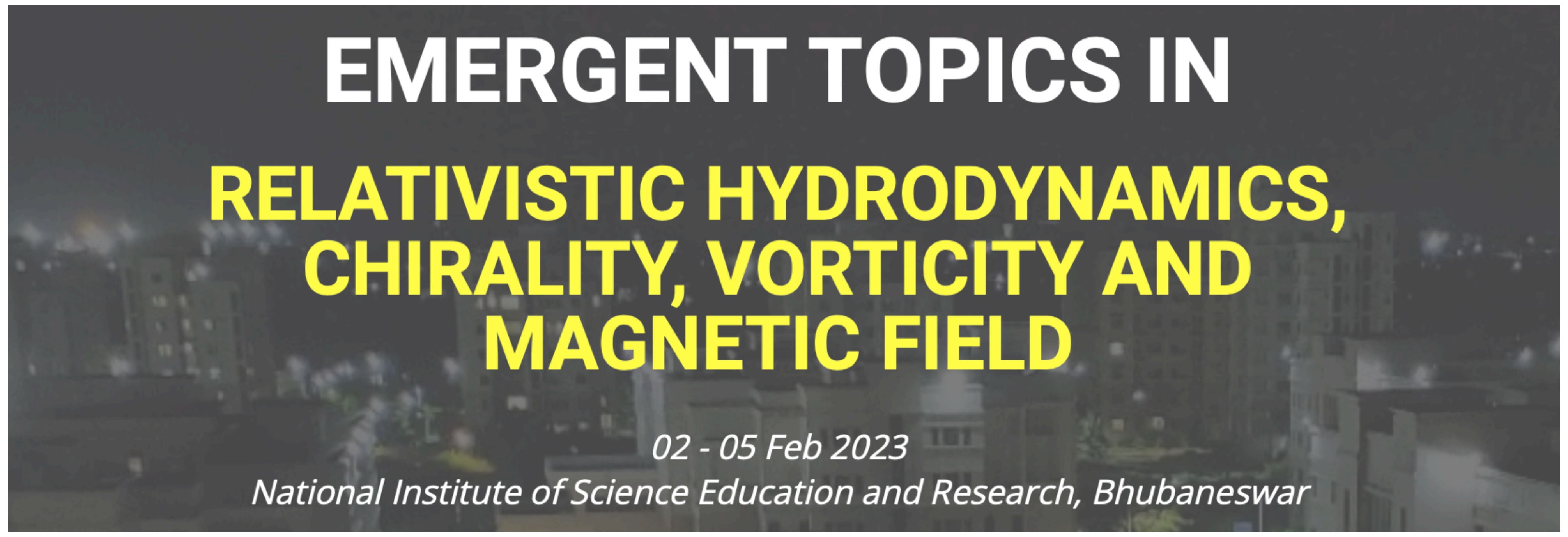

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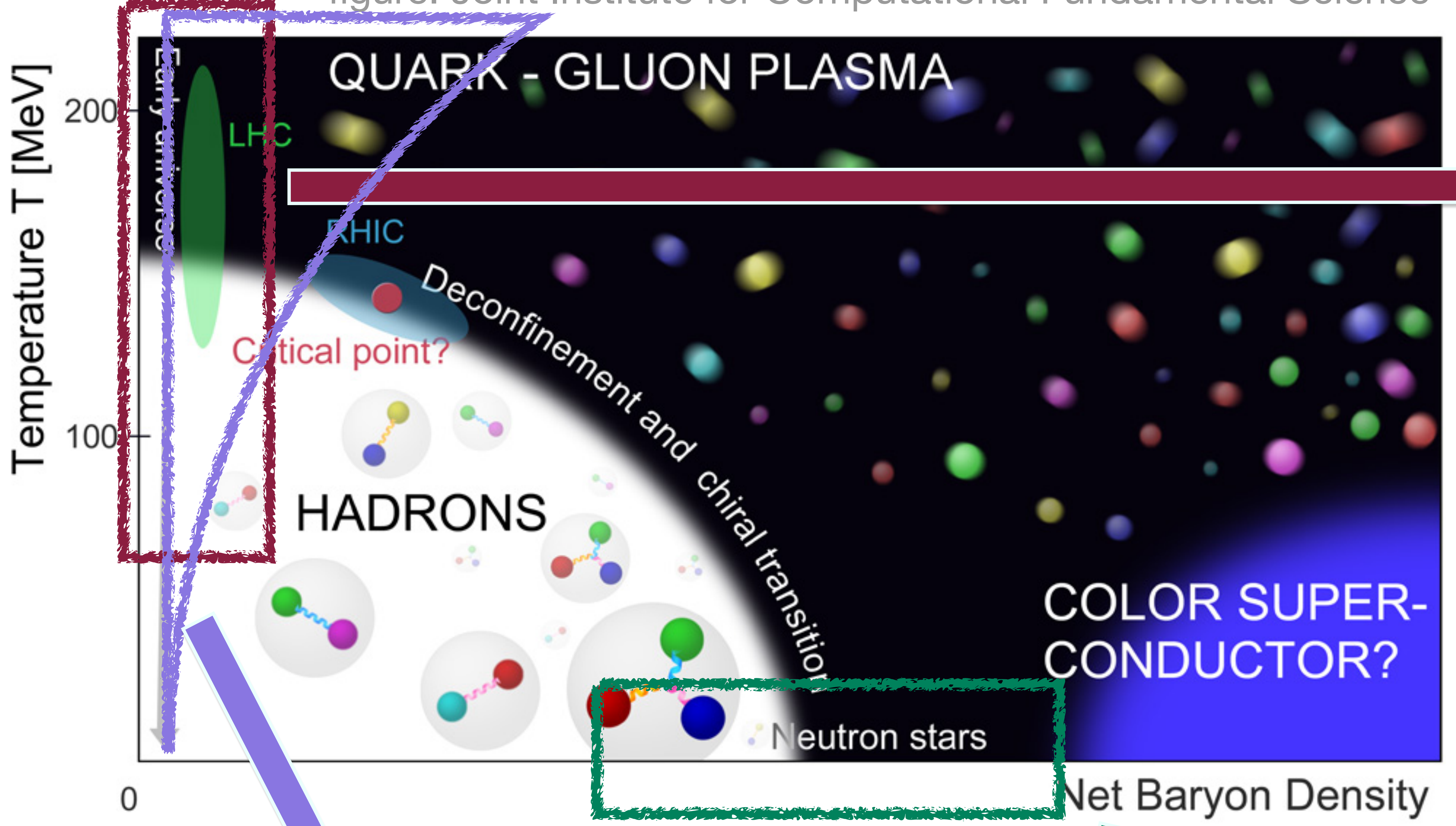
POLISH ACADEMY OF SCIENCES

STUDYING THE PHASE DIAGRAM OF QCD

figure: Joint Institute for Computational Fundamental Science



Heavy-ion collision physics

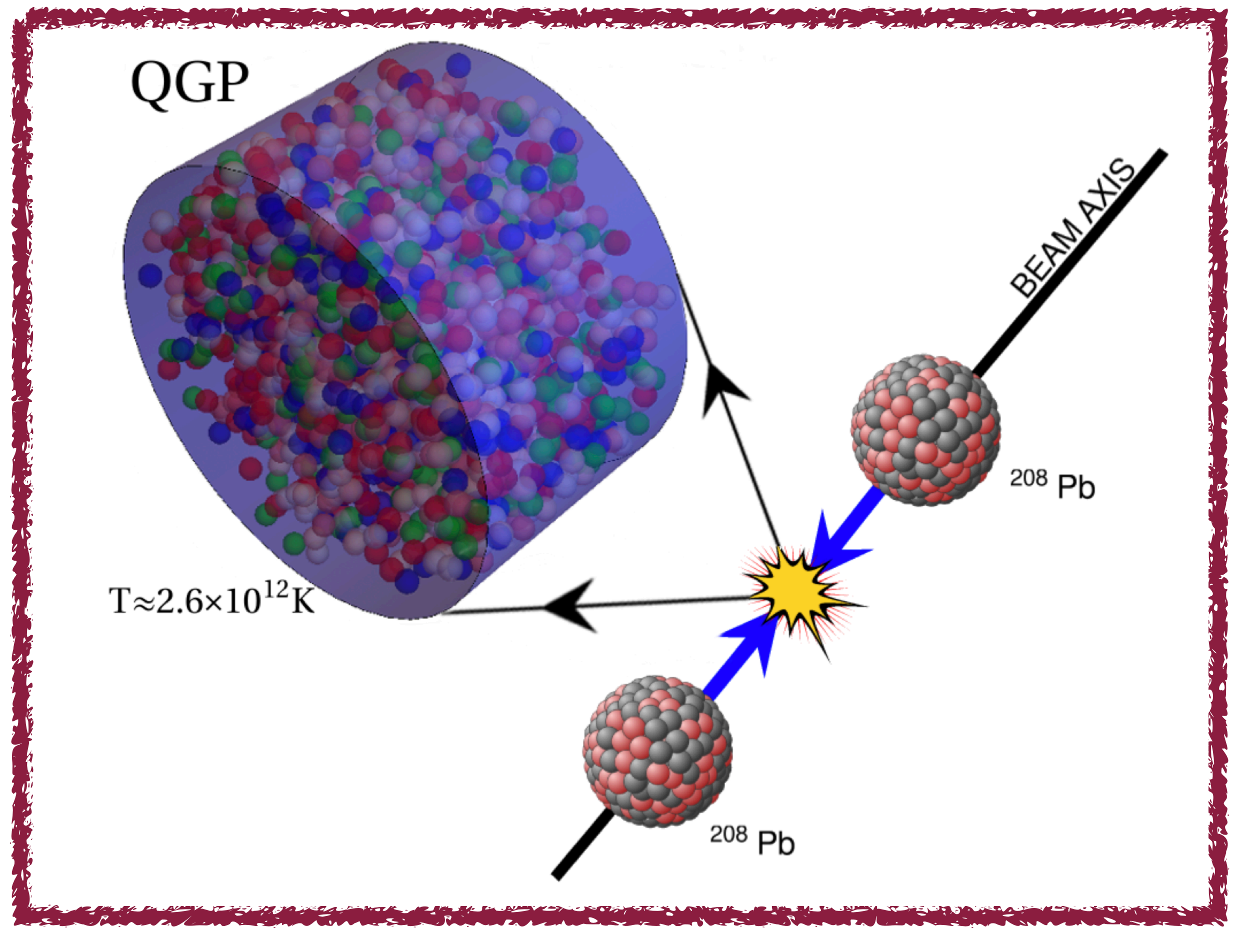


figure: Nature Physics 16, 615–619(2020)

Neutron star physics

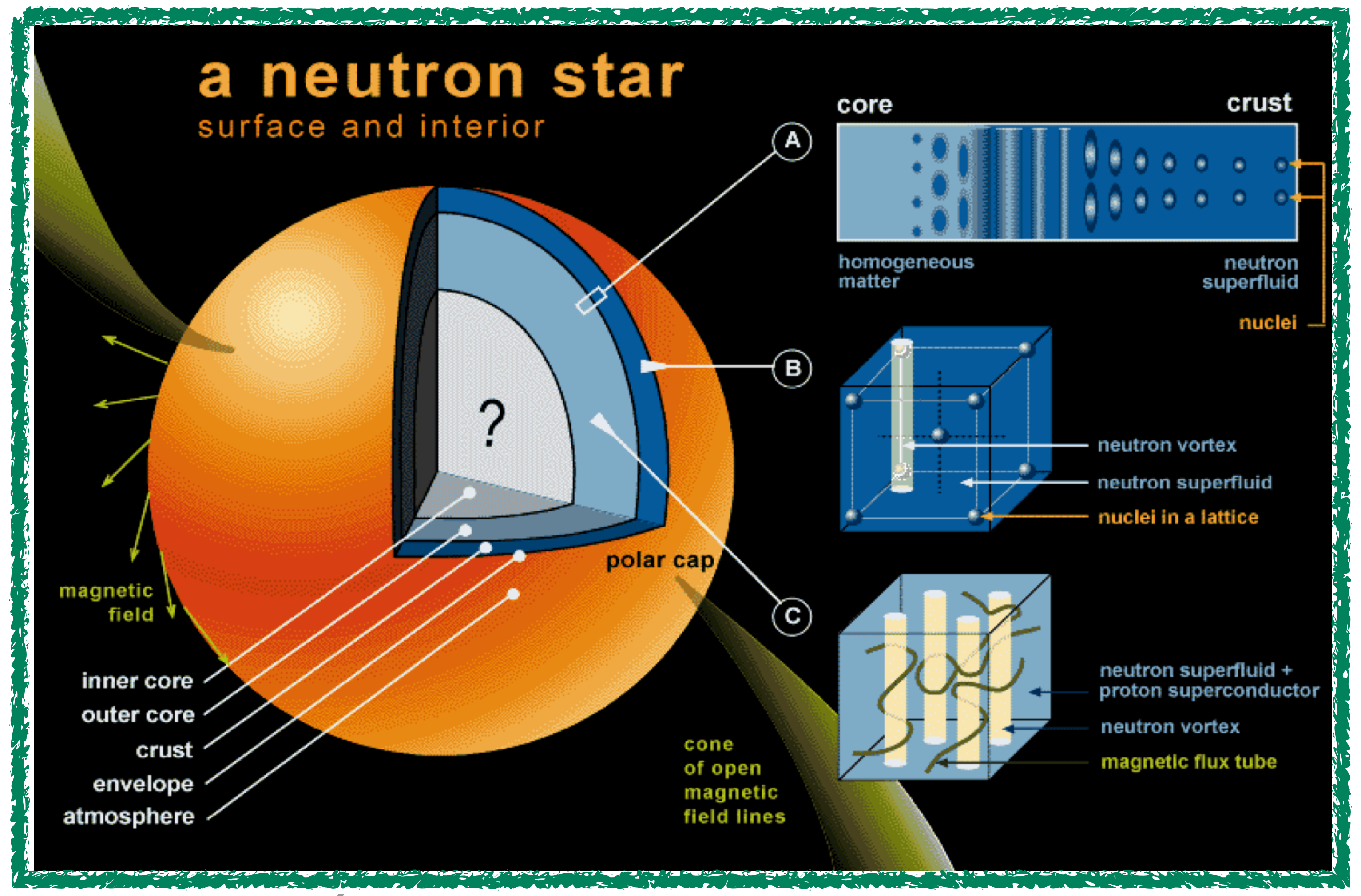


figure: D.E. Á. Castillo, talk @RagTime 22

lattice-QCD simulations

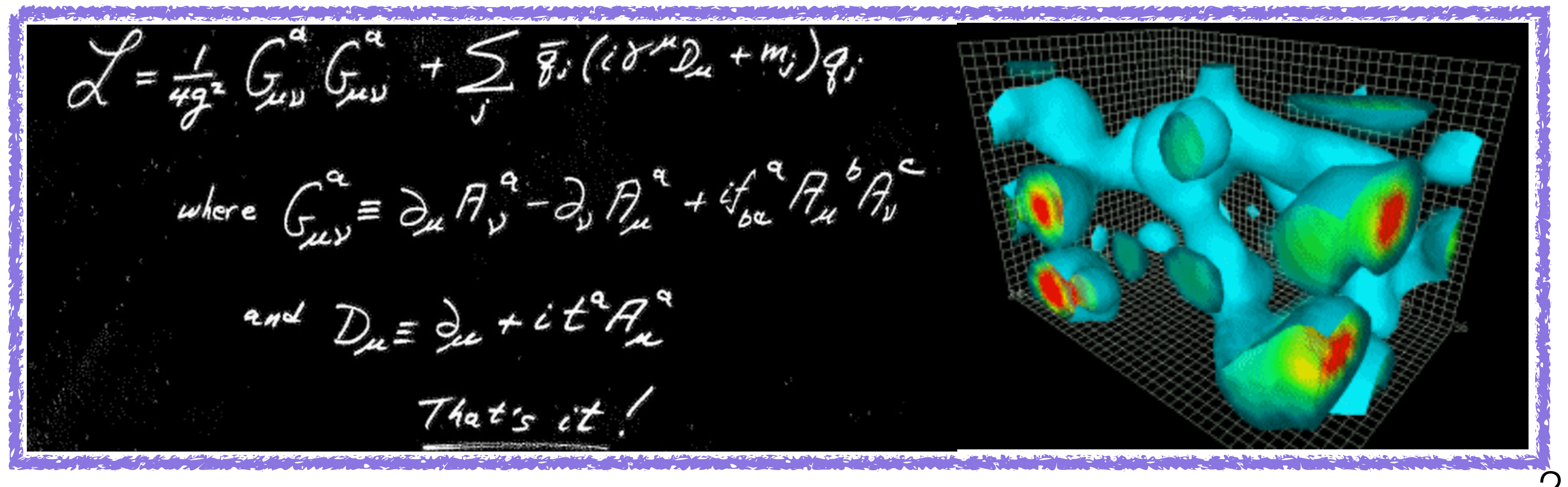


figure: D. Leinweber (www.physics.adelaide.edu.au)

Collectivity of QGP / Standard probes

figure: T. Hirano, N. van der Kolk, A.Bilandzic, LNP 785 (2010) 139-178

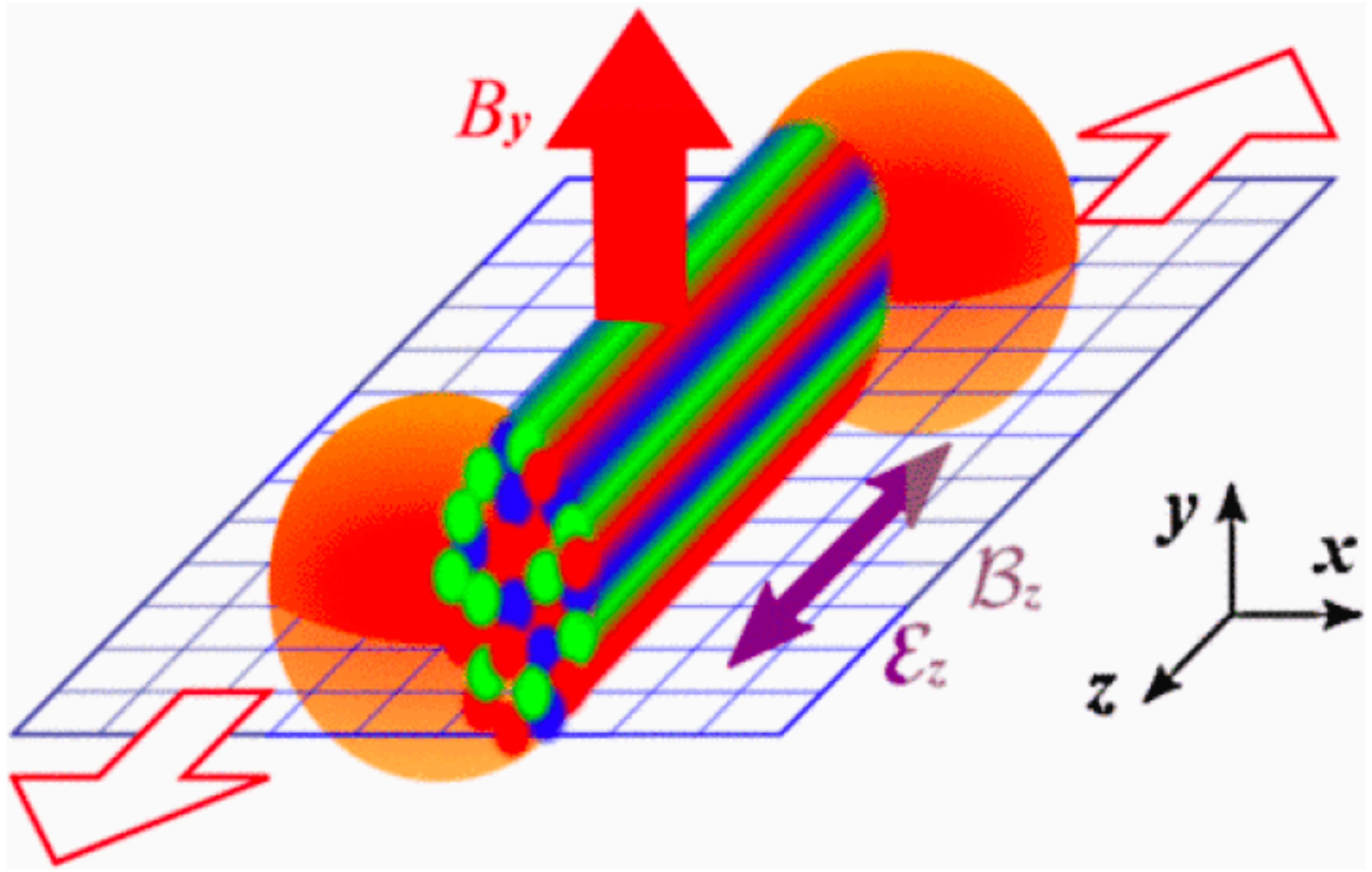
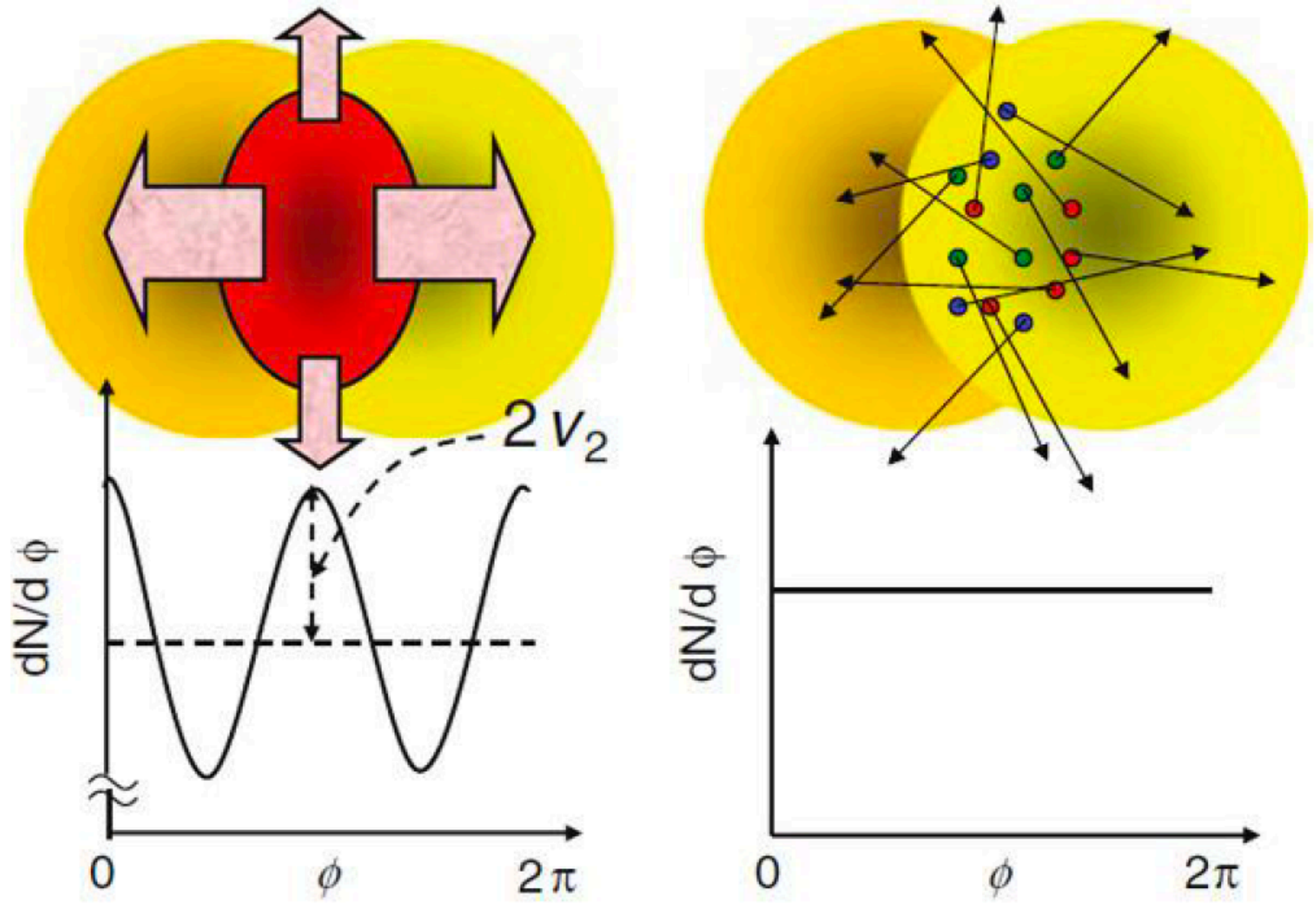


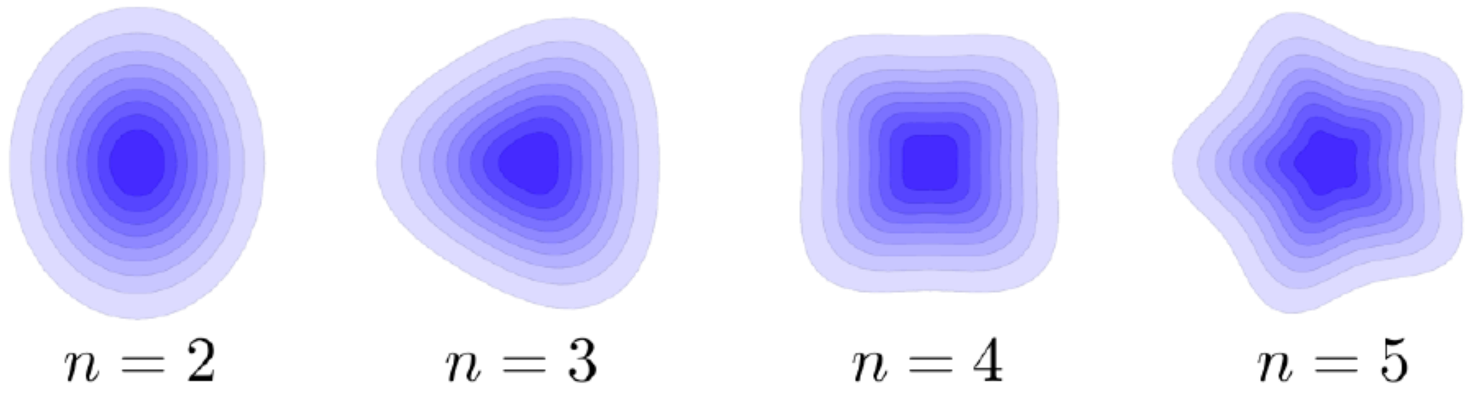
figure: K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRL 104, 212001



Anisotropies in momentum distributions suggest that **QGP is strongly coupled.**

Fluid dynamics modelling applies.

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + ...\right]$$



Nearly perfect fluidity of QGP

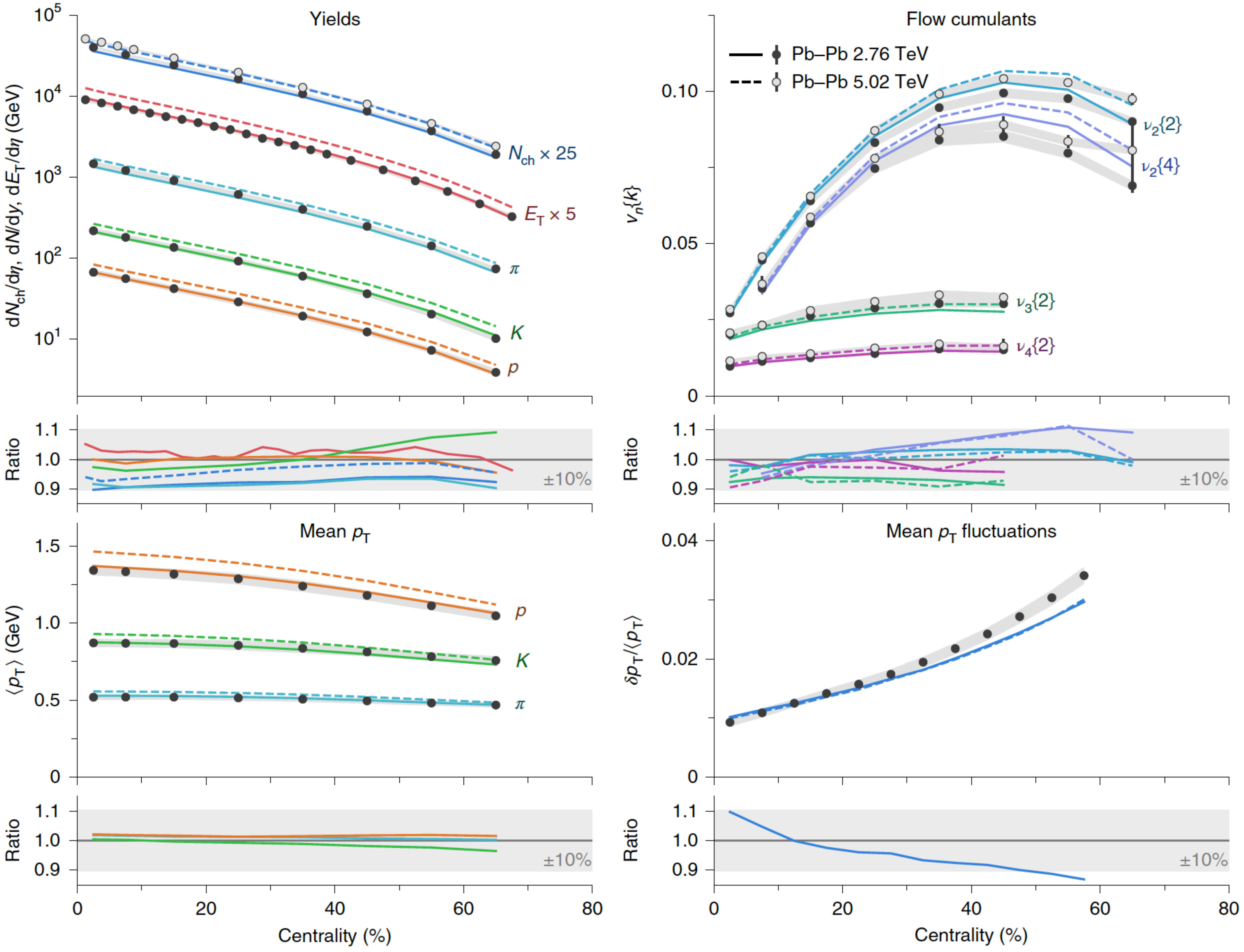
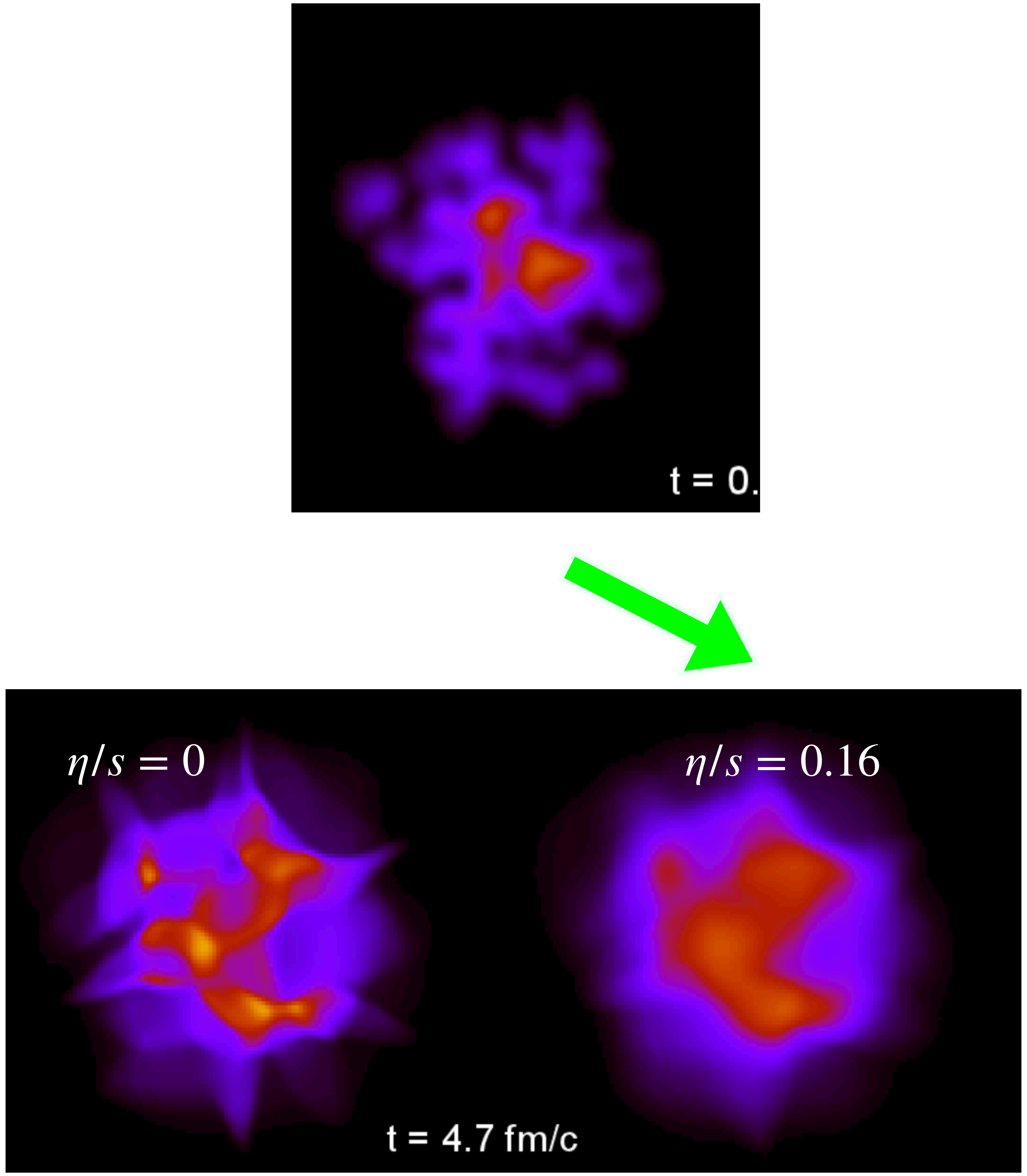


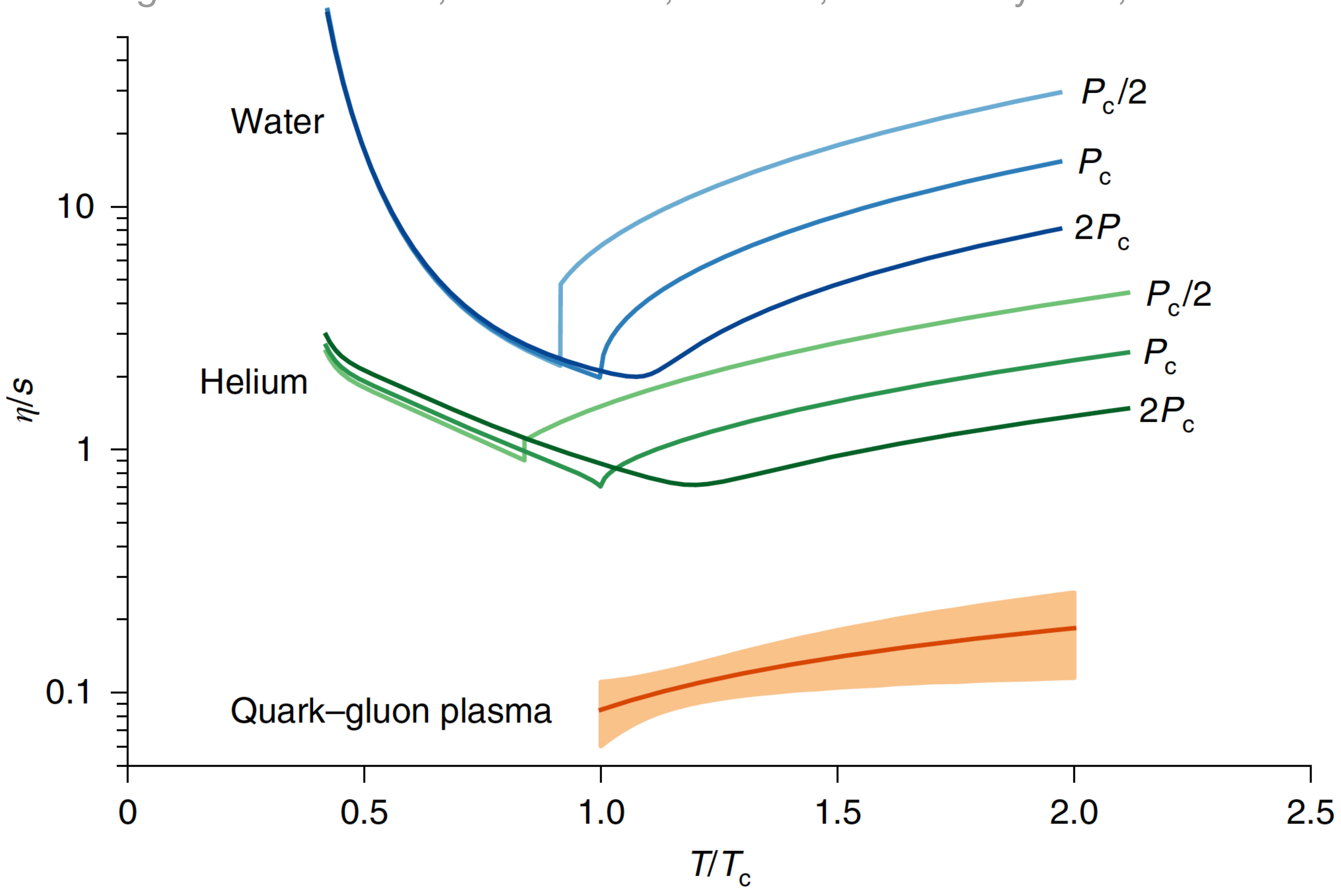
figure: J. Bernhard, J. Moreland, S. Bass, Nat. Phys. 15, 1113–1117 (2019)



figures: B.Schenke

QGP PRECISION STUDIES ERA - CALL FOR NEW OBSERVABLES!

figure: J. Bernhard, J. Moreland, S. Bass, Nature Phys. 15, 1113–1117 (2019)



With the advent of Bayesian analyses
we are entering the precision studies era.

Can we find new observables?

SPIN POLARIZATION DUE TO GLOBAL ORBITAL ANGULAR MOMENTUM

Non-central heavy-ion collisions create fireballs with **large global orbital angular momenta**

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

$$\boldsymbol{L}_{\mathrm{init}} \sim 10^5 \hbar$$

Part of the **angular momentum can be transferred from the orbital to the spin part**

Liang ZT, Wang XN. PRL 94:102301 (2005)
Betz B, Gyulassy M, Torrieri G. PRC 76:044901 (2007)
Gao JH, et al. PRC 77:044902 (2008)
Becattini F, Piccinini F, et al. J. Phys. G 35:054001 (2008)

$$\boldsymbol{J}_{\mathrm{init}} = \boldsymbol{L}_{\mathrm{init}} = \boldsymbol{L}_{\mathrm{final}} + \boldsymbol{S}_{\mathrm{final}}$$

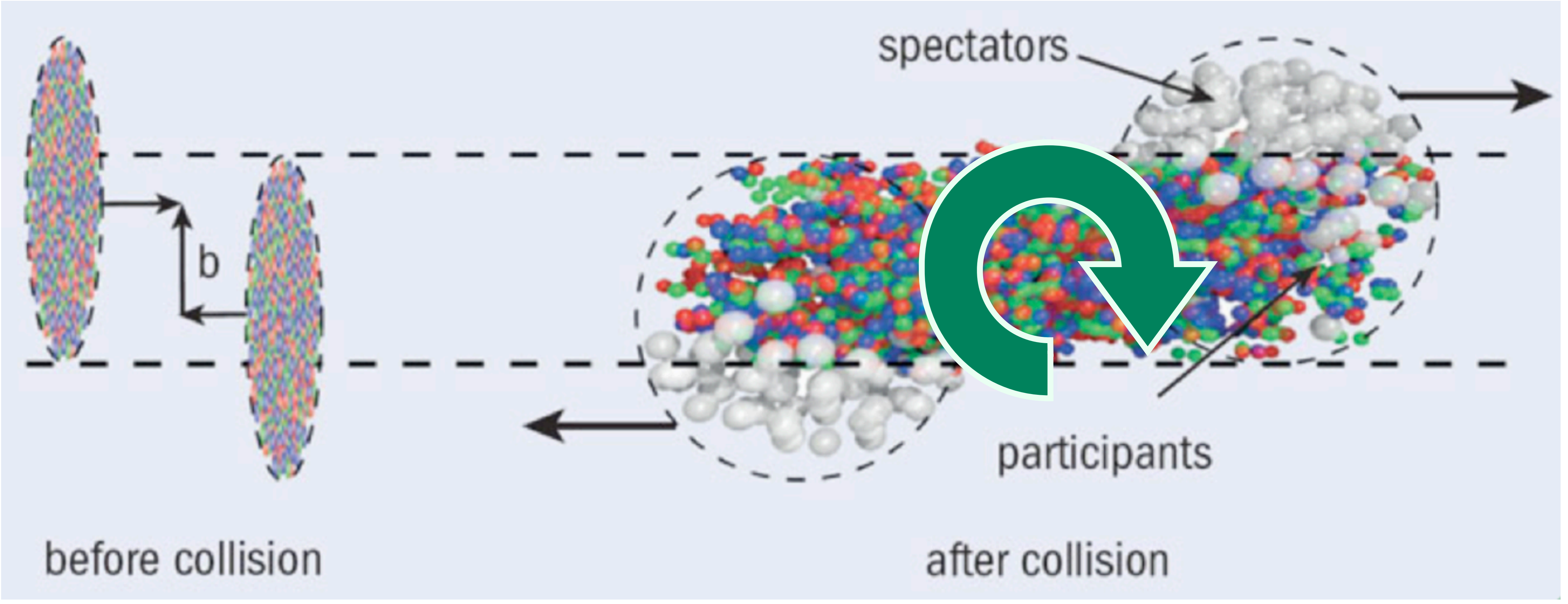
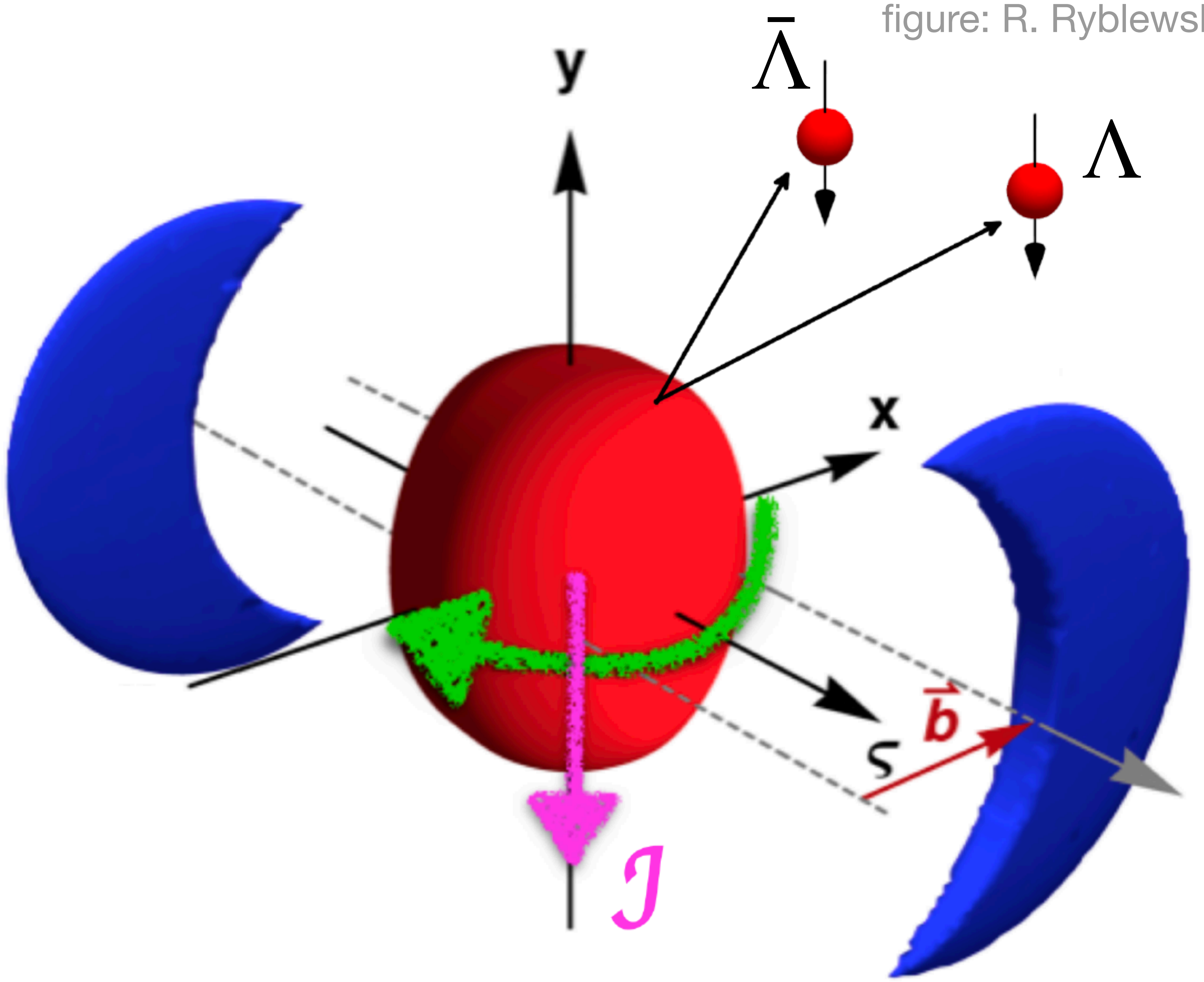


figure: M. Lisa, talk @ "Strangeness in Quark Matter 2016"

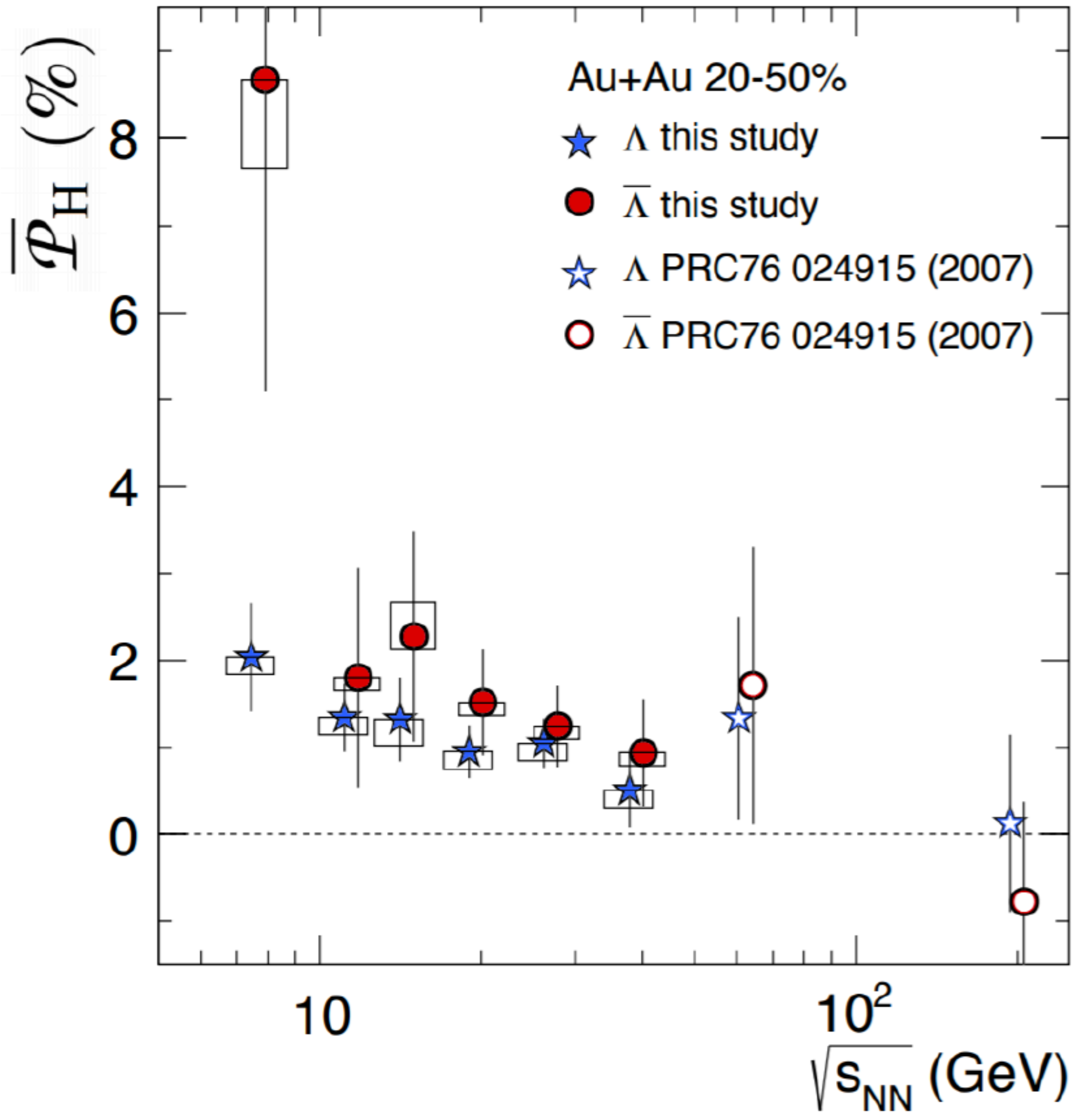
figure: R. Ryblewski



Emitted **particles are expected to be polarized** along the fireball's global angular momentum.

Measurement of Λ and $\bar{\Lambda}$ spin polarization

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ

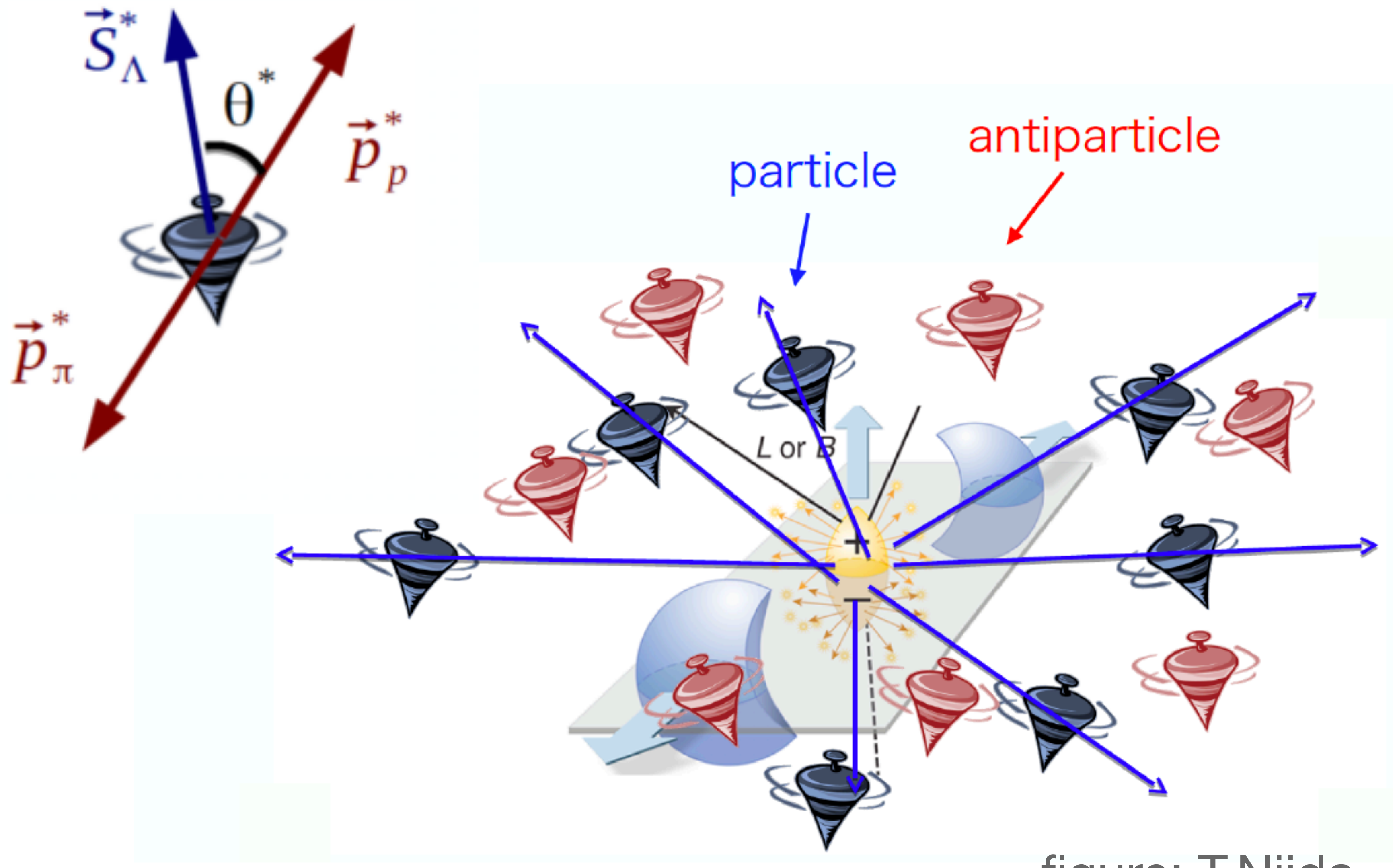


figure: T.Niida

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_{\rm H}\mathbf{P}_{\rm H}\cdot\mathbf{p}^*_{\rm p}\right)$$

Polarization for global thermal equilibrium with rotation is

F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \qquad P_{\overline{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ {\rm s}^{-1}$$

Spin polarization in equilibrated QGP — spin-thermal approach

In thermodynamic equilibrium one can establish a link between spin and vorticity

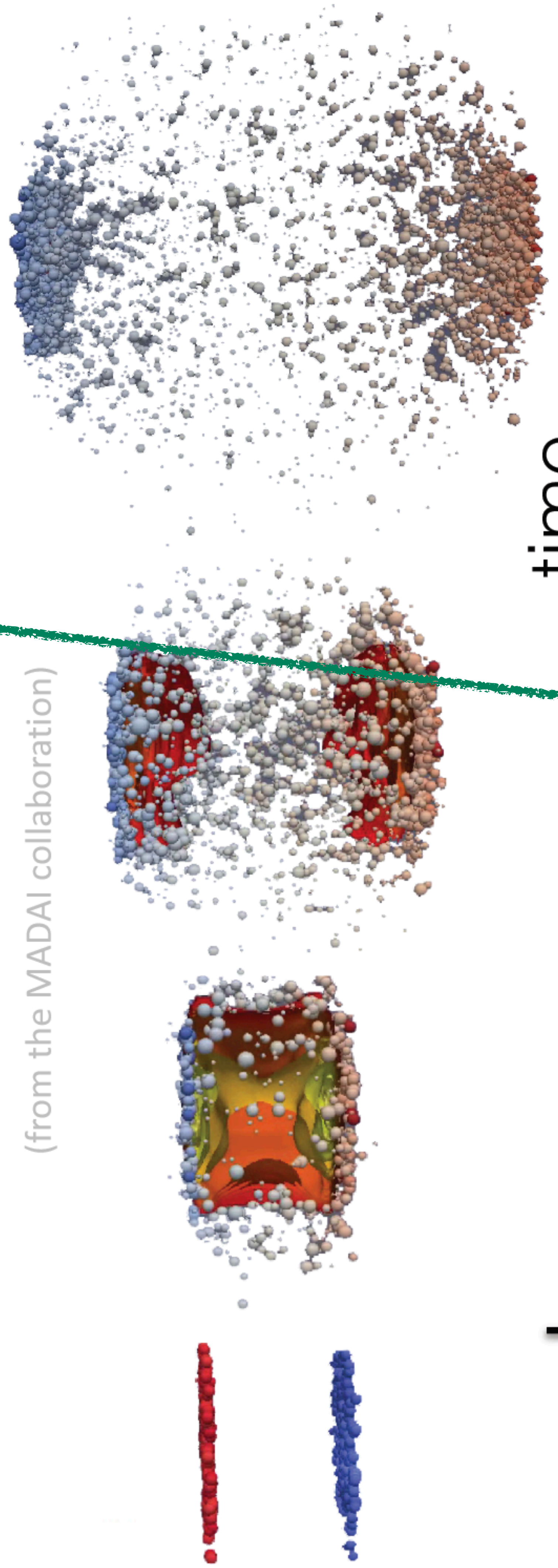
Becattini F, Chandra V, Del Zanna L, Grossi E. AP 338:32 (2013)
F. Becattini, L. Csernai, and D. J. Wang, PRC 88, 034905 (2013)
Fang R, Pang L,Wang Q,Wang X. PRC 94:024904 (2016)
F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F\left(1-n_F\right)\varpi_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n_F}$$

Spin is enslaved to thermal vorticity

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}$$

Allows to extract polarization at the freeze-out hypersurface in any model which provides u^μ, T and μ



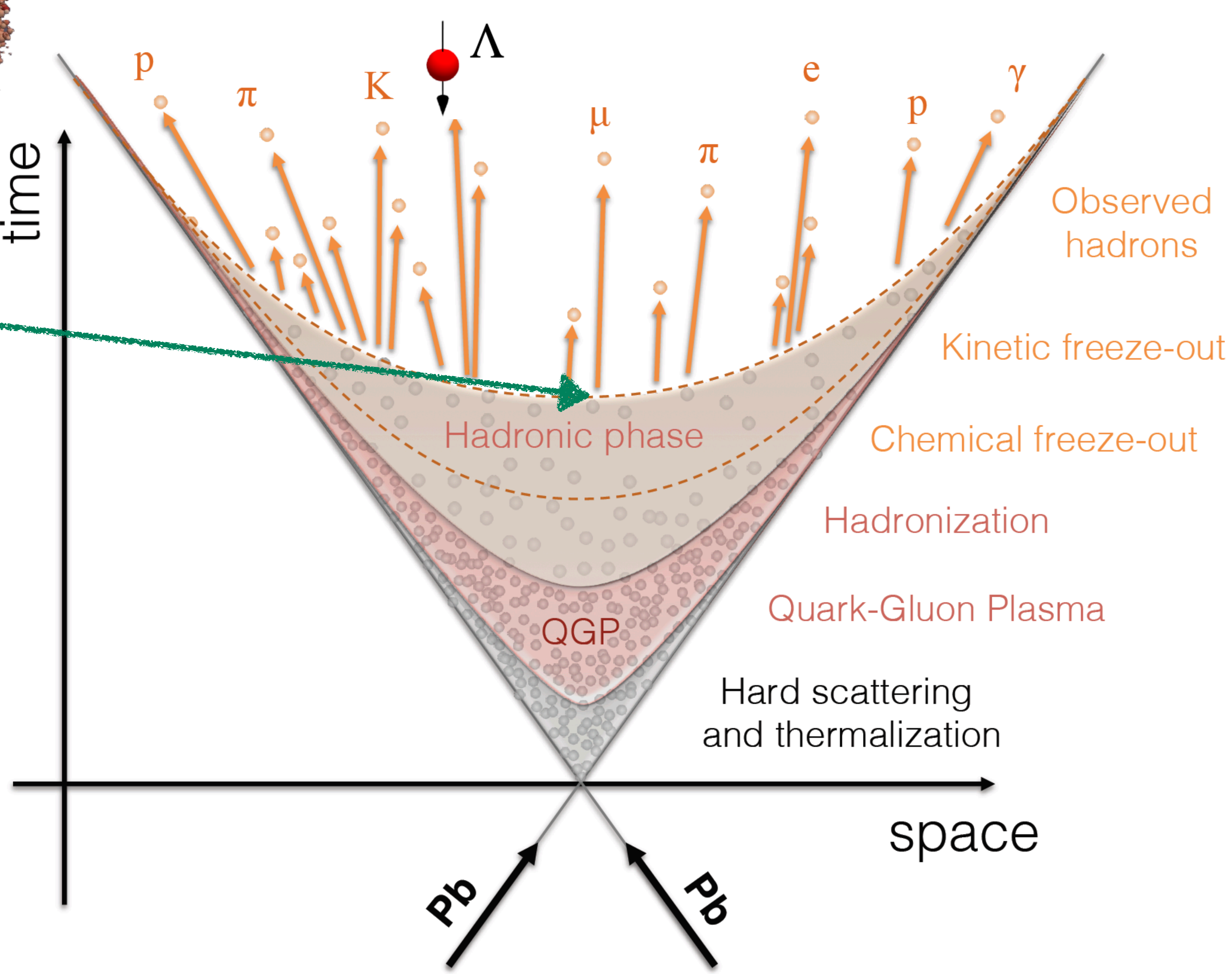


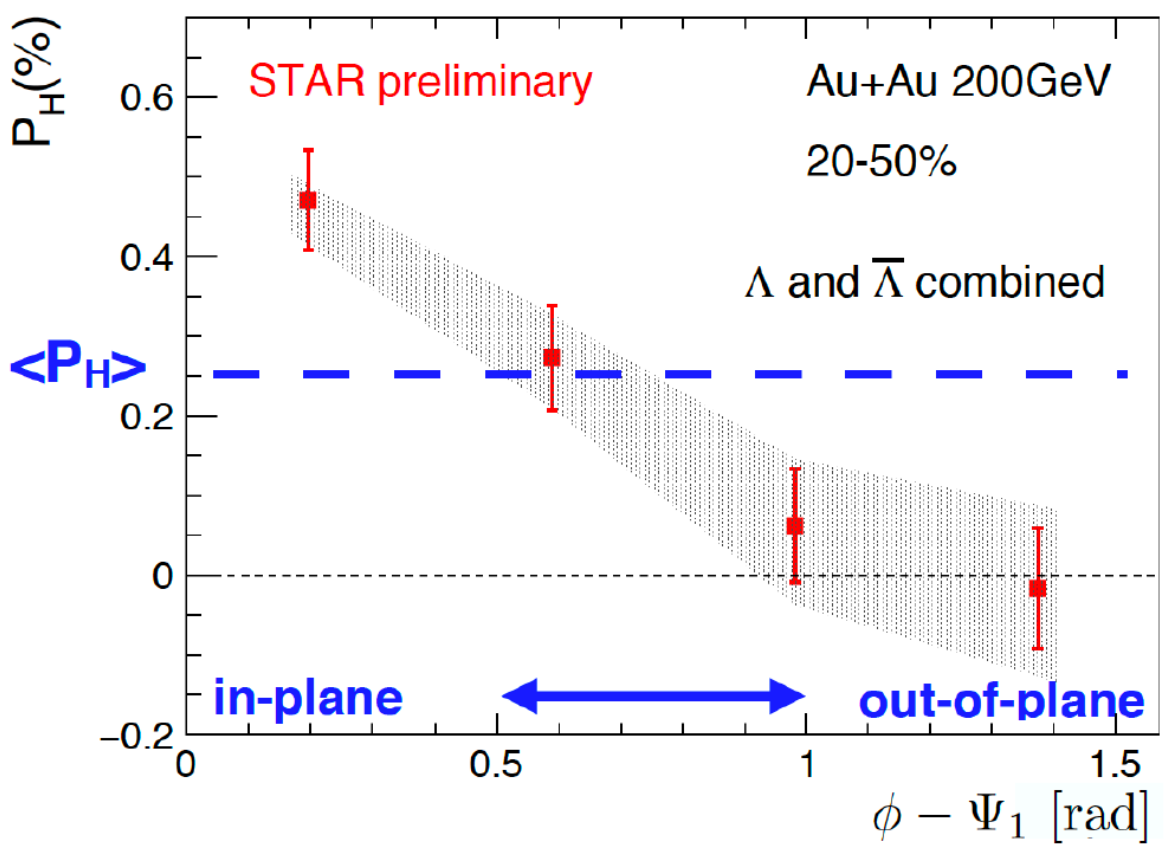
figure: D.D. Chinellato

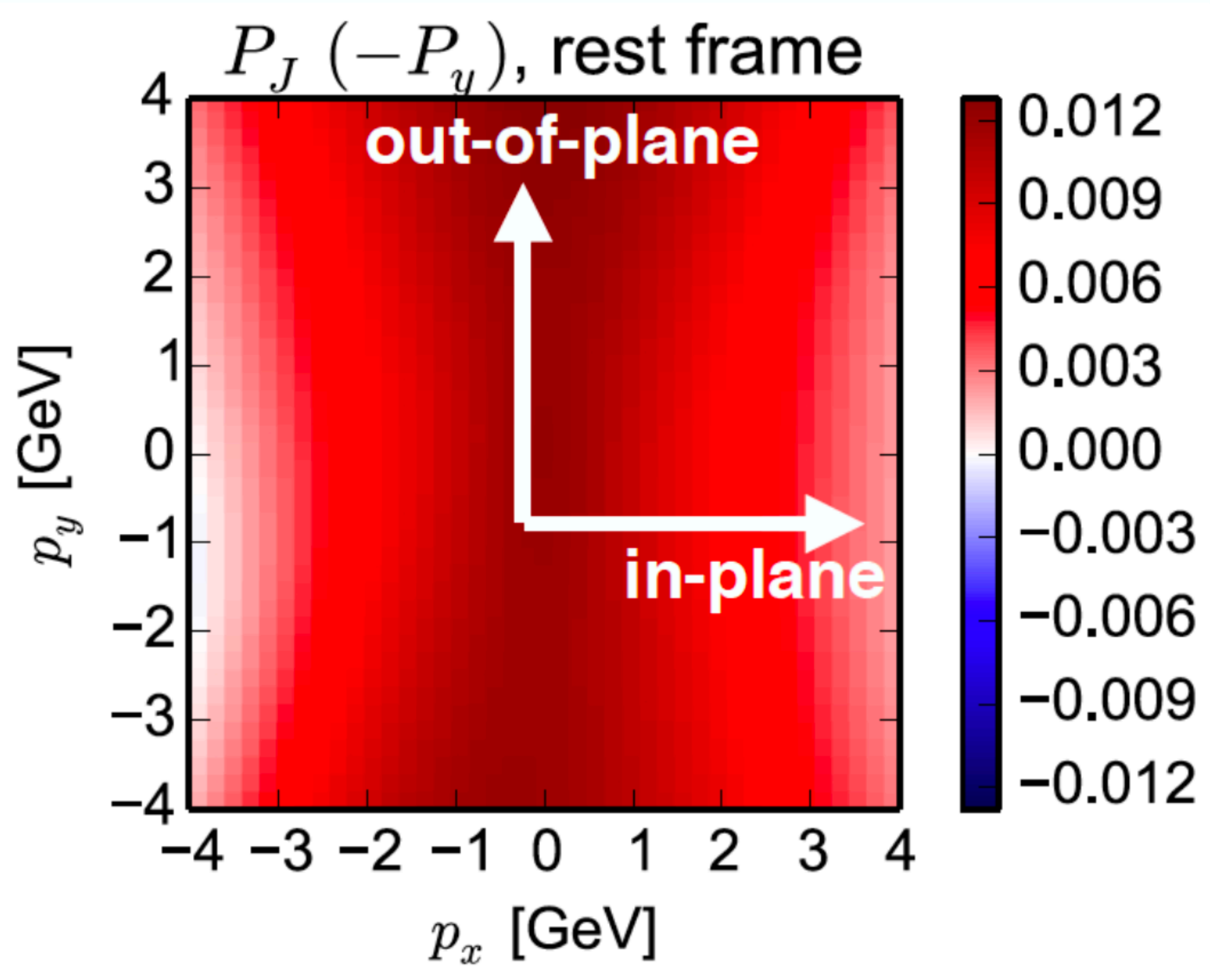
GLOBAL POLARIZATION

Global polarization data supports the spin-thermal approach

Signal is robust and agrees well with predictions of transport and hydrodynamic models

Azimuthal modulation is not captured

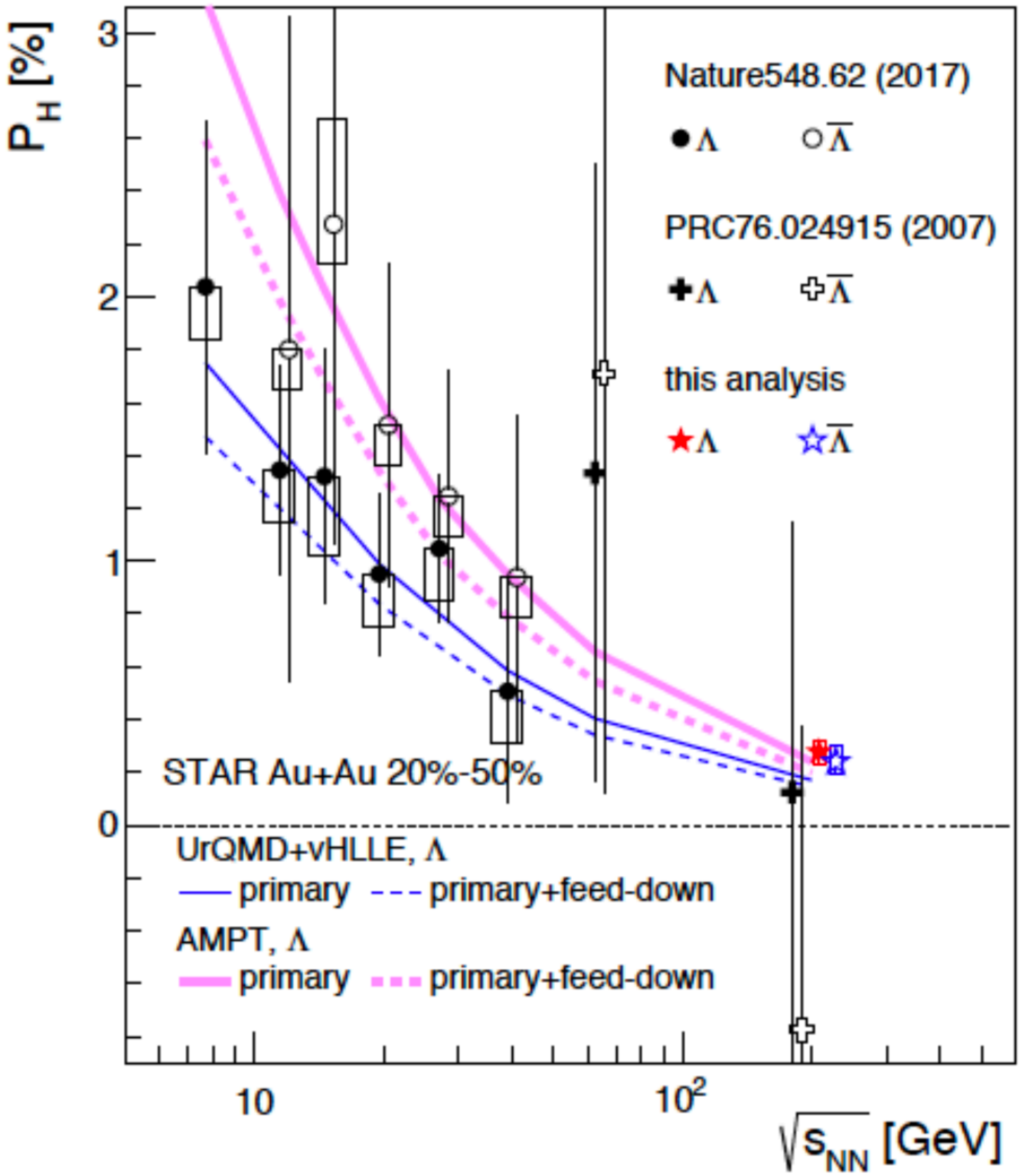




Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019

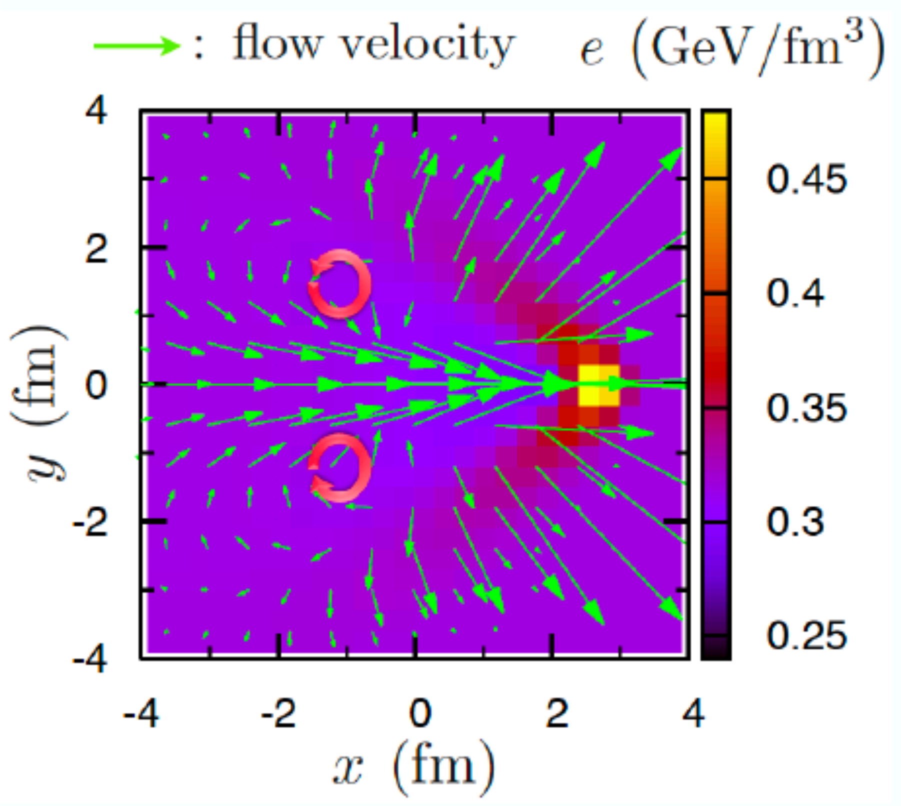
I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

J. Adam, et al., Phys. Rev. C 98, 014910 (2018)

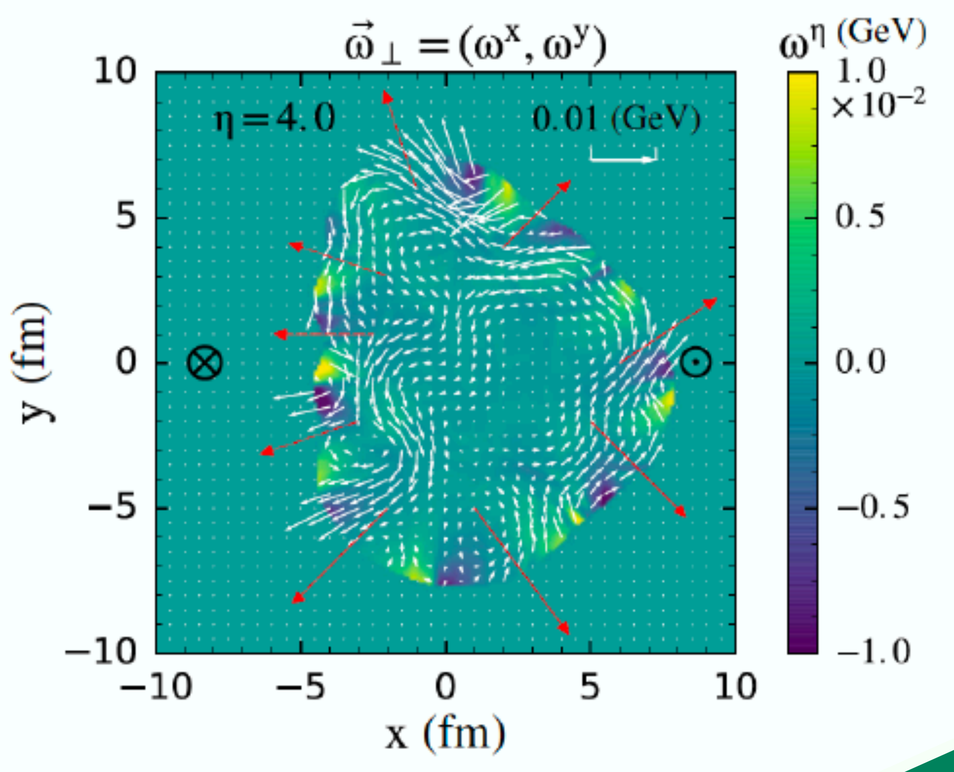


UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017)
AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)

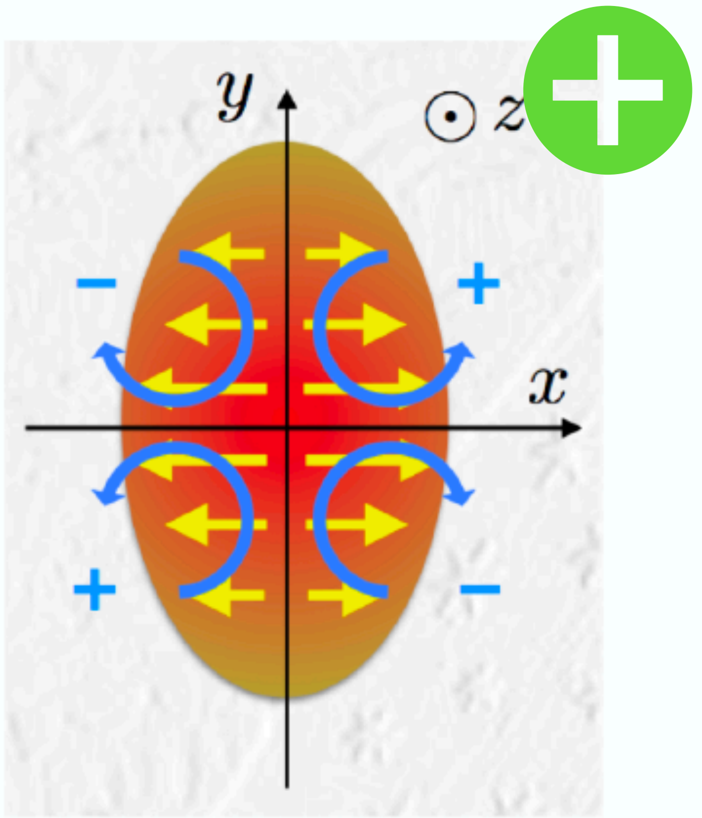
Longitudinal polarization



Y. Tachibana and T. Hirano,
NPA904-905 (2013) 1023

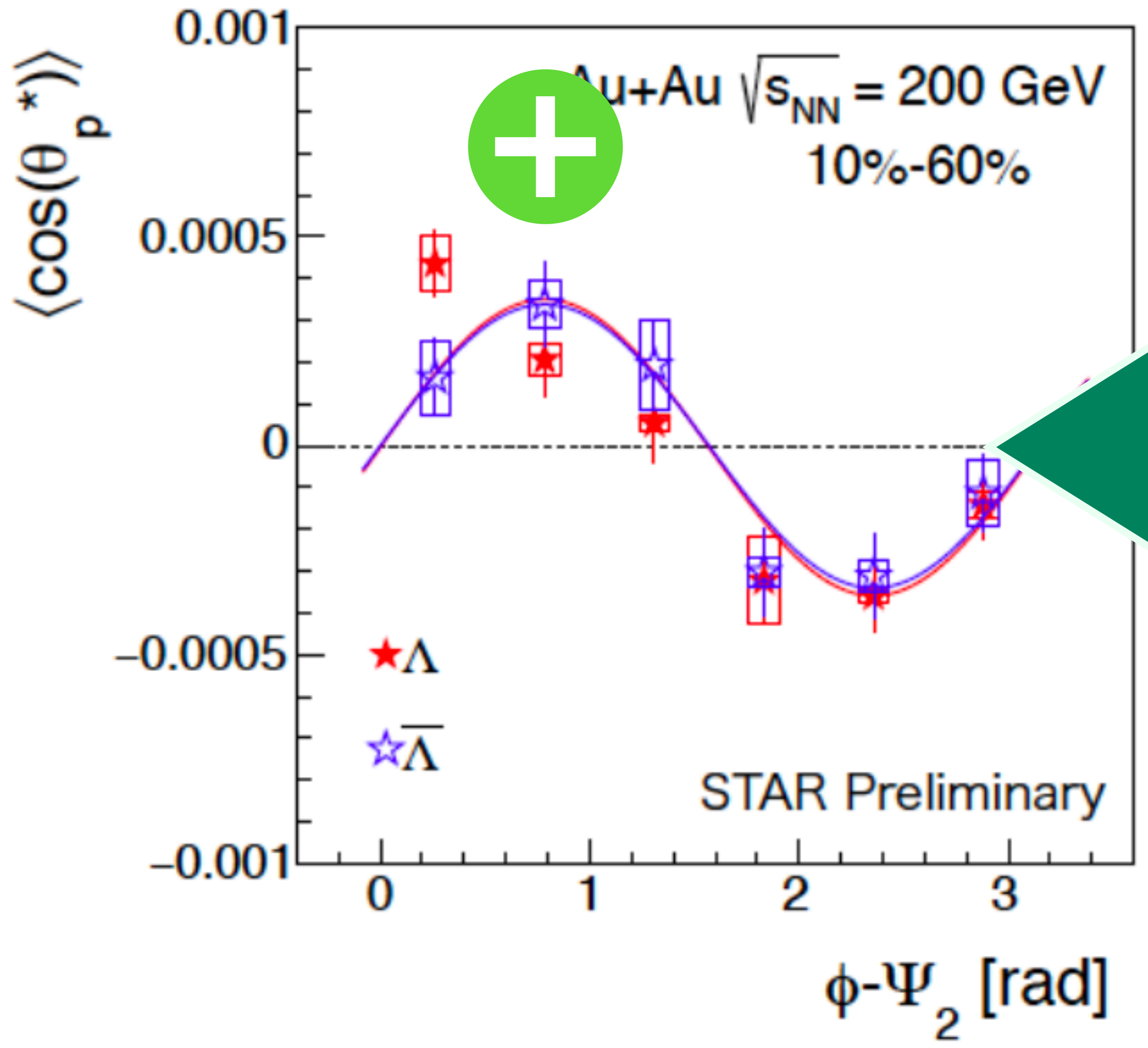


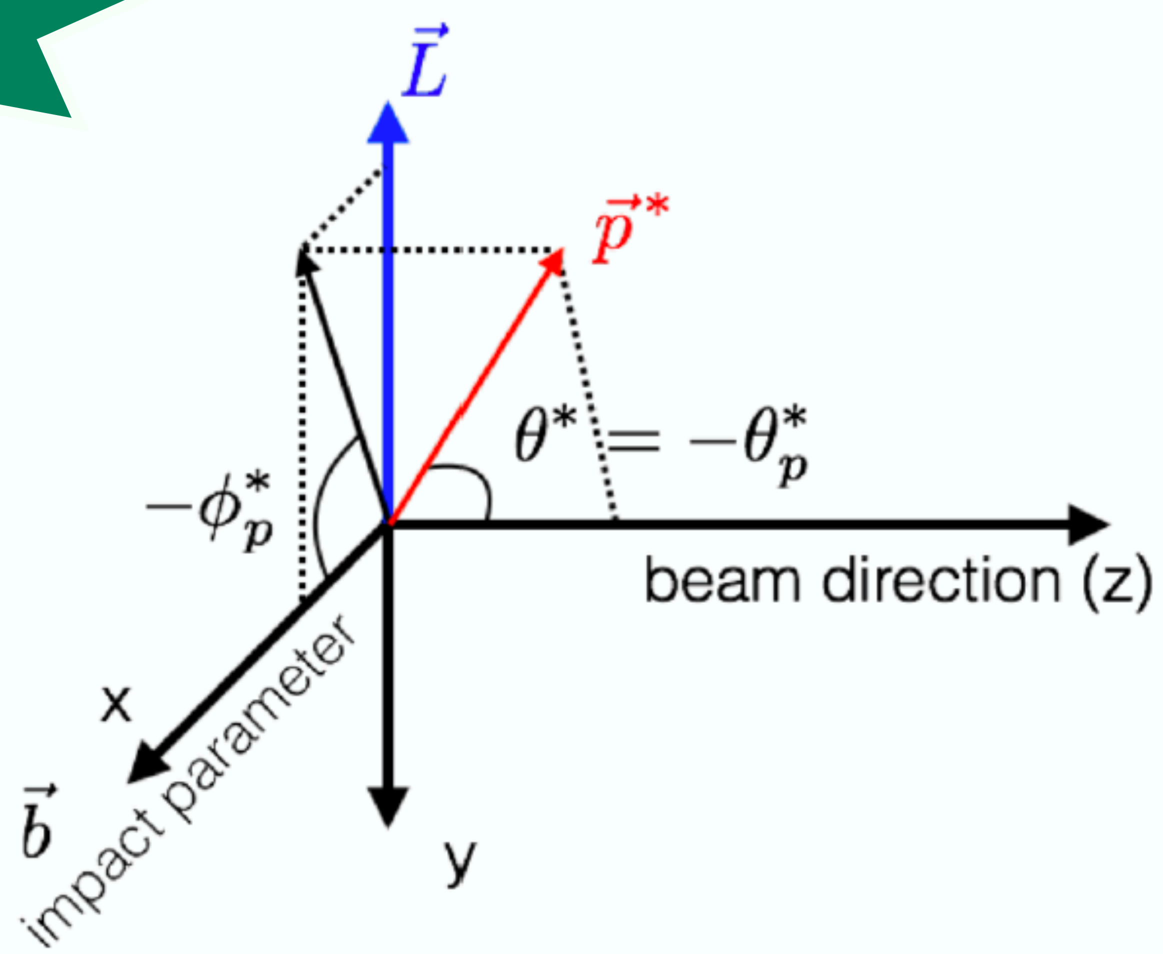
L.-G. Pang, H. Peterson, Q. Wan... Wang
PRL117, 192...



F. Becattini and I. Karpenko, PRL120.012302 (2018)
S. Voloshin, EPJ Web Conf.171, 07002 (2018)

Flow structures in the plane transverse to beam (jet, ebe fluctuations etc.) may generate longitudinal polarization





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\rm H}\mathbf{P_H}\cdot\mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_{\rm H} P_z \langle(\cos\theta_p^*)^2\rangle$$

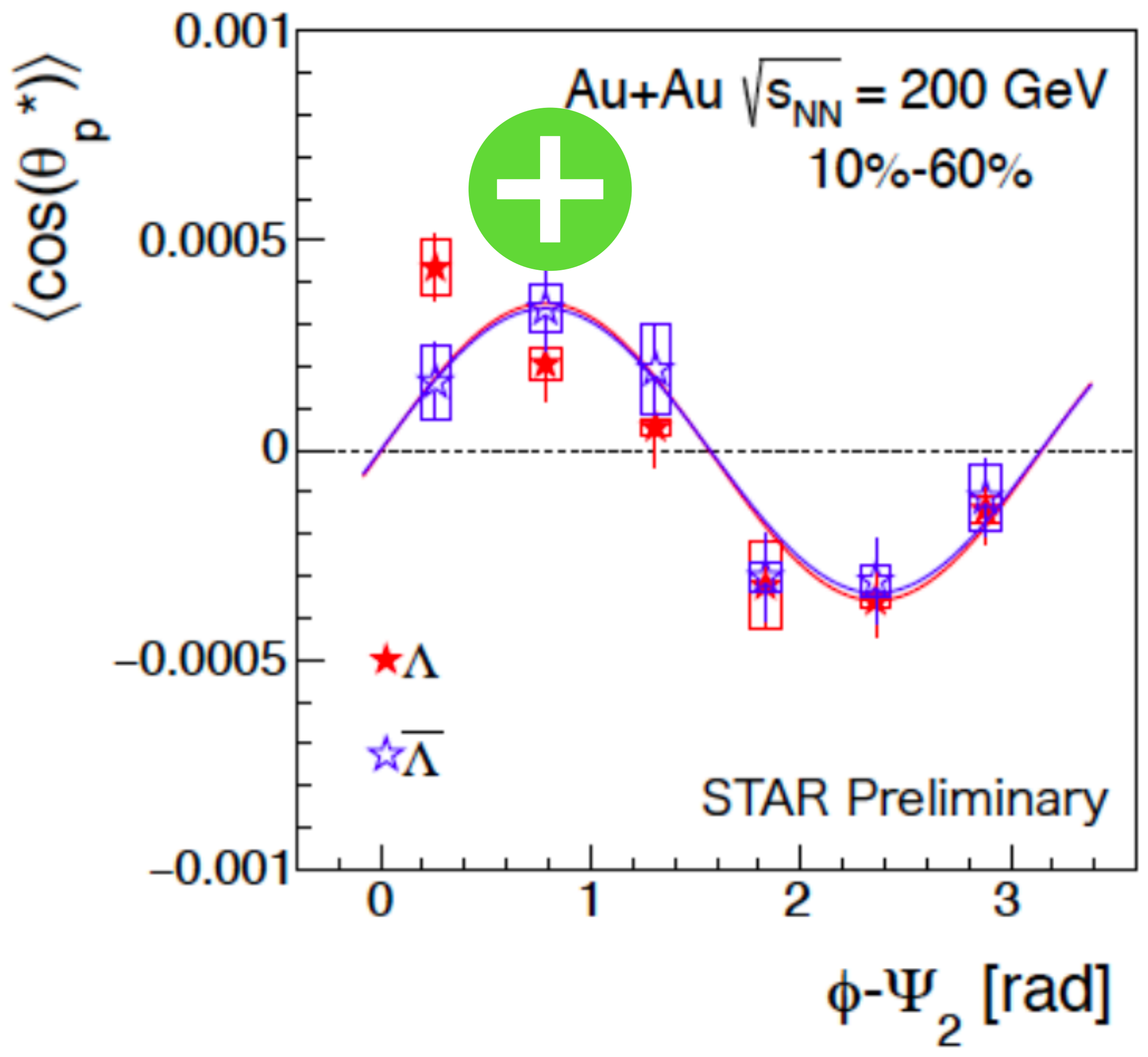
$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}\langle(\cos\theta_p^*)^2\rangle}$$

$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}} \quad \text{(if perfect detector)}$$

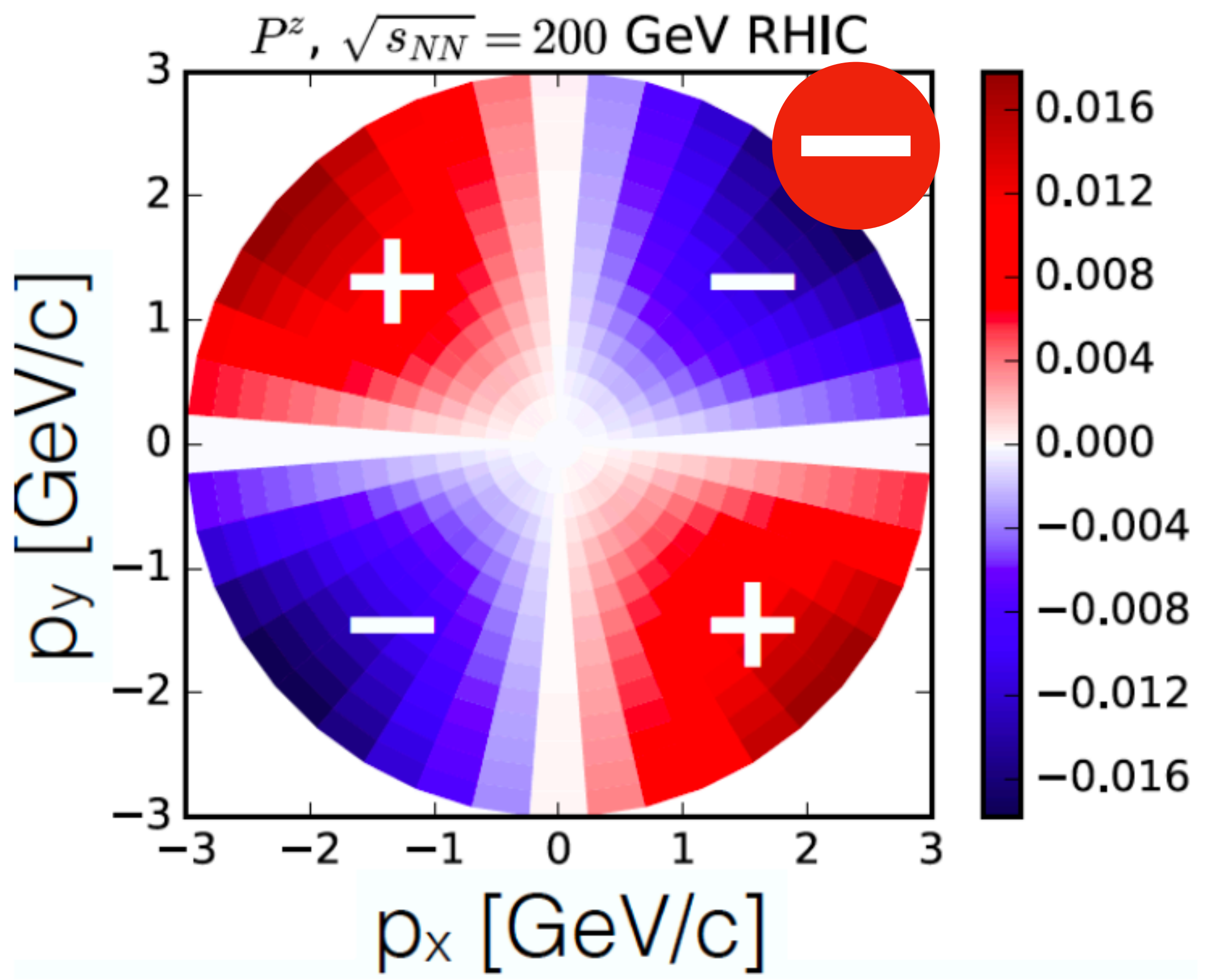
$\alpha_{\rm H}$: hyperon decay parameter

θ_p^*: θ of daughter proton in Λ rest frame

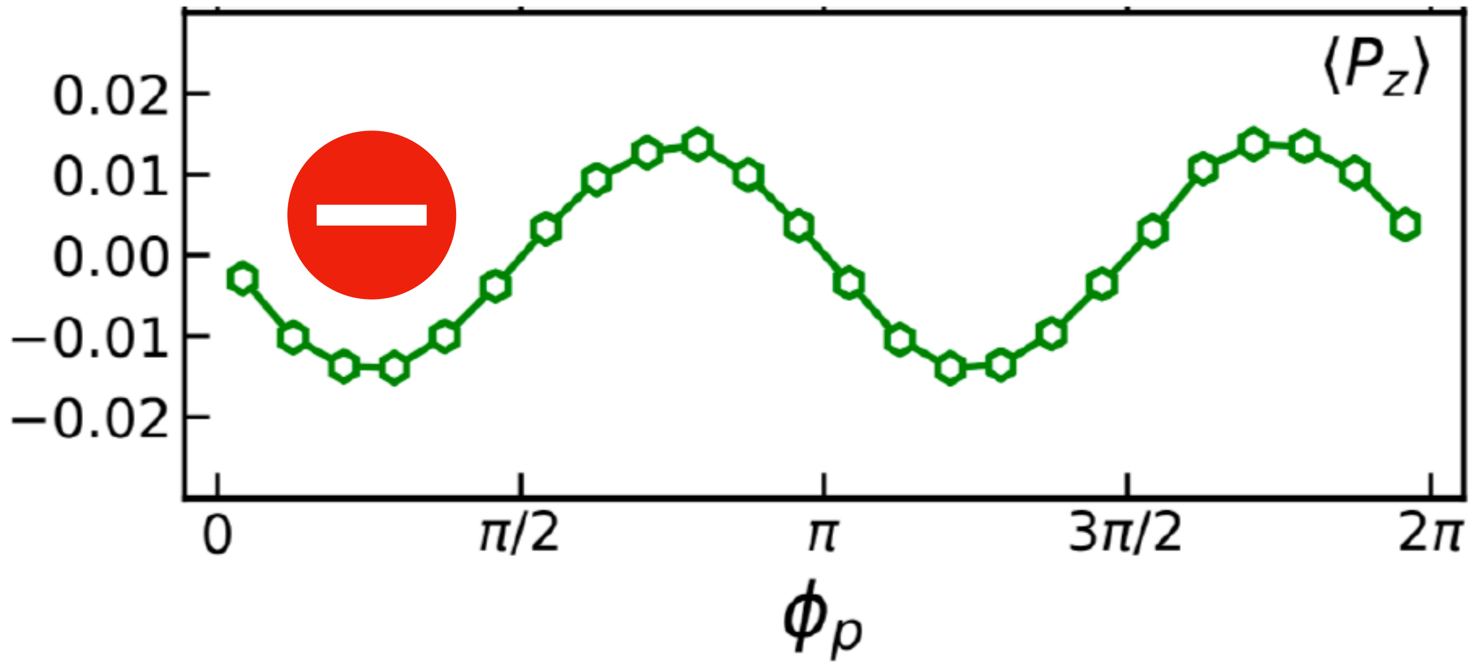
Longitudinal polarization — 'Spin sign' puzzle



T. Niida, NPA 982 (2019) 511514



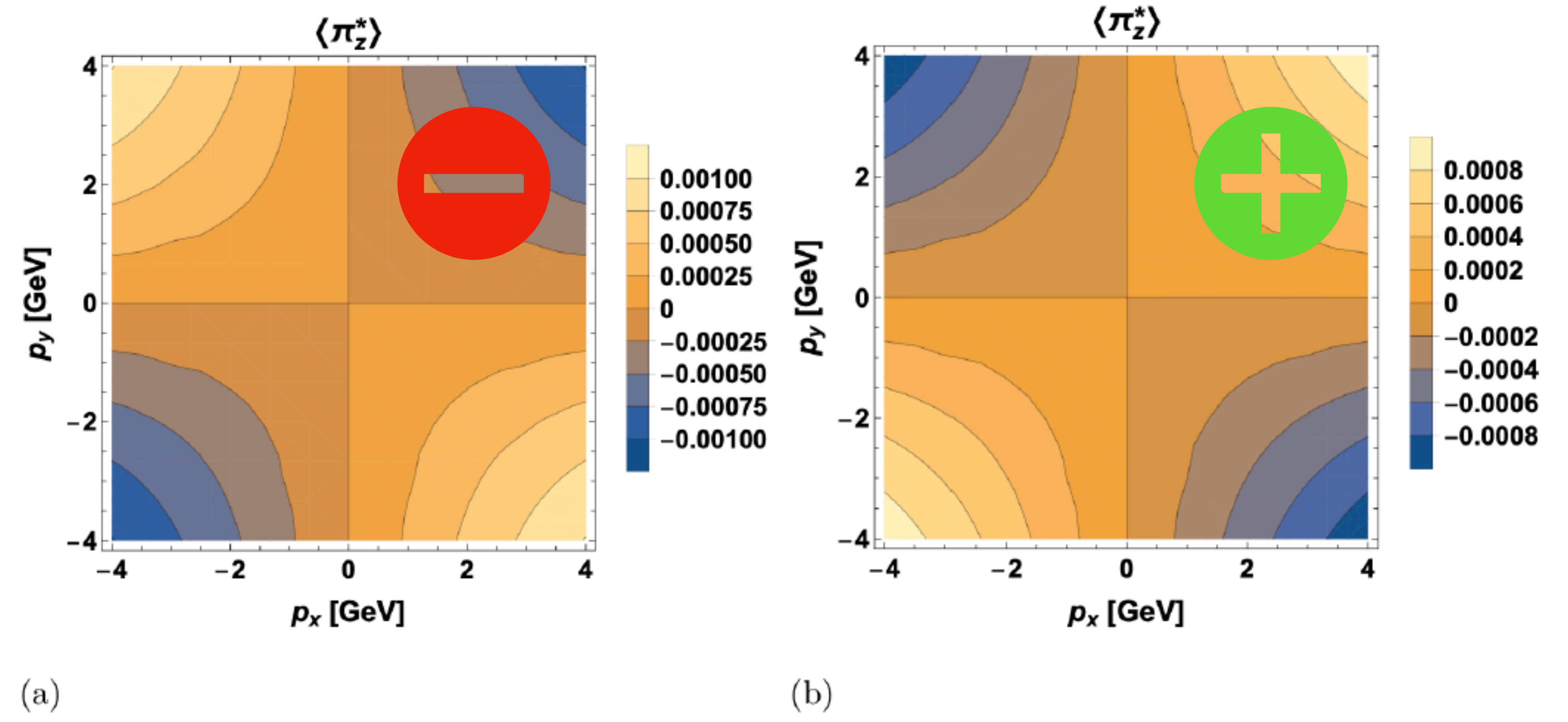
UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

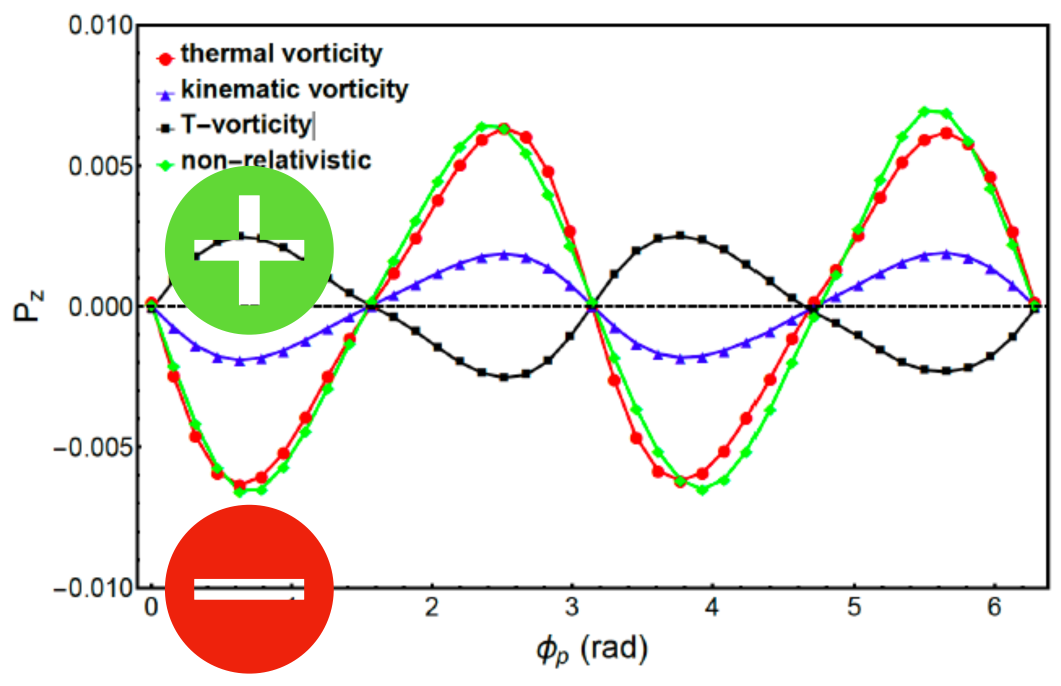
thermal model with projected vorticity $\omega_{\mu\nu} = \varpi_{\alpha\beta}\overline{\Delta}^{\alpha}_{\mu}\overline{\Delta}^{\beta}_{\nu}$

W.Florkowski, A. Kumar, A. Mazeliauskas, R.R., [1904.00002]

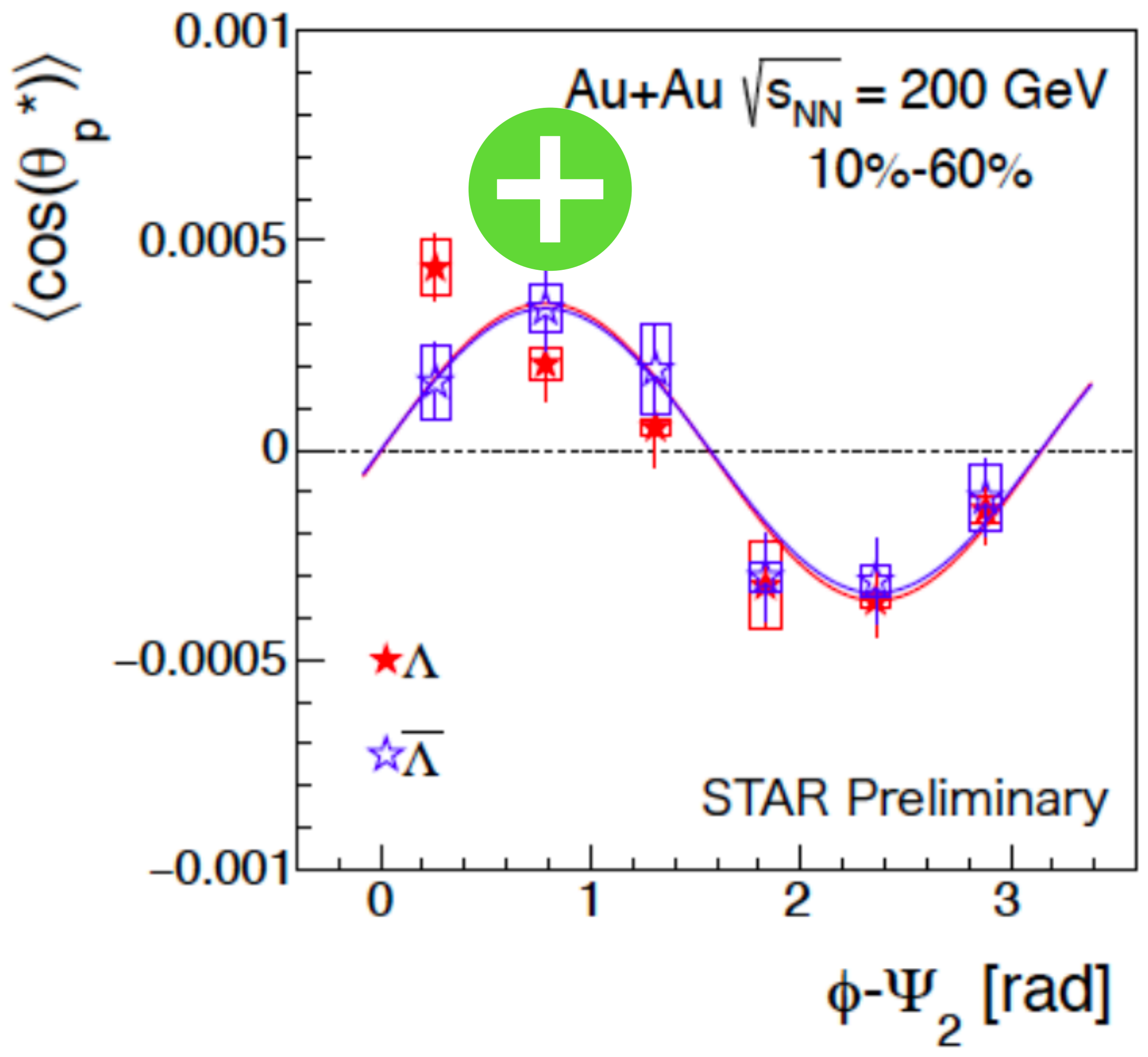


3D VH + AMPT IC with T-vorticity $\omega^{(T)}_{\mu\nu} = -\frac{1}{2}\left[\partial_\mu\left(Tu_\nu\right) - \partial_\nu\left(Tu_\mu\right)\right]$

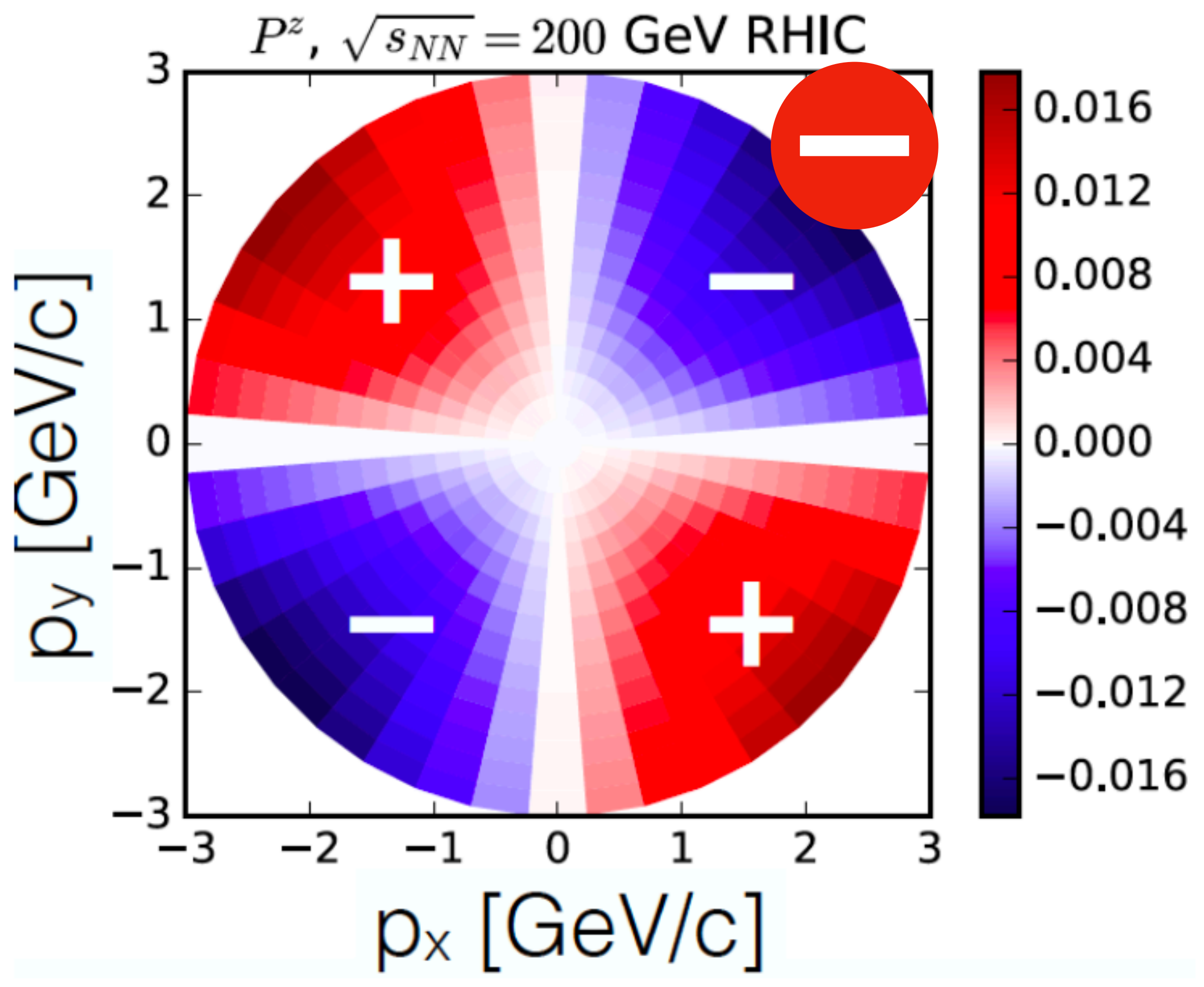
H-Z Wu, L-G Pang, X-G Huang, Q. Wang [1906.09385]



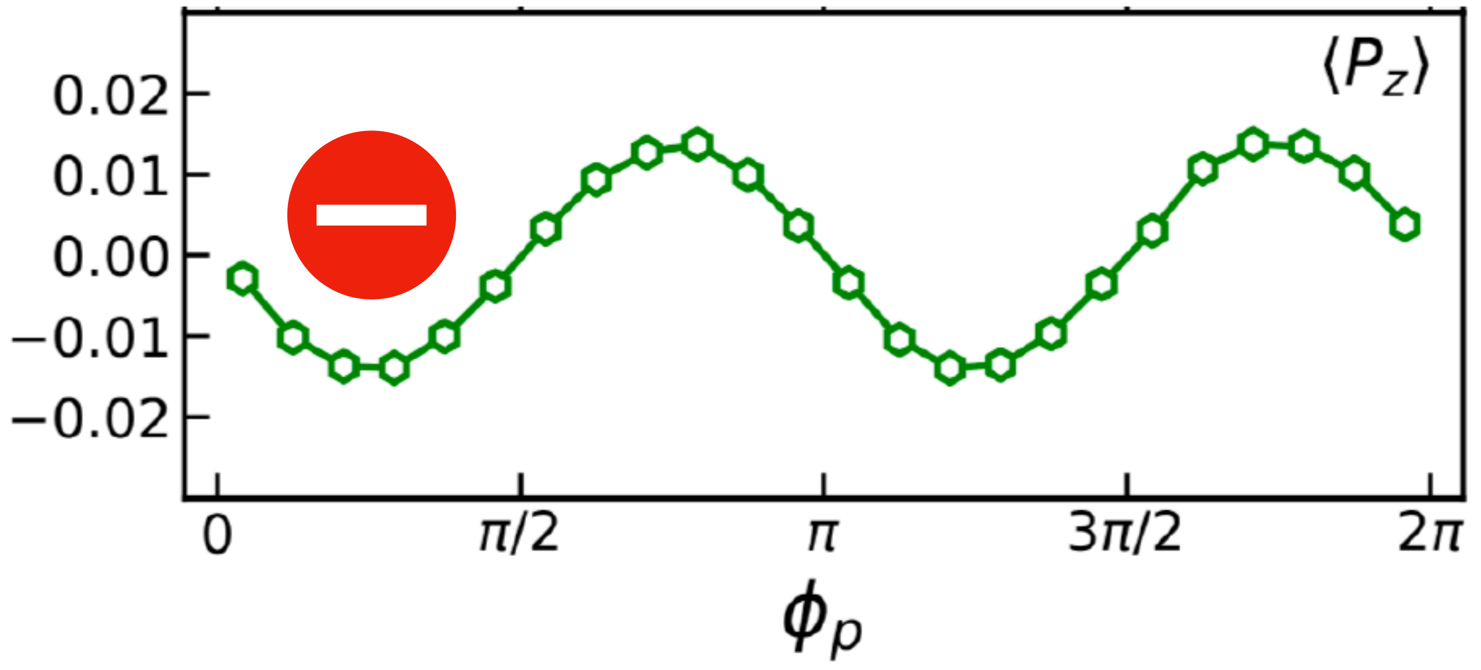
Longitudinal polarization — 'Spin sign' puzzle



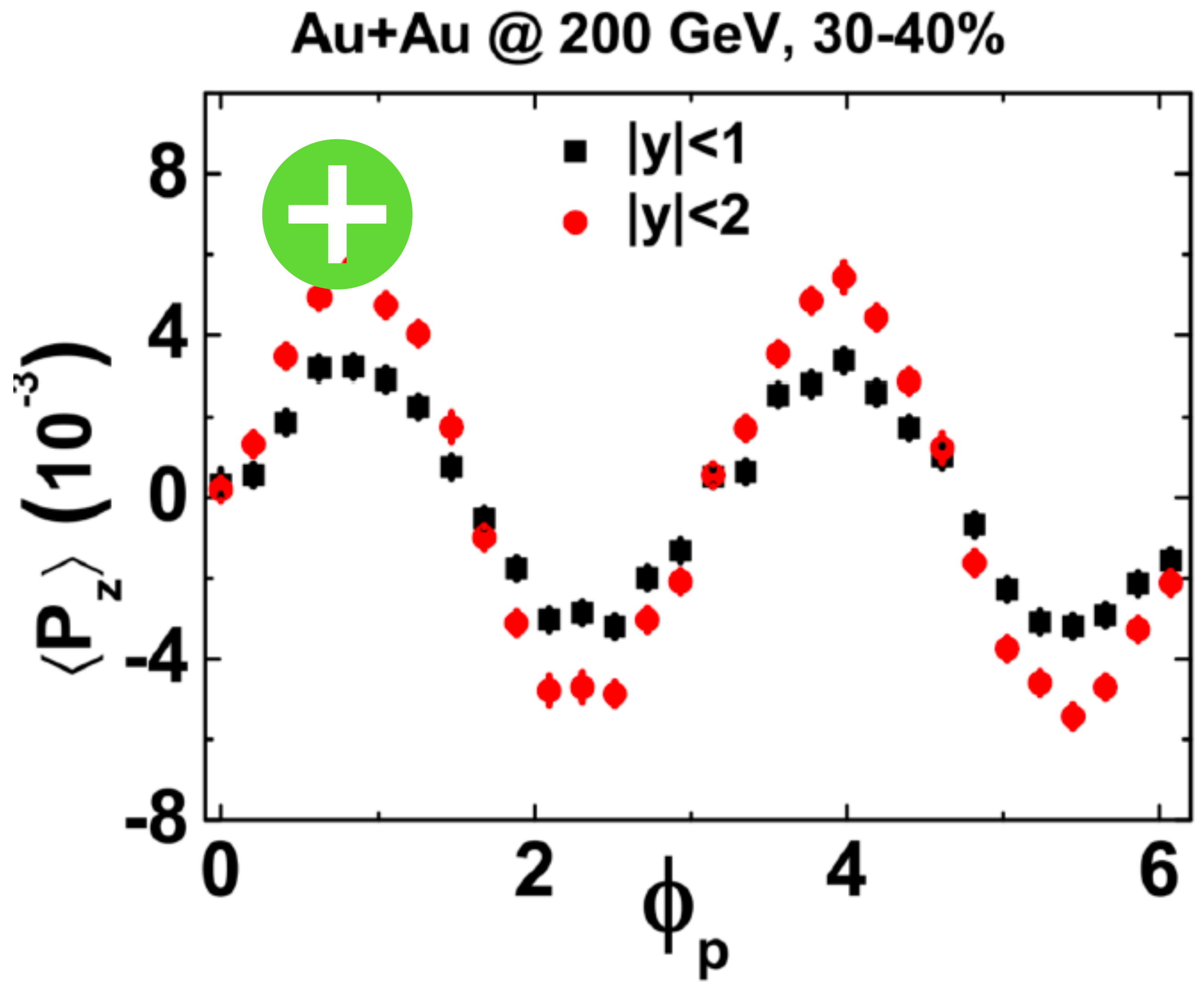
T. Niida, NPA 982 (2019) 511514

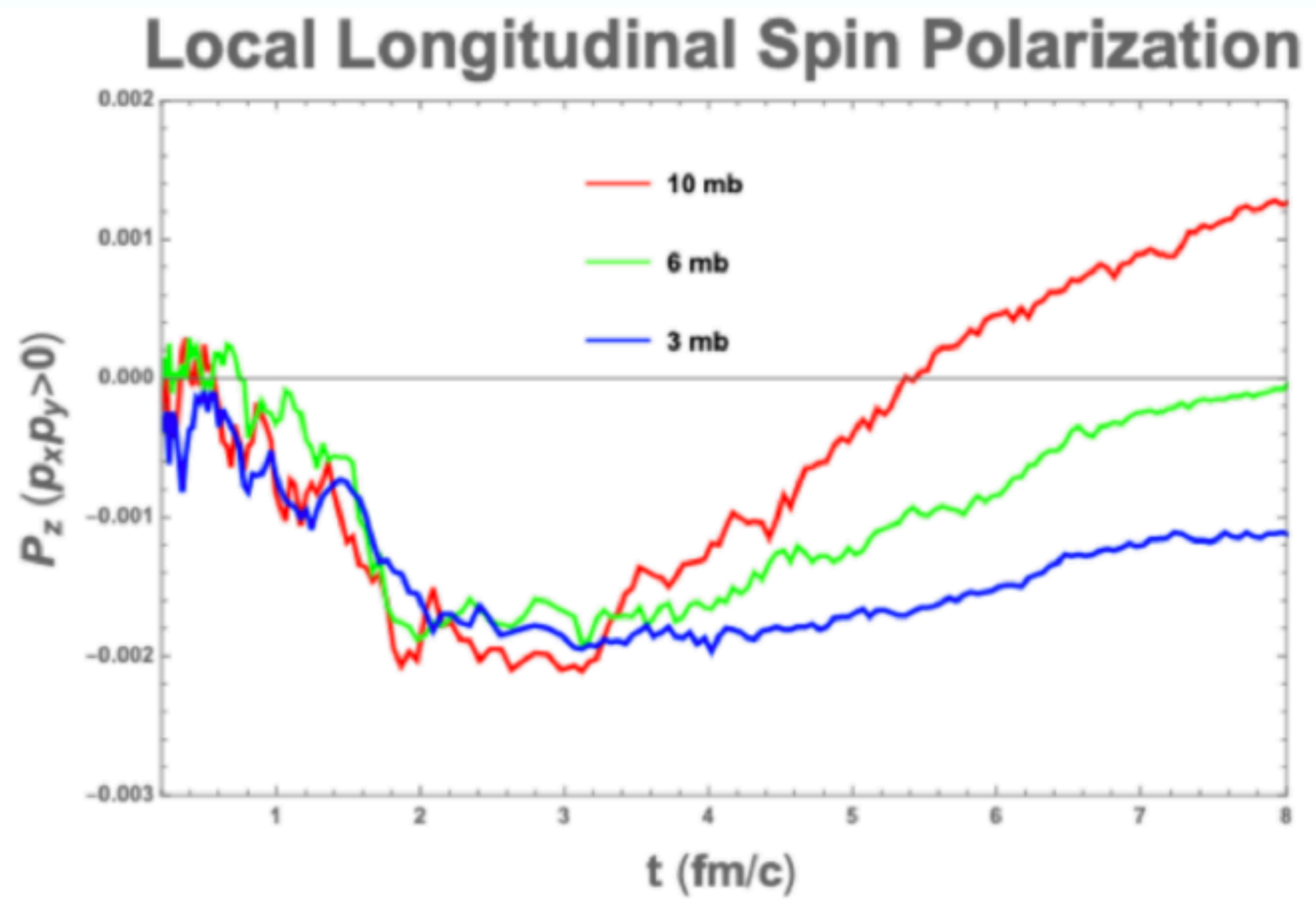


UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)





Y. Sun, C-M. Ko, Phys.Rev. C99 (2019) no.1, 011903

FLUID DYNAMICS OF SPIN?

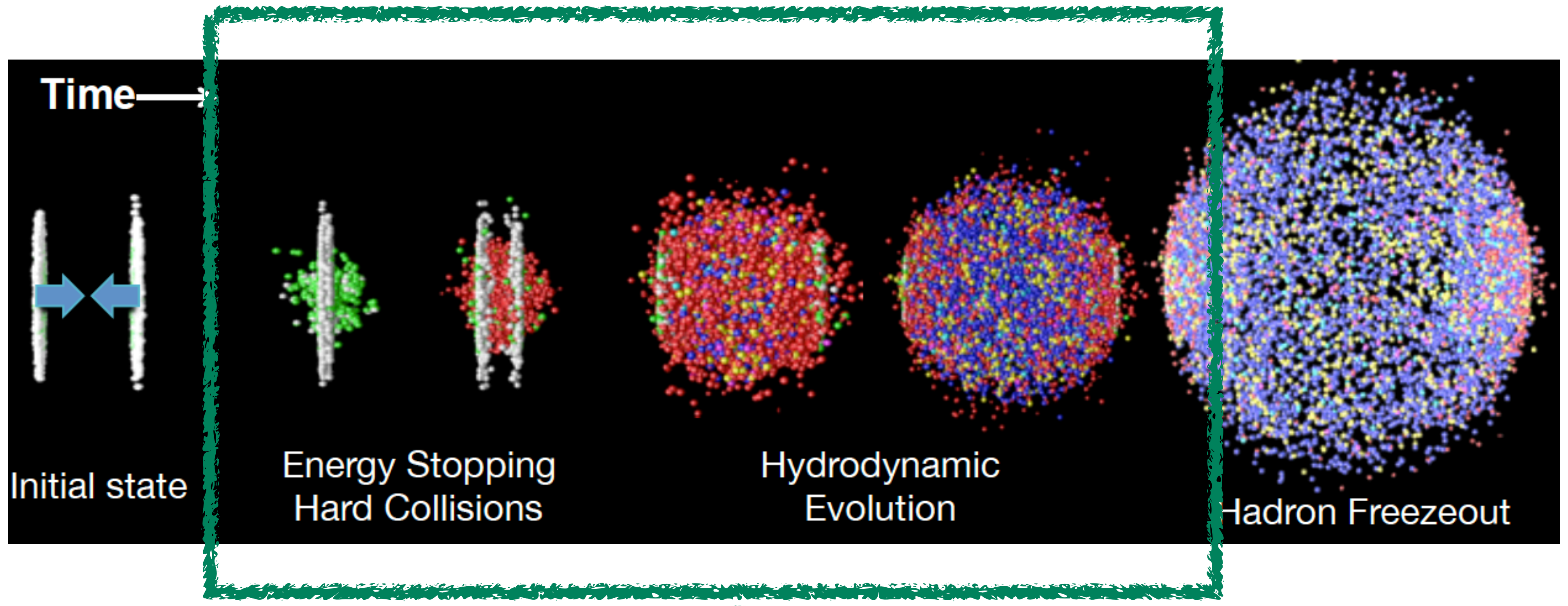
Spin-thermal approach does not capture properly phenomena seen in differential observables.

Nonequilibrium dynamics of spin?

If spin polarization is trully hydrodynamic quantity it should not be enslaved to thermal vorticity.

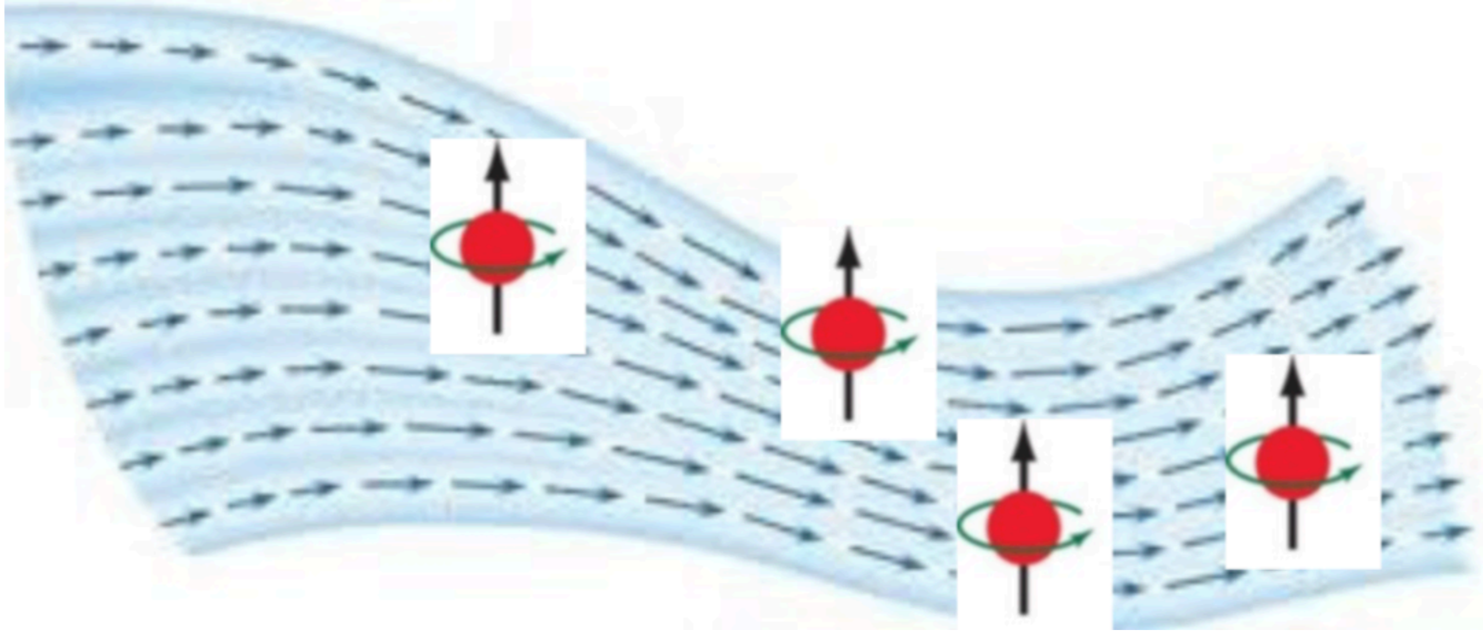
W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901

Relativistic fluid dynamics forms the basis of HIC models



Most of the time close to equilibrium but the dissipation is also important

Fluid dynamics with spin



Basics of spinless relativistic fluid dynamics

perfect fluid dynamics = local equilibrium + conservation laws

figure: Avdhesh Kumar

Ideal	Dissipative
$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$ $N^\mu = n u^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu}_{=6}$	$T^{\mu\nu} = \epsilon u^\mu u^\nu - [P + \Pi]\Delta^{\mu\nu} + \pi^{\mu\nu}$ $N^\mu = n u^\mu + \nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ \nu^\mu}_{=15}$
Equations: $\underbrace{\partial_\mu T^{\mu\nu} = 0,\ \ \partial_\mu N^\mu = 0,\ \ EoS}_{4+1+1=6}$	
Closed set of equations	9 additional equations are needed

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

energy-linear momentum conservation

baryon number conservation

Caution:
Eckart-Landau theory is acausal!

Conservation of angular momentum and spin chemical potential

Conservation of charge (baryon number, electric charge, ...)

$$\partial_\mu \widehat{N}^\mu(x) = 0 \qquad \text{(1 equation/charge)} \quad \Rightarrow \quad \mu \equiv \xi T$$

Conservation of energy and momentum

$$\widehat{J}_C^{\mu,\alpha\beta}(x) = \underbrace{x^\alpha \widehat{T}_C^{\mu\beta}(x) - x^\beta \widehat{T}_C^{\mu\alpha}(x)}_{\widehat{L}_C^{\mu,\alpha\beta}(x)} + \widehat{S}_C^{\mu,\alpha\beta}(x)$$

$$\partial_\mu \widehat{T}_C^{\mu\alpha}(x) = 0 \qquad \text{(4 equations)} \quad \Rightarrow \quad T, u^\nu$$

For particles with spin the conservation of angular momentum implies introduction of new hydrodynamic variables — spin chemical potential

Conservation of total angular momentum

$$\partial_\mu \widehat{J}_C^{\mu,\alpha\beta}(x) = 0 \qquad \Rightarrow \qquad \partial_\mu \widehat{S}_C^{\mu,\alpha\beta}(x) = \widehat{T}_C^{\beta\alpha}(x) - \widehat{T}_C^{\alpha\beta}(x) \quad \Rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu}$$

Pseudogauges and the problem of energy and spin localization

Pseudo-gauge transformation

W. Hehl, Rept. Math. Phys. 9 (1976) 55–82;
F. Becattini, L. Tinti, PRD 84 (2011) 025013; PRD 87(2) (2013) 025029

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

- ⤳ preserve $\widehat{P}^{\mu} = \int d^3\Sigma_\lambda \, \widehat{T}^{\lambda\mu}(x)$ $\quad \widehat{J}^{\mu\nu} = \int d^3\Sigma_\lambda \, \widehat{J}^{\lambda,\mu\nu}(x)$
- ⤳ conservation laws unchanged

Belinfante-Rosenfeld pseudo-gauge (choosing superpotential $\widehat{\Phi} = \widehat{S}_C^{\lambda,\mu\nu}$)

Belinfante, F. J. (1939): Physica 6. 887-898, (1940); Rosenfeld, L. (1940): Mem. Acad. Roy. Belgique, cl. SC., tome 18, fasc. 6

$$\widehat{T}_B^{\mu\nu} = \widehat{T}_C^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\widehat{S}_C^{\lambda,\mu\nu} + \widehat{S}_C^{\mu,\nu\lambda} - \widehat{S}_C^{\nu,\lambda\mu}\right) \qquad \widehat{S}_B^{\lambda,\mu\nu} = 0$$

- ⤳ gives exactly symmetric Hilbert $T^{\mu\nu}$ acting as the source of gravity in GR
- ⤳ long-standing problem of physical significance of the spin tensor
- ⤳ spin tensor is used by the community that studies the spin of proton
  X.S. Chen, X.F. Lu, W.M. Sun, F. Wang, T. Goldman, PRL 100 (2008) 232002;
  E. Leader, C. Lorce, Phys. Rep. 541 (2014) 163.

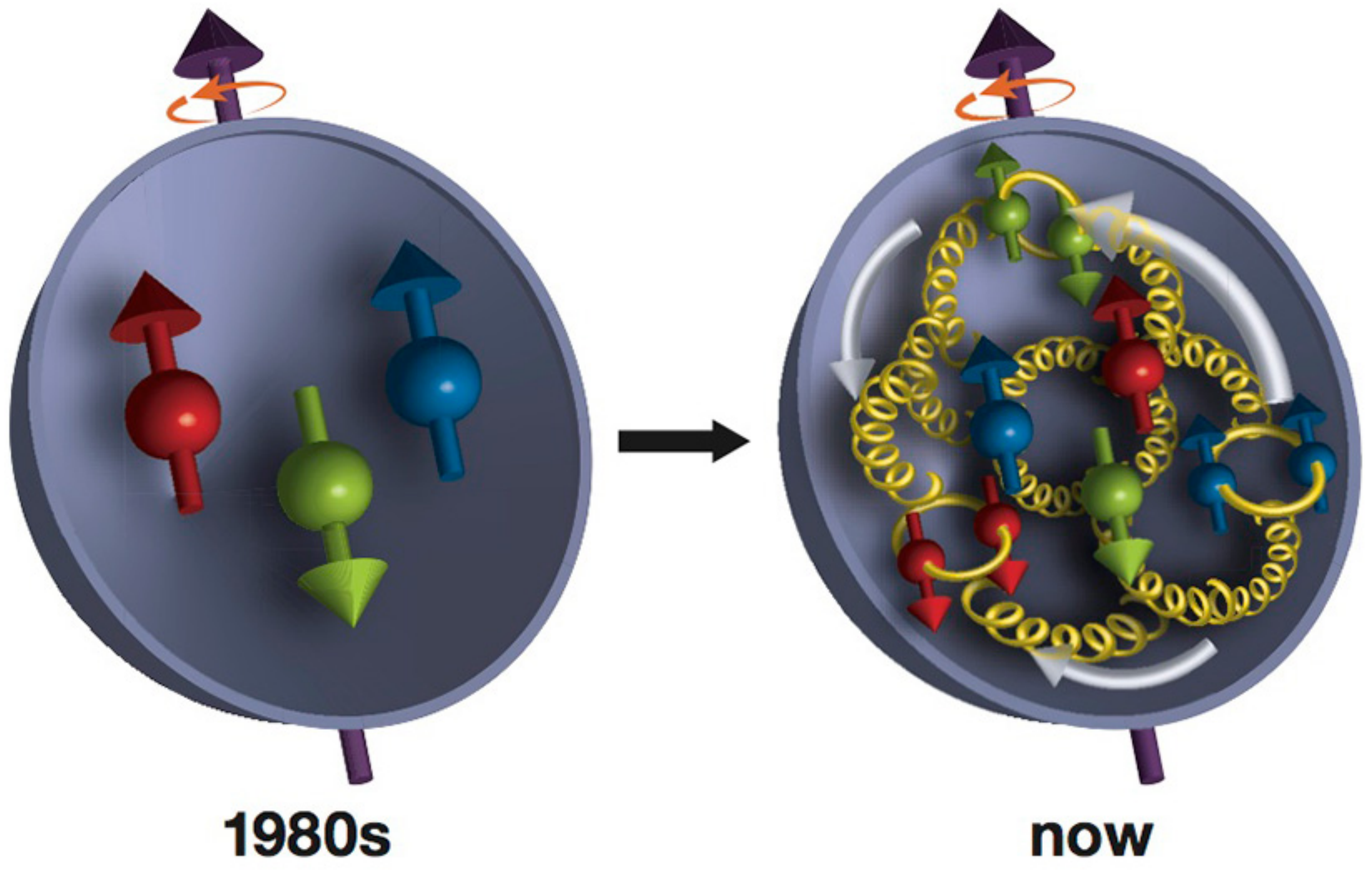


figure: Physics World

Ideal fluid dynamics with spin

If the energy-momentum tensor is symmetric the spin tensor is conserved

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017
F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425
W. Florkowski, A. Kumar, R. R., Prog. Part. Nucl. Phys. 108 (2019) 103709

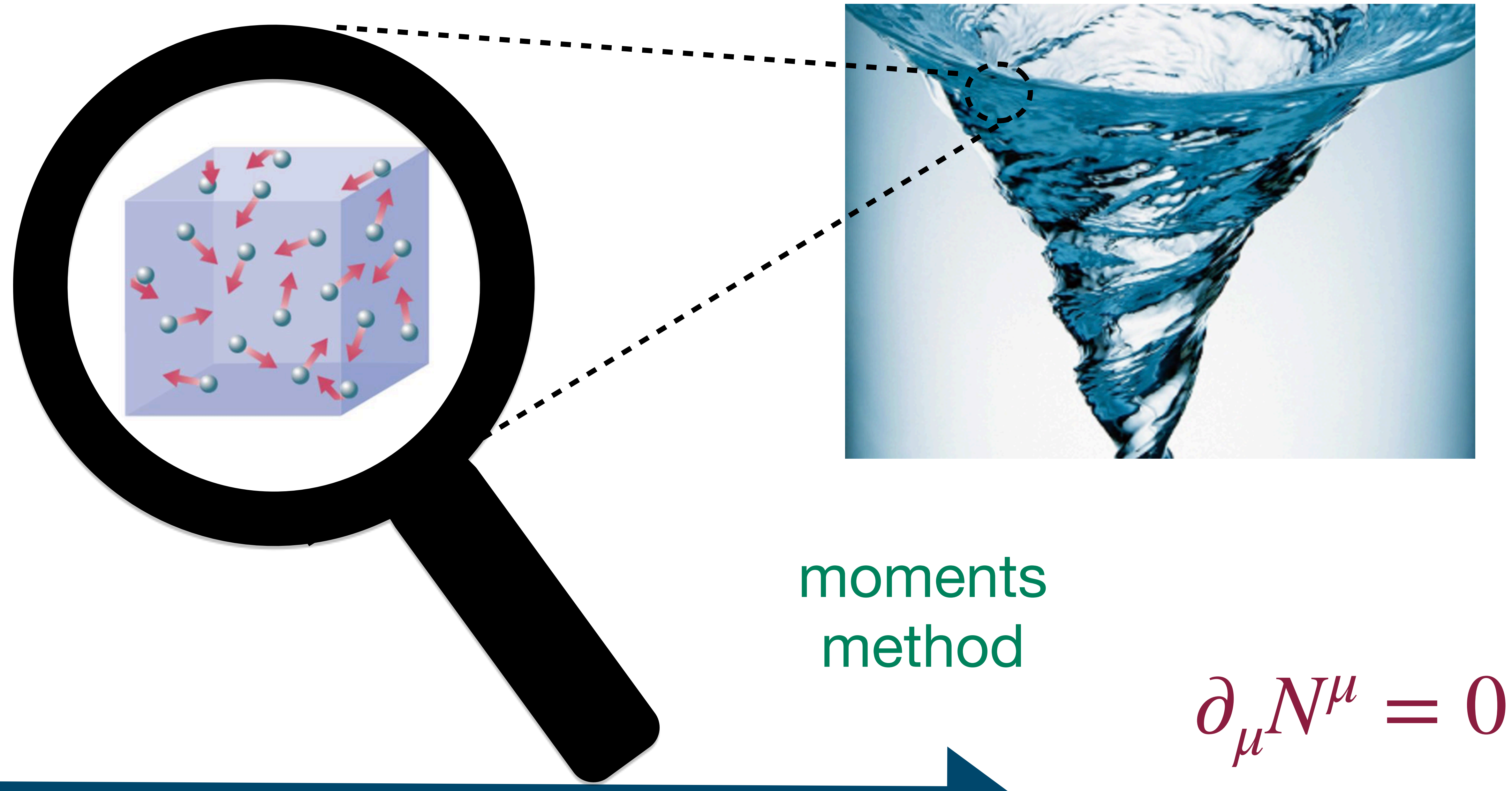
$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

What are the constitutive relations which enter equations of motion?

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Fluid dynamics with spin should tell how the spin chemical potential evolves but not its origin — need for modeling of initial conditions!

Relativistic kinetic theory formulation of perfect fluid dynamics



For dilute systems, the fluid dynamics can be derived from relativistic kinetic theory (RKT)

W. Florkowski, A. Kumar, and R. R., PRC 98, 044906 (2018)
W. Florkowski, A. Kumar, R. R., Prog. Part. Nucl. Phys. 108 (2019) 103709

classical RKT

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)]$$

moments method →

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

quantum RKT

$$\left(\gamma_\mu K^\mu - m\right) \mathscr{W}(x,k) = C[\mathscr{W}(x,k)]$$

$$K^\mu = k^\mu + \frac{i}{2}\left(\hbar \partial^\mu\right)$$

semi-classical expansion →

$$k^\mu \partial_\mu \mathscr{F}_{\rm eq}(x,k) = 0$$
$$k^\mu \partial_\mu \mathscr{A}^\nu_{\rm eq}(x,k) = 0$$

moments method →

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

LOCAL EQUILIBRIUM DISTRIBUTION FUNCTIONS

System without spin

$$f^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]$$

System with spin

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi , AP 338 (2013) 32
W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (11) (2018) 116017

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^+u_s(p)$$

$$f^-_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^-v_r(p)$$

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

De Groot, van Leeuwen, van Weert (GLW) pseudogauge

De Groot, van Leeuwen, van Weert: Relativistic Kinetic Theory. Principles and Applications, 1980.
W. Florkowski, A. Kumar, R. R., PRC 98 (2018) 044906

$$\mathcal{W}^+_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k-p)u^r(p)\bar{u}^s(p)f^+_{rs}(x,p)$$

$$\mathcal{W}^-_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k+p)v^s(p)\bar{v}^r(p)f^-_{rs}(x,p)$$

$$\mathcal{W}_{\rm eq}(x,k) = \mathcal{W}^+_{\rm eq}(x,k) + \mathcal{W}^-_{\rm eq}(x,k)$$

$$T^{\beta\alpha}_{\rm eq}(x) = T^{\alpha\beta}_{\rm eq}(x)$$

Spin is conserved separately!

Classical approach to spin hydrodynamics

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles M. Mathisson, APPB 6 (1937) 163-2900

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_\gamma s_\delta .$$

$s^{\alpha\beta}$ is antisymmetric *i.e.* $s^{\alpha\beta} = -s^{\beta\alpha}$ and satisfies Frenkel (or Weyssenhoff) $p_\alpha s^{\alpha\beta} = 0$.

The spin four vector can be obtained by above equation,

$$s^\alpha = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta}p_\beta s_{\gamma\delta}$$

In particle rest frame (PRF) where $p^\mu = (m, 0, 0, 0)$, $s^\alpha = (0, \mathbf{s}_*)$ with the length of spin vector given by $-s^2 = -s^\alpha s_\alpha = |\mathbf{s}_*|^2 = \mathfrak{s}^2 = \frac{1}{2}\left(1+\frac{1}{2}\right) = \frac{3}{4}$.


M.Mathisson


J. Weyssenhoff

Classical approach to spin hydrodynamics - perfect fluid

Distribution function in extended phase-space

W. Florkowski, R. R., A. Kumar, Prog. Part. Nucl. Phys. 108 (2019) 103709 ;

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)s^{\alpha\beta}\right)$$

Definitions of the currents

$$N^{\mu}_{\rm eq} = \int dP \int dS \; p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right]$$

$$T^{\mu\nu}_{\rm eq} = \int dP \int dS \; p^{\mu}p^{\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda\mu\nu}_{\rm eq} = \int dP \int dS \; p^{\lambda}s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

$$\int dS \ldots = \frac{m}{\pi \mathfrak{s}} \int d^4s \, \delta(s\cdot s + \mathfrak{s}^2)\, \delta(p\cdot s) \ldots$$

Explicit constitutive relations

$$N^{\alpha}_{\rm eq} = n u^{\alpha}$$

$$T^{\alpha\beta}_{\rm eq}(x) = \varepsilon u^{\alpha}u^{\beta} - P\Delta^{\alpha\beta}$$

$$S^{\lambda,\mu\nu}_{\rm eq} = S^{\lambda,\mu\nu}_{\rm GLW} = \mathcal{C}\left(n_0(T)u^{\lambda}\omega^{\mu\nu} + S^{\lambda,\mu\nu}_{\Delta\rm GLW}\right)$$

$$S^{\alpha,\beta\gamma}_{\Delta\rm GLW} = \mathcal{A}_0\, u^{\alpha}u^{\delta}u^{[\beta}\omega^{\gamma]}{}_{\delta} + \mathcal{B}_0\left(u^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_{\delta} + u^{\alpha}\Delta^{\delta[\beta}\omega^{\gamma]}{}_{\delta} + u^{\delta}\Delta^{\alpha[\beta}\omega^{\gamma]}{}_{\delta}\right)$$

Important extensions to dissipative systems using RTA

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R.R. PLB 814, 136096 (2021)
S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R.R, PRD 103, 014030 (2021).
S. Bhadury, J. Bhatt, A. Jaiswal, and A. Kumar,, EPJ ST 230, 655 (2021).
S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R.R, PRL 129 (2022) 19, 192301

Spin dynamics in Bjorken-expanding background

Decomposing into electric- and magnetic-like components one has

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$$

$$\kappa \cdot U = 0\,, \qquad \omega \cdot U = 0$$

One can introduce orthogonal basis $I \in \{U, X, Y, Z\}$ which allows us to write

$$\begin{aligned} \kappa^\alpha &= C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha \\ \omega^\alpha &= C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha \end{aligned}$$

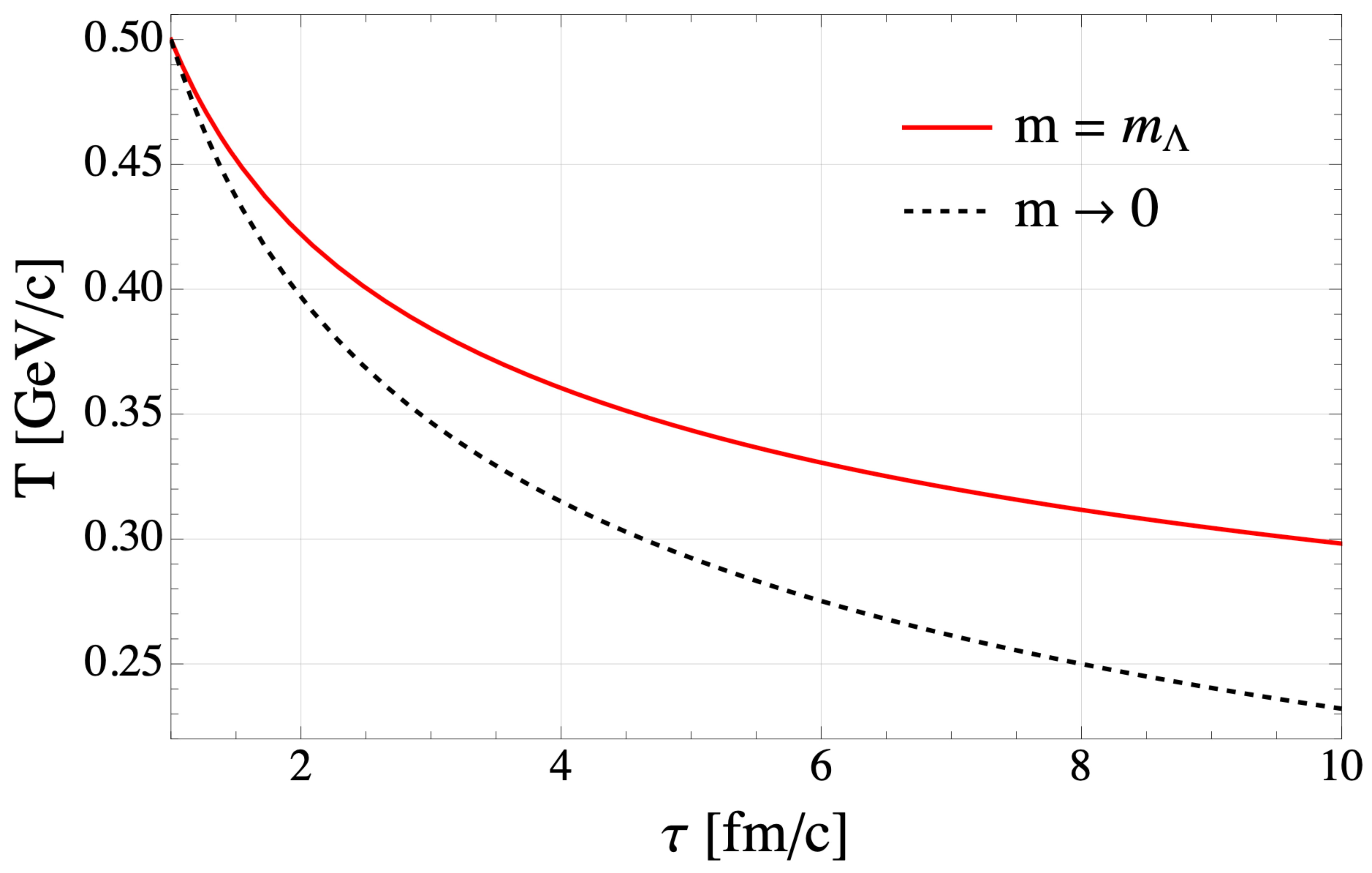
For Bjorken-expanding system we may choose

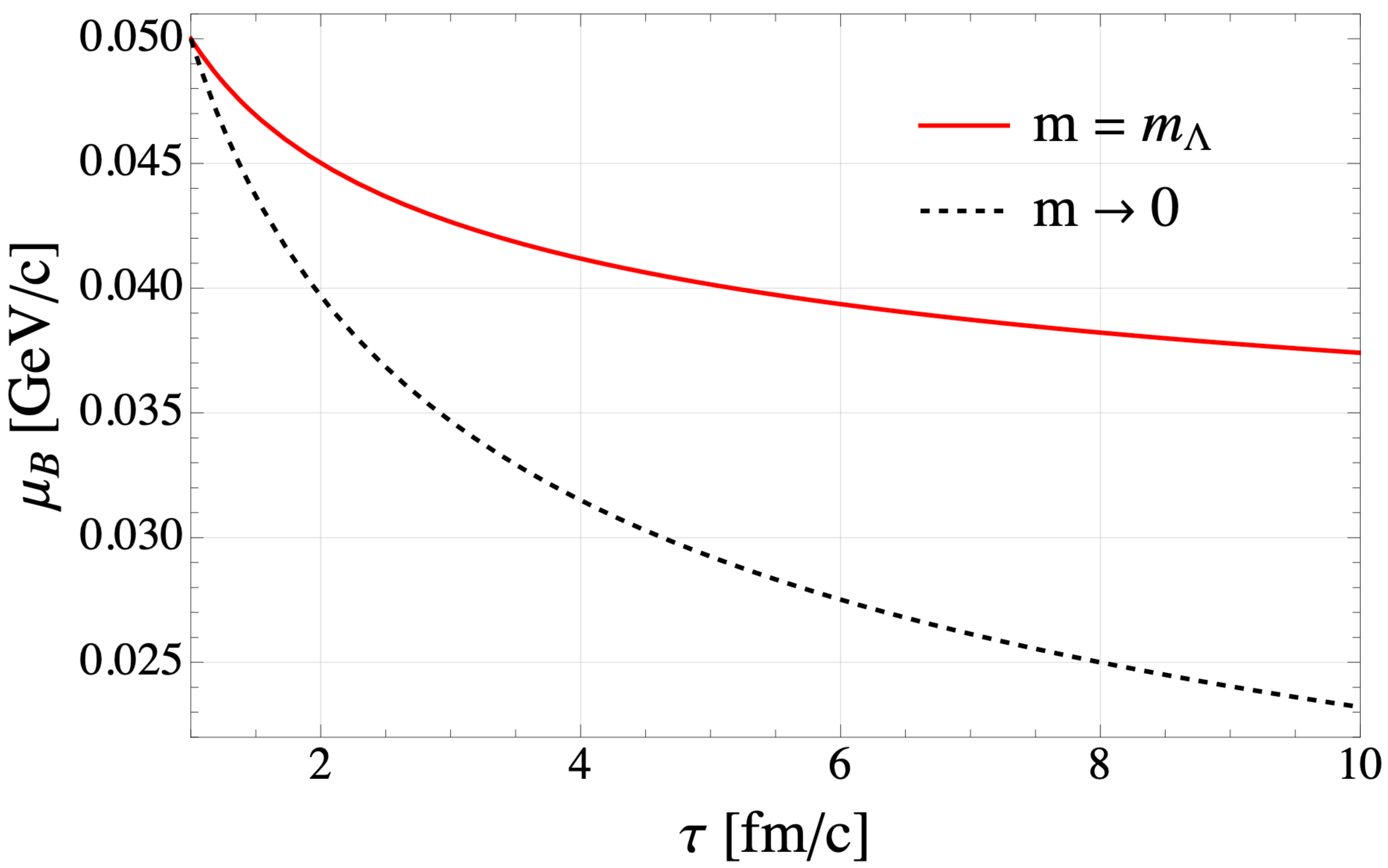
$$\begin{aligned} U^\alpha &= (\cosh(\eta),\, 0,\, 0,\, \sinh(\eta))\,, \qquad & Z^\alpha &= (\sinh(\eta),\, 0,\, 0,\, \cosh(\eta)) \\ X^\alpha &= (0,\, 1,\, 0,\, 0)\,, \qquad & Y^\alpha &= (0,\, 0,\, 1,\, 0)\,. \end{aligned}$$

Spin dynamics in Bjorken-expanding background

$$\frac{\partial \mathcal{N}}{\partial \tau} + \frac{\mathcal{N}}{\tau} = 0 \qquad \frac{\partial \mathcal{E}}{\partial \tau} + \frac{\mathcal{E} + \mathcal{P}}{\tau} = 0$$

The conservation laws for energy-linear momentum and baryon number yield well known results

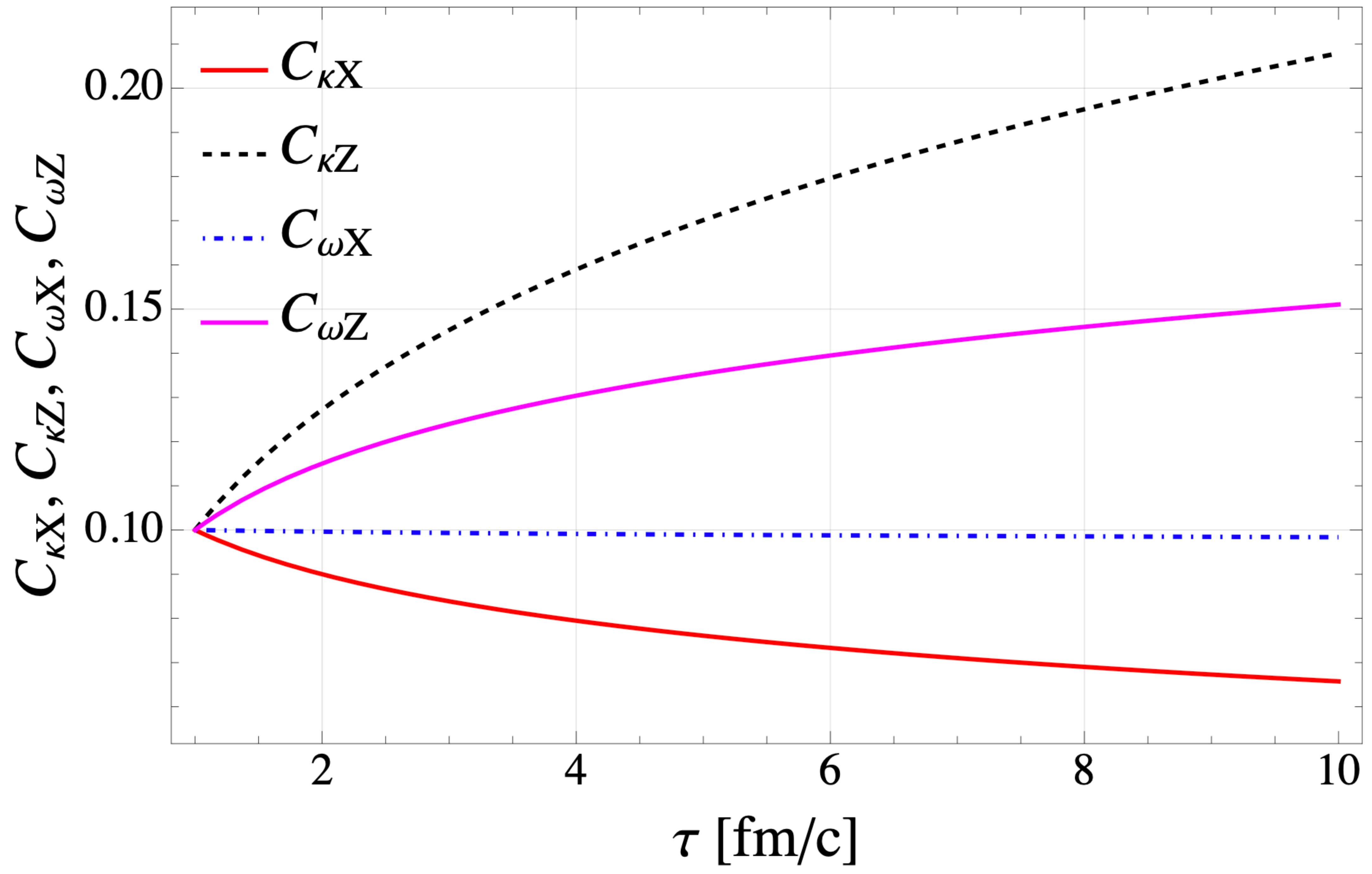




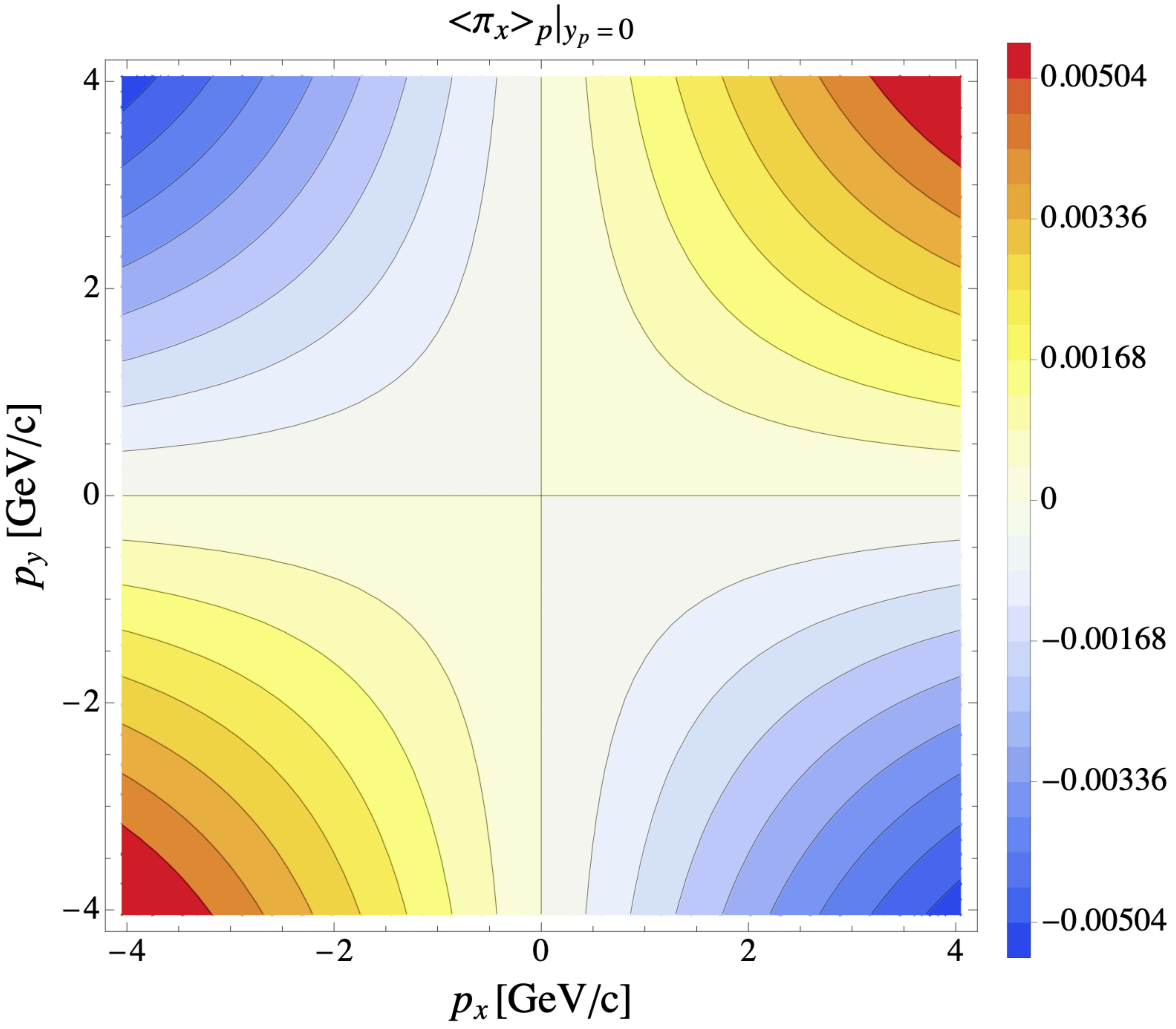
Spin dynamics in Bjorken-expanding background

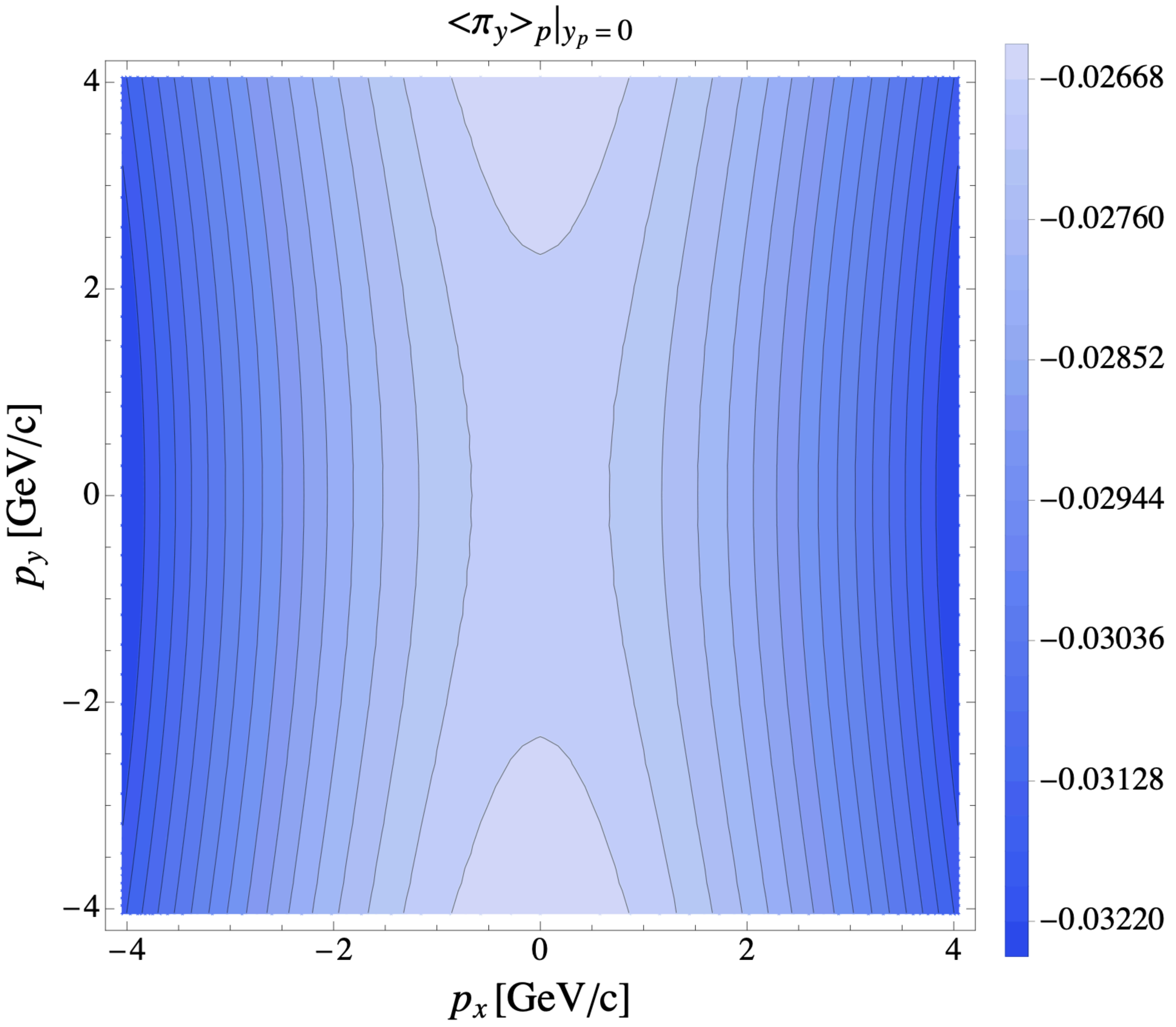
The conservation law for spin gives

$$
\begin{aligned}
\overset{\bullet}{\alpha}_{x1} &= -\alpha_{x1}\,\theta_U - \frac{\alpha_{x2}}{2}\,U\overset{\circ}{Z}\,, \\
\overset{\bullet}{\alpha}_{y1} &= -\alpha_{y1}\,\theta_U - \frac{\alpha_{y2}}{2}\,U\overset{\circ}{Z}\,, \\
\overset{\bullet}{\alpha}_{z1} &= -\alpha_{z1}\,\theta_U\,, \\
\overset{\bullet}{\beta}_{x2} &= -\beta_{x2}\,\theta_U - \beta_{x1}\,Z\overset{\circ}{U}\,, \\
\overset{\bullet}{\beta}_{y2} &= -\beta_{y2}\,\theta_U - \beta_{y1}\,Z\overset{\circ}{U}\,, \\
\overset{\bullet}{\beta}_{z2} &= -\beta_{z2}\,\theta_U\,,
\end{aligned}
$$



Spin dynamics in Bjorken-expanding background





$$\langle \pi_\mu \rangle = \frac{\int \mathrm{dP} \, \langle \pi_\mu \rangle_p \, E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int \mathrm{dP} \, E_p \frac{d\mathcal{N}(p)}{d^3p}} \equiv \frac{\int d^3p \, \frac{d\Pi^*_\mu(p)}{d^3p}}{\int d^3p \, \frac{d\mathcal{N}(p)}{d^3p}}$$

$$E_p \frac{d\Pi^*_\mu(p)}{d^3p} = -\frac{1}{(2\pi)^3 m} \int \cosh(\xi) \, \Delta\Sigma_\lambda \, p^\lambda \, e^{-\beta \cdot p} \, (\overset{\star}{\omega}_{\mu\beta} p^\beta)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh(\xi) \, \Delta\Sigma_\lambda \, p^\lambda \, e^{-\beta \cdot p}$$

Other developments towards hydrodynamics with spin

Lagrangian effective field theory approach

D. Montenegro, G. Torrieri, PRD 94 (2016) no.6, 065042; PRD 100, 056011 (2019)
D. Montenegro, L. Tinti, G. Torrieri, PRD 96(5) (2017) 056012; PRD 96(7) (2017) 076016

Hydrodynamics with spin based on entropy-current analysis

K. Hattori, M. Hongo, X-G Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100-106

Hydrodynamics of spin currents using presence of torsion

D. Gallegos, U. Gursoy, A. Yarom SciPost Phys. 11 (2021) 041

Relativistic viscous hydrodynamics with spin using Navier-Stokes type gradient expansion analysis

D. She, A. Huang, D. Hou, J. Liao, *Sci.Bull.* 67 (2022) 2265-2268

Relativistic viscous spin hydrodynamics from chiral kinetic theory

S. Shi, C. Gale, and S. Jeon, PRC 103, 044906 (2021)

Spin polarization generation from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, PRL 127, 052301 (2021), PRD 104, 016022 (2021)

Spin polarisation due to thermal shear

F. Becattini, M. Buzzegoli, and A. Palermo, *Phys.Lett.B* 820 (2021) 136519
S. Y. F. Liu and Y. Yin, *JHEP* 07 (2021) 188

Relativistic second-order dissipative spin hydrodynamics from the method of moments

N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, *PRD* 106 (2022) 9, 096014; *PRD* 106 (2022) 9, L091901

Summary and Outlook

The spin polarization provides a new probe of the QGP properties

The disagreements between spin-thermal approach and data motivates developments of dynamical models

The fluid dynamics with spin is a natural framework one should seek for QGP

Presented ideal spin hydro formulation is readily applicable

The theory is developing fast - future looks interesting!

Thank you for your attention!

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