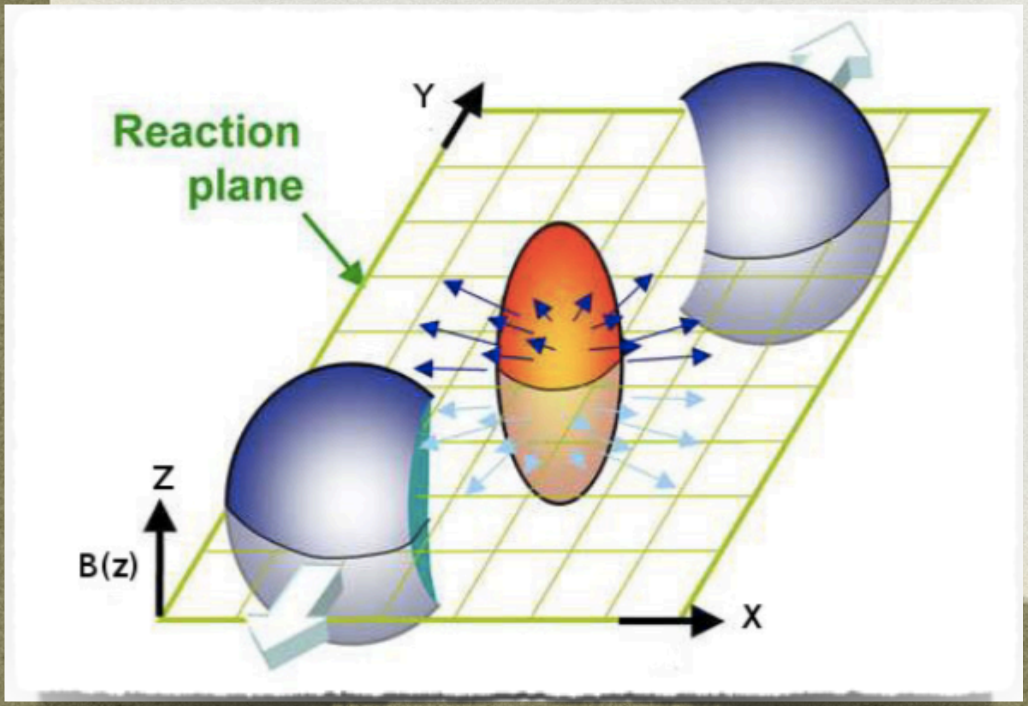
COLLECTIVE MODES AND INSTABILITIES IN ANISOTROPIC THERMO-MAGNETIC MEDIUM

Ritesh Ghosh



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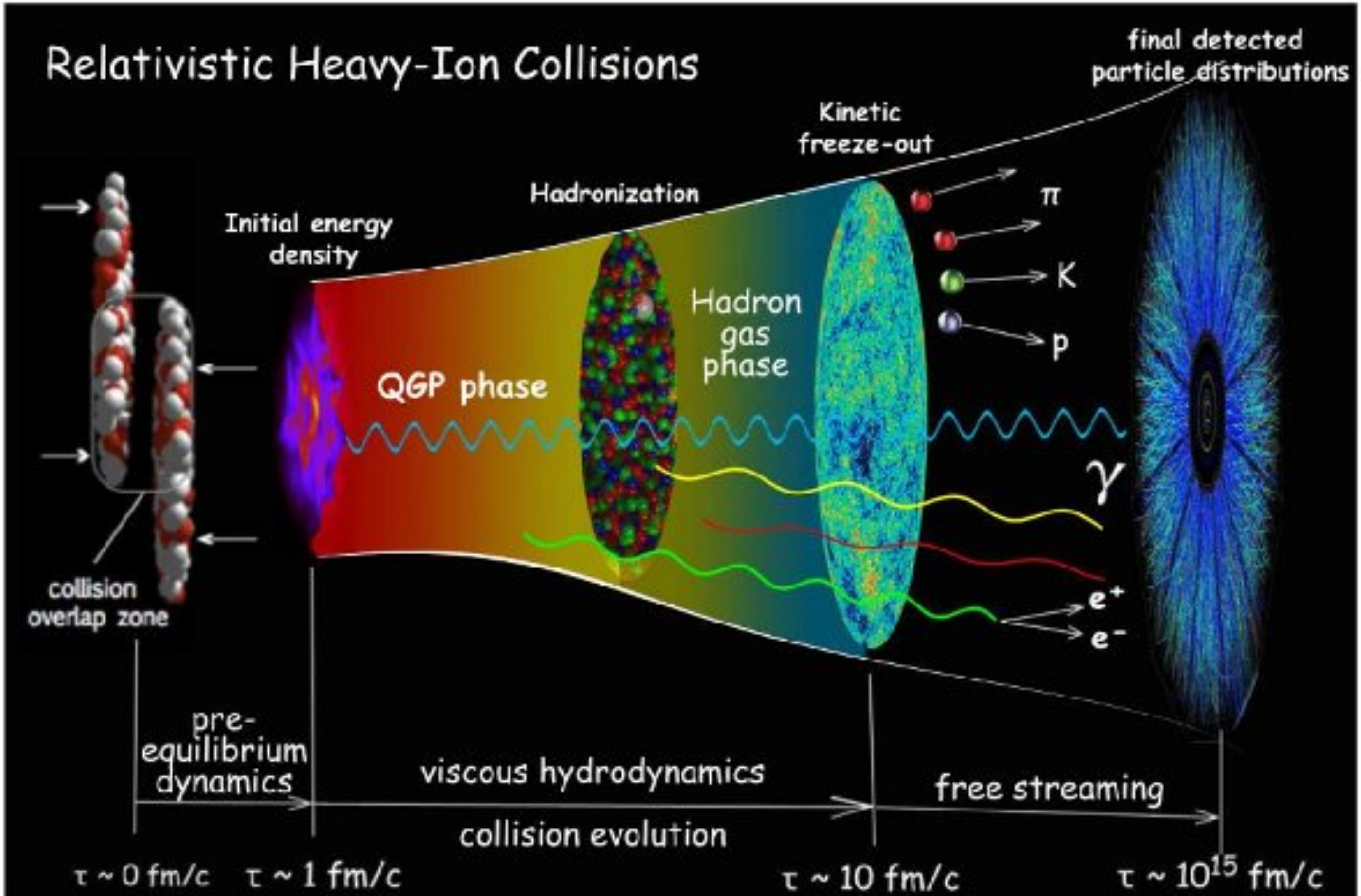

Relativistic Heavy-Ion Collisions
final detected
particle distributions
Kinetic
freeze-out
Hadronization
Initial energy
density
π
K
p
Hadron
gas
phase
QGP phase
γ
e+
e−
collision
overlap zone
pre-
equilibrium
dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Chun Shen

Momentum Space Anisotropy

- The deconfined quark-gluon plasma(QGP) matter produced in the heavy ion collision experiments is most likely to possess substantial deviation from perfect local isotropic equilibrium.

- QGP produced in URHIC is not momentum space isotropic.

Phys.Lett.B 314 (1993) 118-121

arXiv:1603.08946v2

JHEP08(2003)002

- The collective modes that possess a positive imaginary part in their mode frequencies result an exponential growth in the chromomagneic and chromoelectric fields.

- Romatschke and Strickland introduced an elegant Ansatz to model anisotropic distributions by squeezing or stretching isotropic ones

$$f_{\text{aniso}}(\mathbf{k}) \equiv f_{\text{iso}}\left(\frac{1}{\Lambda}\sqrt{\mathbf{k}^2 + \xi(\mathbf{k}\cdot\mathbf{n})^2}\right)$$

Phys. Rev. D **68**, 036004

$$f_{\text{aniso}}(\mathbf{k}) \equiv f_{\text{iso}}\left(\frac{1}{\Lambda}\sqrt{\mathbf{k}^2 + \xi_a(\mathbf{k}\cdot\mathbf{a})^2 + \xi_b(\mathbf{k}\cdot\mathbf{b})^2}\right)$$

PHYSICAL REVIEW D 97, 054022 (2018)

- The large momentum space anisotropy in early stages can be efficiently incorporated in the aHydro framework.

Phys. Rev. Lett. 119, 042301 (2017).

Kharzeev et al, 2008

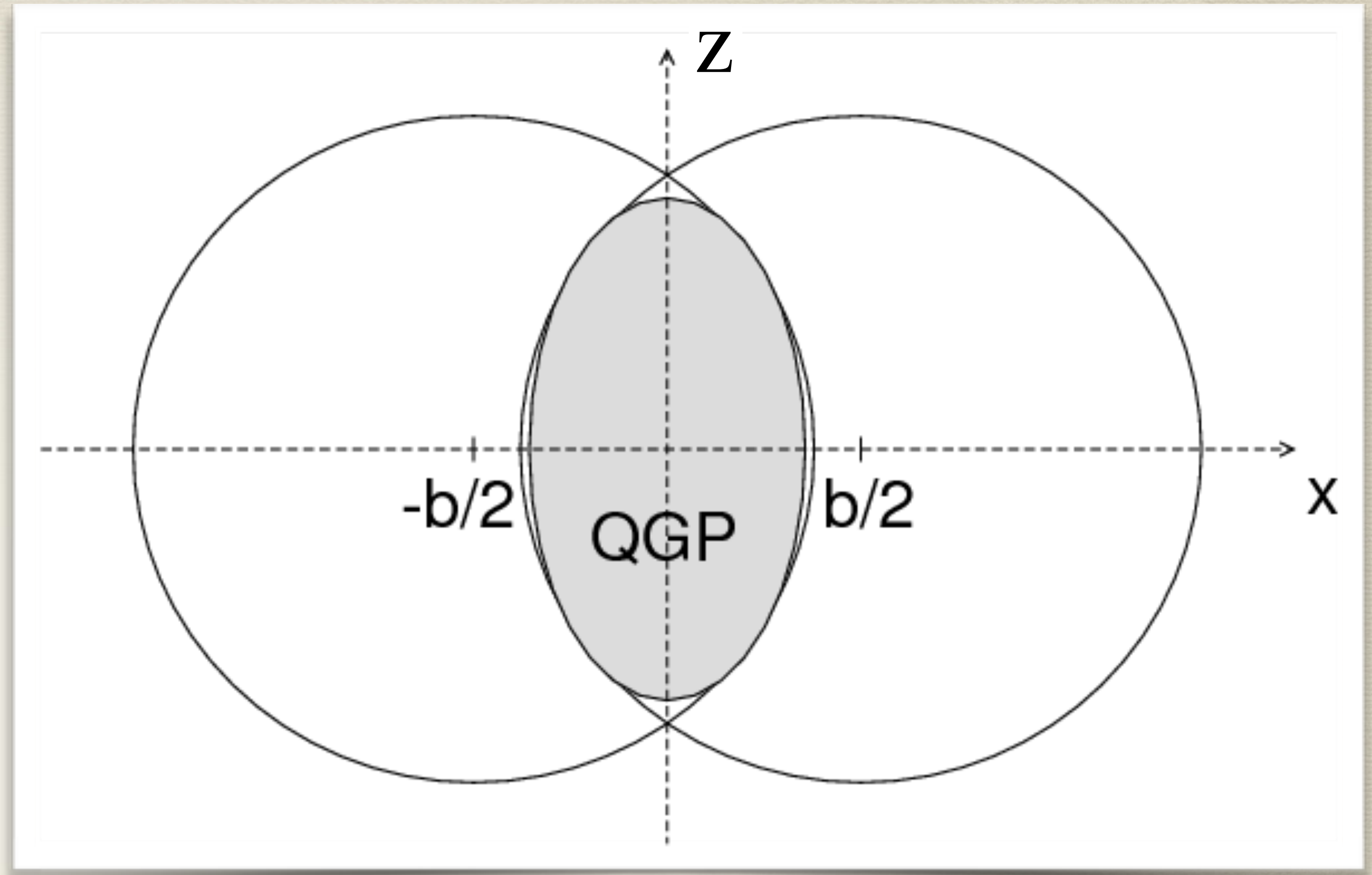
Skokov et al 2009

Magnetic field

Non-central heavy ion collision

Magnetic field strength $(10-30)m_\pi^2$

Decreases rapidly $(1-2)m_\pi^2$ after $(4-5)fm/c$



$m_\pi^2 \sim 10^{18}G$

$1fm/c = 3.3 \times 10^{-24}\ sec$

RHIC

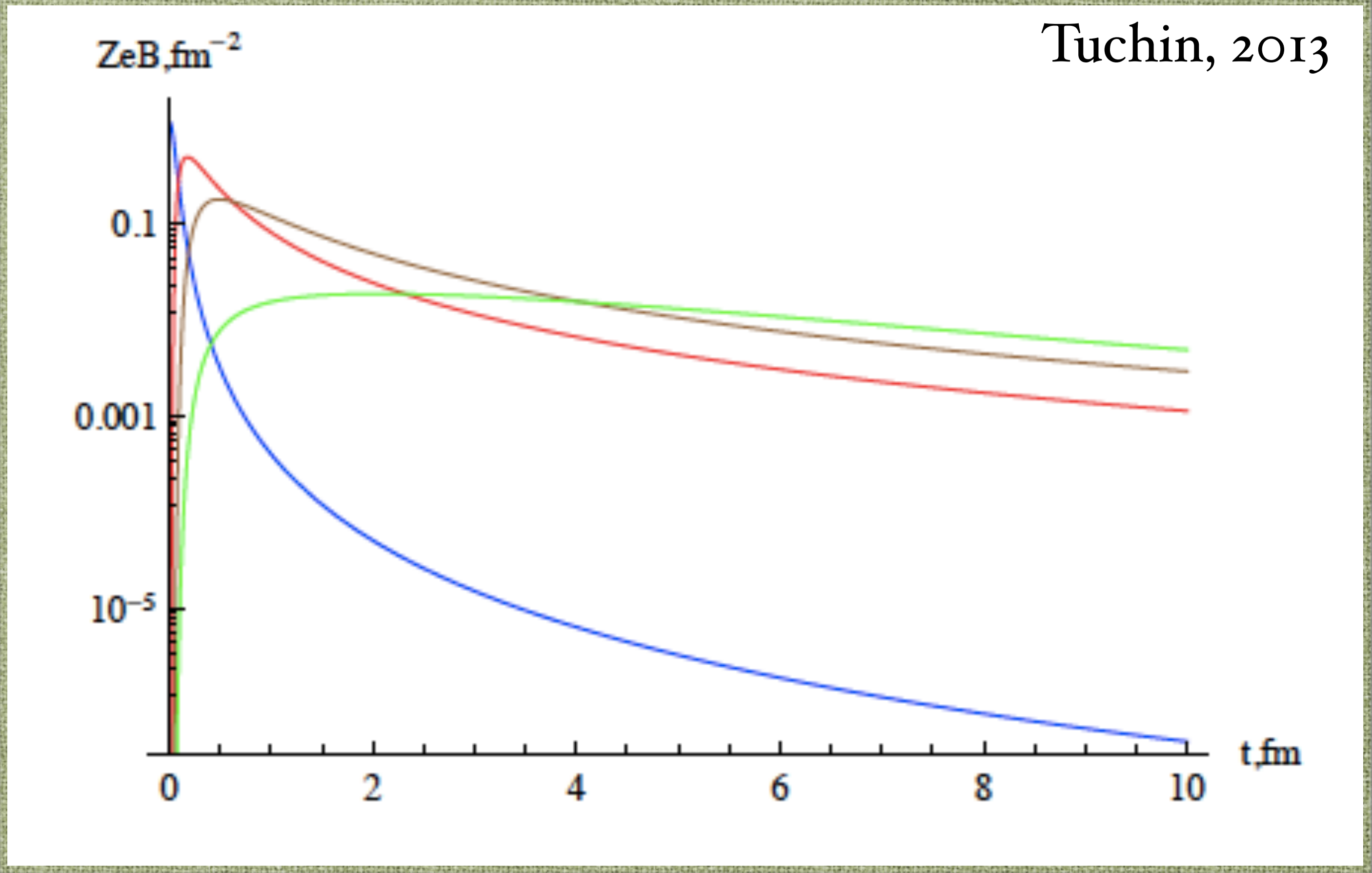
$\sim 10^{18} G$

Earth magnetic field

$\sim 10^{-1} G$

Magnetic field in core of neutron star

$\sim (10^{10} - 10^{13})\ G$



In vacuum (**blue line**)

In expanding medium (**Green line**)

In Static conducting medium (other lines)

So, the production of strong magnetic field at early stage of heavy ion collisions motivates to investigate the magnetic field effects on anisotropic QGP.

FORMALISM

Constructing general structure of the gauge boson self-energy

R.Ghosh, B. Karmakar, A. Mukherjee PHYSICAL REVIEW D 102, 114002 (2020)

- Anisotropic momentum distribution characterized by two independent four vectors a^μ and b^μ.

- Heat bath velocity u^μ and gluon momentum P^μ.

- Set of ten independent symmetric tensors:

  $P^\mu P^\nu, u^\mu u^\nu, b^\mu b^\nu, a^\mu a^\nu, P^\mu u^\nu + P^\nu u^\mu, P^\mu b^\nu + P^\nu b^\mu, P^\mu a^\nu + P^\nu a^\mu,$

  $u^\mu b^\nu + b^\mu u^\nu, u^\mu a^\nu + u^\nu a^\mu$ and $b^\mu a^\nu + a^\mu b^\nu$

- The transversality condition $P^\mu \Pi_{\mu\nu} = 0$ further reduce the number of independent basis tensors to six.

- In the rest frame of the heat bath with $u^\mu = (1,0,0,0)$, one of the anisotropy directions can be taken along z, say $b^\mu = (0,0,0,1)$, whereas the other anisotropy direction can be assumed to lie in the xz plane without any loss of generality.

- The general structure of the gauge boson self-energy in vacuum

$$\Pi^{\mu\nu} = \left(\eta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)\Pi(P^2) = V^{\mu\nu}\Pi(P^2)\,.$$

- Using the tensor $V^{\mu\nu}$, we obtain $\tilde{u}^\mu = V^{\mu\nu}u_\nu$. First basis tensor $A^{\mu\nu} = \dfrac{\tilde{u}^\mu\tilde{u}^\nu}{\tilde{u}^2}$.

- $U^{\mu\nu} = V^{\mu\nu} - A^{\mu\nu}$ is used to obtain $\tilde{b}^\mu$ defined as $\tilde{b}^\mu = U^{\mu\nu}b_\nu$ such that it becomes orthogonal to $\tilde{u}^\mu$ by construction.

$$B^{\mu\nu} = \frac{\tilde{b}^\mu\tilde{b}^\nu}{\tilde{b}^2}\,.$$

- $R^{\mu\nu} = U^{\mu\nu} - B^{\mu\nu}$. Then we obtain the $\tilde{a}^\mu$ from a^μ as

  $\tilde{a}^\mu = R^{\mu\nu} a_\nu$.

  $$D^{\mu\nu} = \frac{\tilde{a}^\mu \tilde{a}^\nu}{\tilde{a}^2}.$$

- All the four vectors of the set $\tilde{u}^\mu$, $\tilde{b}^\mu$ and $\tilde{a}^\mu$ are orthogonal to the gluon four momentum P^μ.

- $$C^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{b}^\nu + \tilde{b}^\mu \tilde{u}^\nu}{\sqrt{\tilde{u}^2}\sqrt{\tilde{b}^2}},\ E^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{a}^\nu + \tilde{a}^\mu \tilde{u}^\nu}{\sqrt{\tilde{u}^2}\sqrt{\tilde{a}^2}},\ F^{\mu\nu} = \frac{\tilde{a}^\mu \tilde{b}^\nu + \tilde{b}^\mu \tilde{a}^\nu}{\sqrt{\tilde{a}^2}\sqrt{\tilde{b}^2}}.$$

- The general structure of the gauge boson self-energy in presence of an ellipsoidal anisotropic medium can be expressed as a linear combination of the six basis tensors as

  $$\Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu} + \sigma E^{\mu\nu} + \lambda F^{\mu\nu}.$$

Effective propagator

- The Dyson-Schwinger equation $i\mathscr{D}^{\mu\nu} = i\mathscr{D}_0^{\mu\nu} + i\mathscr{D}_0^{\mu\rho}(i\Pi_{\rho\rho'})i\mathscr{D}^{\rho'\nu}$, where the inverse bare propagator is given by $(\mathscr{D}_0^{-1})^{\mu\nu} = -P^2\eta^{\mu\nu} - \frac{1-\zeta}{\zeta}P^\mu P^\nu$ with ζ representing the gauge fixing parameter.

- **Now the gluon propagator:**

$$\mathscr{D}^{\mu\nu} = -\frac{\beta\delta - (\beta+\delta)P^2 - \lambda^2 + P^4}{\Delta}A^{\mu\nu} - \frac{\delta\alpha - (\delta+\alpha)P^2 - \sigma^2 + P^4}{\Delta}B^{\mu\nu} - \frac{\gamma\left(P^2-\delta\right)+\sigma\lambda}{\Delta}C^{\mu\nu} - \frac{\alpha\beta - (\alpha+\beta)P^2 - \gamma^2 + P^4}{\Delta}D^{\mu\nu} - \frac{\sigma\left(P^2-\beta\right)+\lambda\gamma}{\Delta}E^{\mu\nu} - \frac{\lambda\left(P^2-\alpha\right)+\gamma\sigma}{\Delta}F^{\mu\nu} - \zeta\frac{P^\mu P^\nu}{P^4}$$

- Denominator of basis tensors:

$$\Delta = P^6 - (\alpha+\beta+\delta)P^4 - (\gamma^2+\sigma^2+\lambda^2-\alpha\beta-\beta\delta-\delta\alpha)P^2 + \alpha\lambda^2 + \beta\sigma^2 + \delta\gamma^2 - \alpha\beta\delta - 2\gamma\sigma\lambda\,.$$

- $\Delta = (P^2-\Omega_0)(P^2-\Omega_+)(P^2-\Omega_-)\,.$

- Once the form factors are extracted from the polarization function, the desired collective modes of the gluon can be obtained from the pole of the effective propagator.

One-Loop

- $\Pi^{\mu\nu}(p^0 = \omega, \mathbf{p}) = g_s^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{K^\mu}{E_k} \frac{\partial f(\mathbf{k})}{\partial K^\rho} \left[\eta^{\rho\nu} - \frac{K^\rho P^\nu}{P \cdot K + i0^+}\right]$
- Distribution function: $f(\mathbf{k}) = 2N_c n_g(\mathbf{k}) + N_f\left[n_q(\mathbf{k}) + n_{\bar{q}}(\mathbf{k})\right]$.
- In our case we use the ellipsoidal momentum distribution parametrized as $f_{\text{aniso}}(\mathbf{k}) \equiv f_{\text{iso}}\left(\frac{1}{\Lambda}\sqrt{\mathbf{k}^2 + \xi_a(\mathbf{k}\cdot\mathbf{a})^2 + \xi_b(\mathbf{k}\cdot\mathbf{b})^2}\right)$
- $\Pi^{\mu\nu}(\omega, \mathbf{p}, \xi) = m_D^2 \int \frac{d\Omega}{4\pi} v^\mu \frac{v^l + \xi_a(\mathbf{v}\cdot\mathbf{a})a^l + \xi_b(\mathbf{v}\cdot\mathbf{b})b^l}{(1 + \xi_a(\mathbf{v}\cdot\mathbf{a})^2 + \xi_b(\mathbf{v}\cdot\mathbf{b})^2)^2} \left[\eta^{\nu l} - \frac{v^\nu P^l}{\omega - \mathbf{p}\cdot\mathbf{v} + i0^+}\right]$
- $m_D^2 = (N_c + N_f/2)\frac{g_s^2\Lambda^2}{3}$ corresponds to the QCD Debye mass scale. Λ represents a temperature-like scale which, in the equilibrium limit, corresponds to the temperature.

One anisotropic case:

$$\Omega_0 = \frac{1}{2}\left(\alpha + \beta + \sqrt{(\alpha - \beta)^2 + 4\gamma^2}\right)$$

$$\Omega_+ = \frac{1}{2}\left(\alpha + \beta - \sqrt{(\alpha - \beta)^2 + 4\gamma^2}\right)$$

$$\Omega_- = \delta$$

Without anisotropy:

$$\beta = \delta = \Pi_T = \frac{m_D^2}{2}\frac{\omega^2}{p^2}\left[1 - \frac{\omega^2 - p^2}{2\omega p}\ln\frac{\omega + p}{\omega - p}\right]$$

$$\alpha = \Pi_L = \frac{m_D^2}{\tilde{u}^2}\left[1 - \frac{\omega}{2p}\ln\frac{\omega + p}{\omega - p}\right]$$

$$\Omega_0 = \alpha$$

$$\Omega_+ = \Omega_- = \beta = \delta$$

Small anisotropy

$$\alpha = \frac{1}{24}(\hat{p}_0^2 - 1)(-3\cos(2\theta_p)(\xi_1 - 2\xi_2)(6\hat{p}_0^2 + (3\hat{p}_0^2 - 2)\hat{p}_0(\log(\hat{p}_0 - 1) - \log(\hat{p}_0 + 1)) - 2)$$
$$+ 6\xi_1 \sin^2\theta_p(6\hat{p}_0^2 + (3\hat{p}_0^2 - 2)\hat{p}_0(\log(\hat{p}_0 - 1) - \log(\hat{p}_0 + 1)) - 2)\cos(2\phi_p)$$
$$- 3\hat{p}_0(\log(\hat{p}_0 - 1) - \log(\hat{p}_0 + 1))(\xi_1(\hat{p}_0^2 - 2) - 2\xi_2\hat{p}_0^2 + 4) + \xi_1(10 - 6\hat{p}_0^2) + 4(\xi_2 + 3\xi_2\hat{p}_0^2 - 6)),$$

$$\beta = \frac{1}{192}(2(\hat{p}_0^2 - 1)(\cos(2\theta_p)(42\hat{p}_0^2 + 3(7\hat{p}_0^2 - 5)\hat{p}_0(\log(\hat{p}_0 - 1) - \log(\hat{p}_0 + 1)) - 16)(\xi_1\cos(2\phi_p) + \xi_1 - 2\xi_2)$$
$$+ 3\hat{p}_0(\log(\hat{p}_0 - 1) - \log(\hat{p}_0 + 1))(\xi_1(\hat{p}_0^2 - 3) - 2\xi_2(\hat{p}_0^2 + 1) + 8) - 3\xi_1\hat{p}_0((3\hat{p}_0^2 - 1)\log(\hat{p}_0 - 1)$$
$$+ (1 - 3\hat{p}_0^2)\log(\hat{p}_0 + 1) + 6\hat{p}_0)\cos(2\phi_p)) + 4\hat{p}_0^2(-11\xi_1 - 2\xi_2 + 3\xi_1\hat{p}_0^2 - 6\xi_2\hat{p}_0^2 + 24)),$$

$$\gamma = -\frac{1}{12}\sqrt{\hat{p}_0^2 - 1}(11\hat{p}_0 - 12\hat{p}_0^3 + 3(1 - 5\hat{p}_0^2 + 4\hat{p}_0^4)\coth^{-1}(\hat{p}_0))\cos\theta_p(\xi_1 - 2\xi_2 + \xi_1\cos(2\phi_p))\sin\theta_p.$$

FERMION PROPAGATOR: WEAK AND STRONG FIELD APPROXIMATION

Strong field: lowest Landau level approximation

$$\sqrt{eB} > T$$

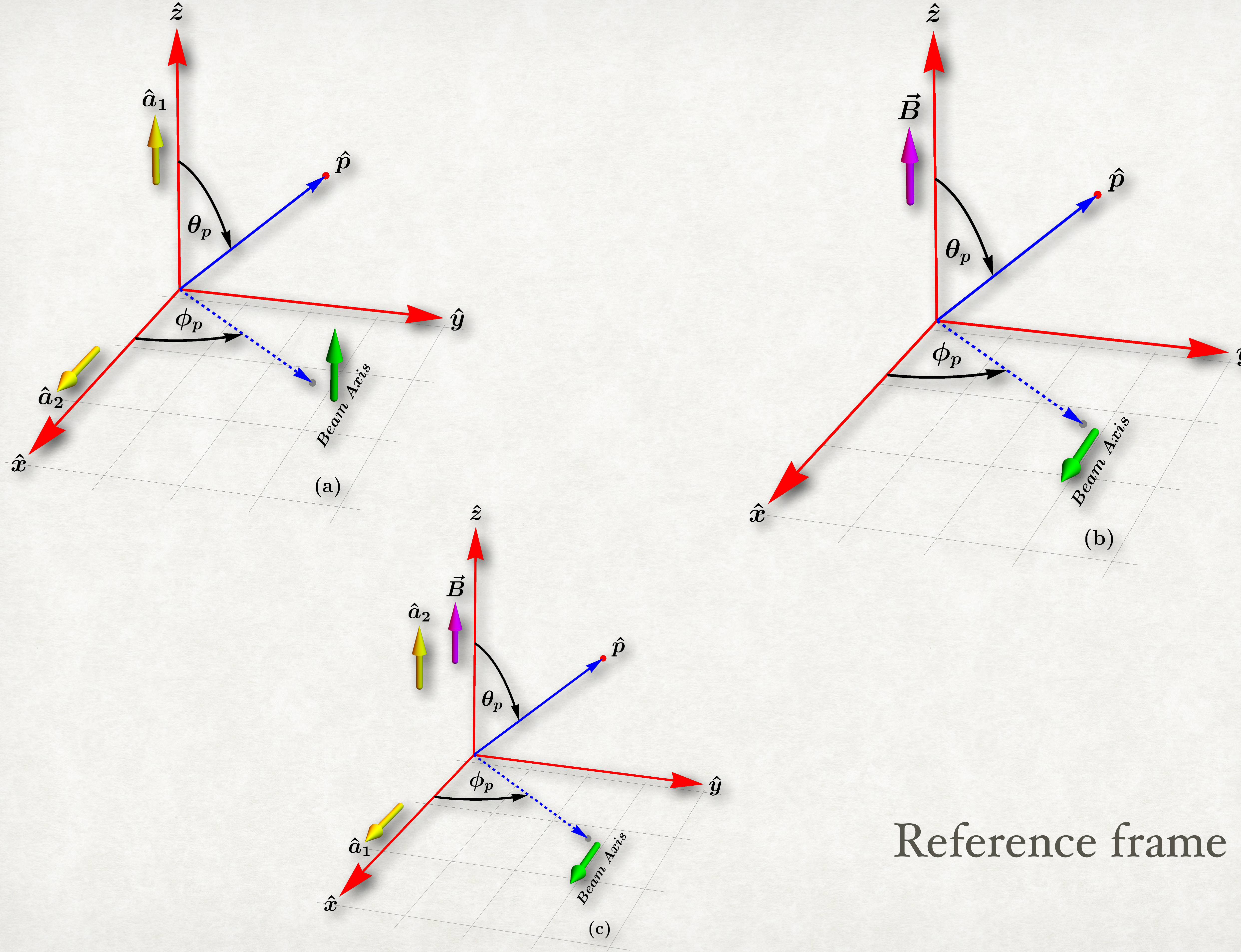
$$iS(K) = ie^{-\frac{k_\perp^2}{qB}} \frac{\not{K}_\parallel + m_f}{K_\parallel^2 - m_f^2}(1 - i\gamma^1\gamma^2)$$

$$K_\parallel^2 = k_0^2 - k_z^2$$

Weak field approximation: $\sqrt{eB} < m_{\rm th} \sim eT < T$

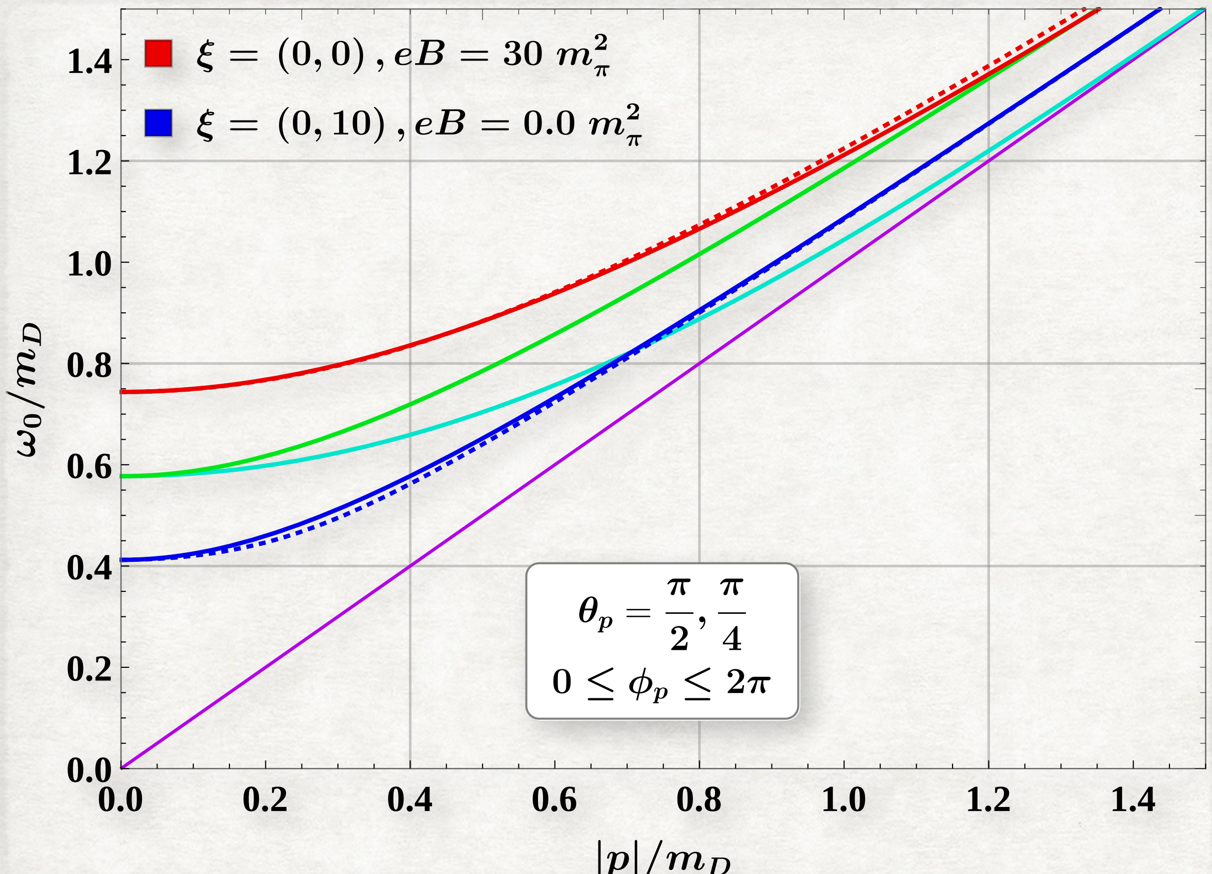
$$S(K) = \frac{\not{K} + m_f}{K^2 - m_f^2} + i\gamma^1\gamma^2 \frac{\not{K}_\parallel + m_f}{(K^2 - m_f^2)^2}(eB) + 2\left[\frac{\left\{(K\cdot u)\not{u} - (K\cdot n)\not{n}\right\} - \not{K}}{(K^2 - m_f^2)^3} - \frac{k_\perp^2(\not{K} + m_f)}{(K^2 - m_f^2)^4}\right](eB)^2 + \mathcal{O}\left[(eB)^3\right]$$

- One loop gluon self-energy in obtained using HTL approximation.
- $$\Pi^{\mu\nu} = \sum_f \frac{g^2}{2} \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu S(K) \gamma^\nu S(Q)]$$



Reference frame

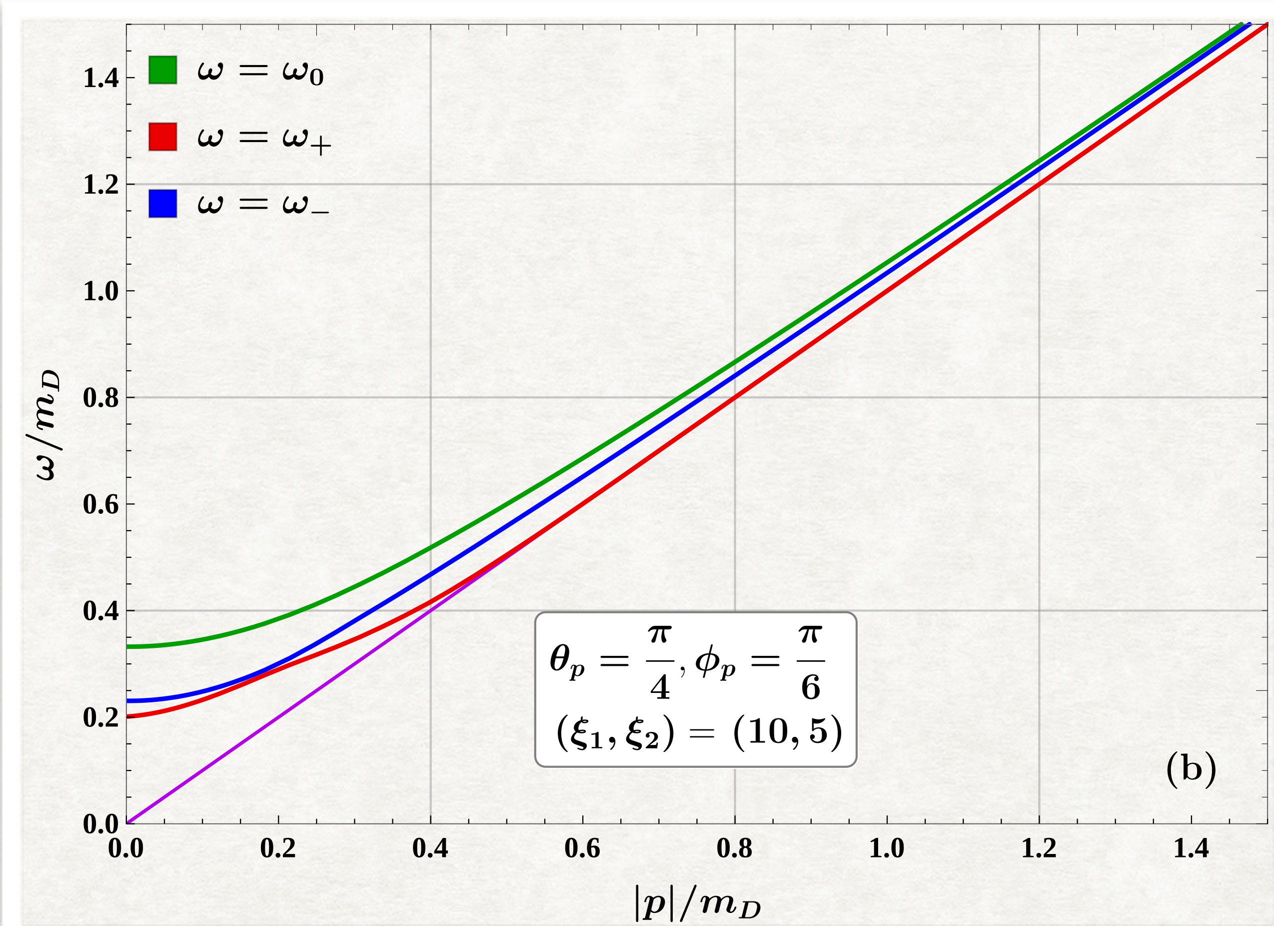
Collective modes



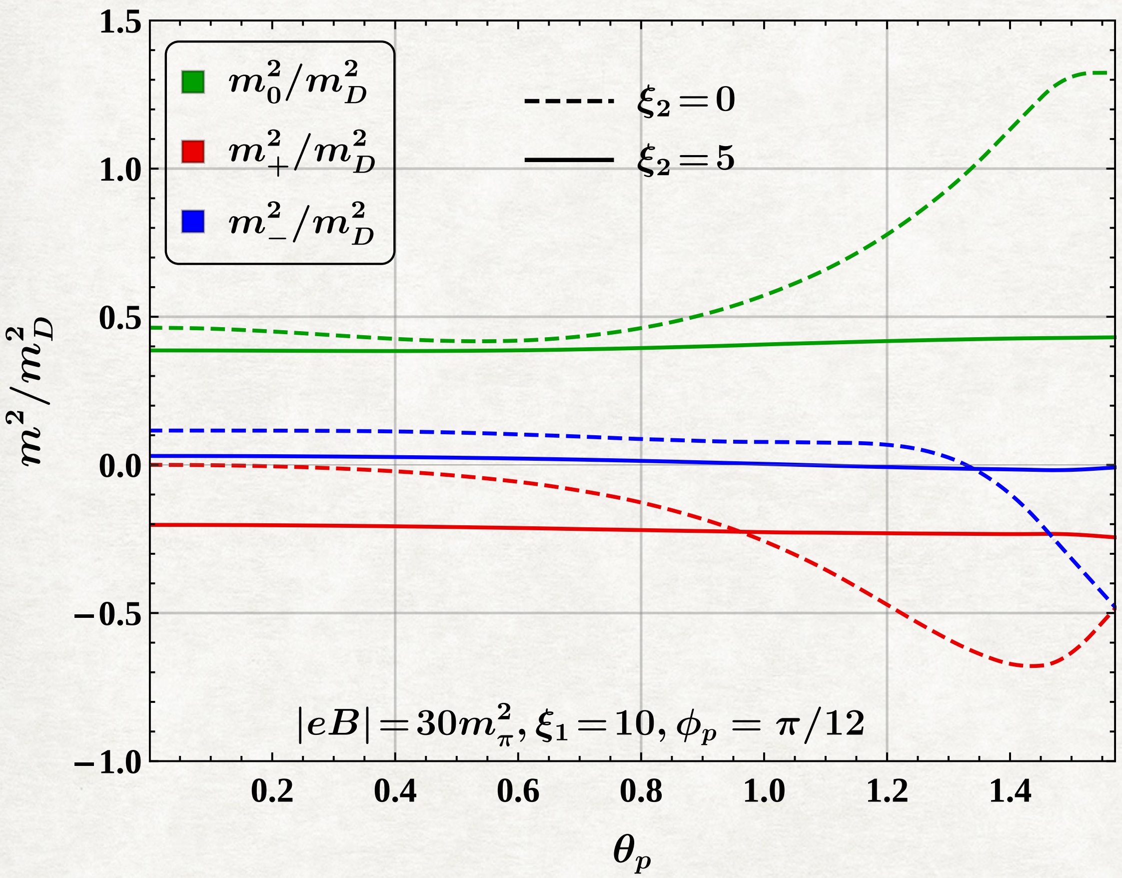
Solid line: $\theta_p = \pi/2$

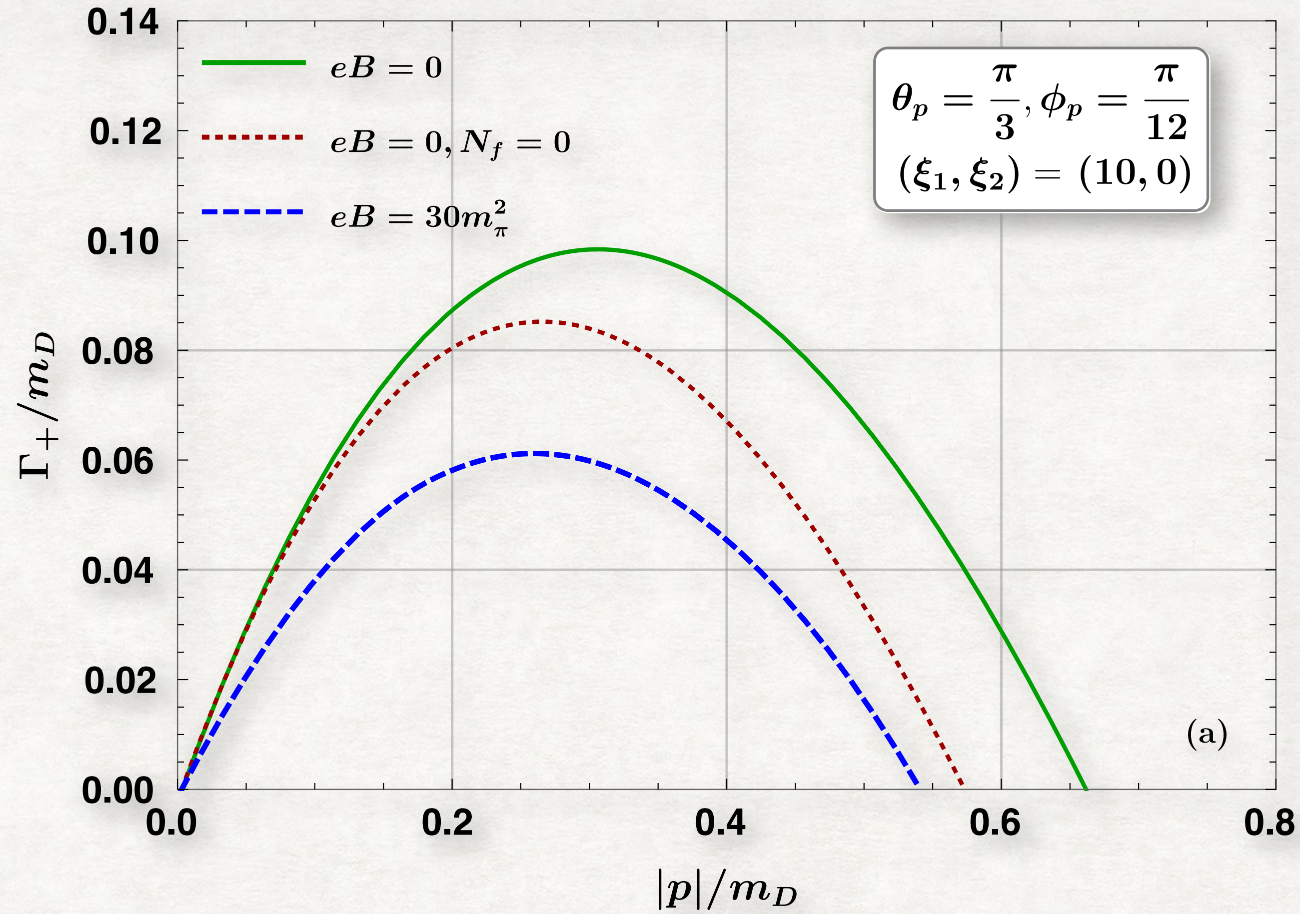
Dotted line: $\theta_p = \pi/4$

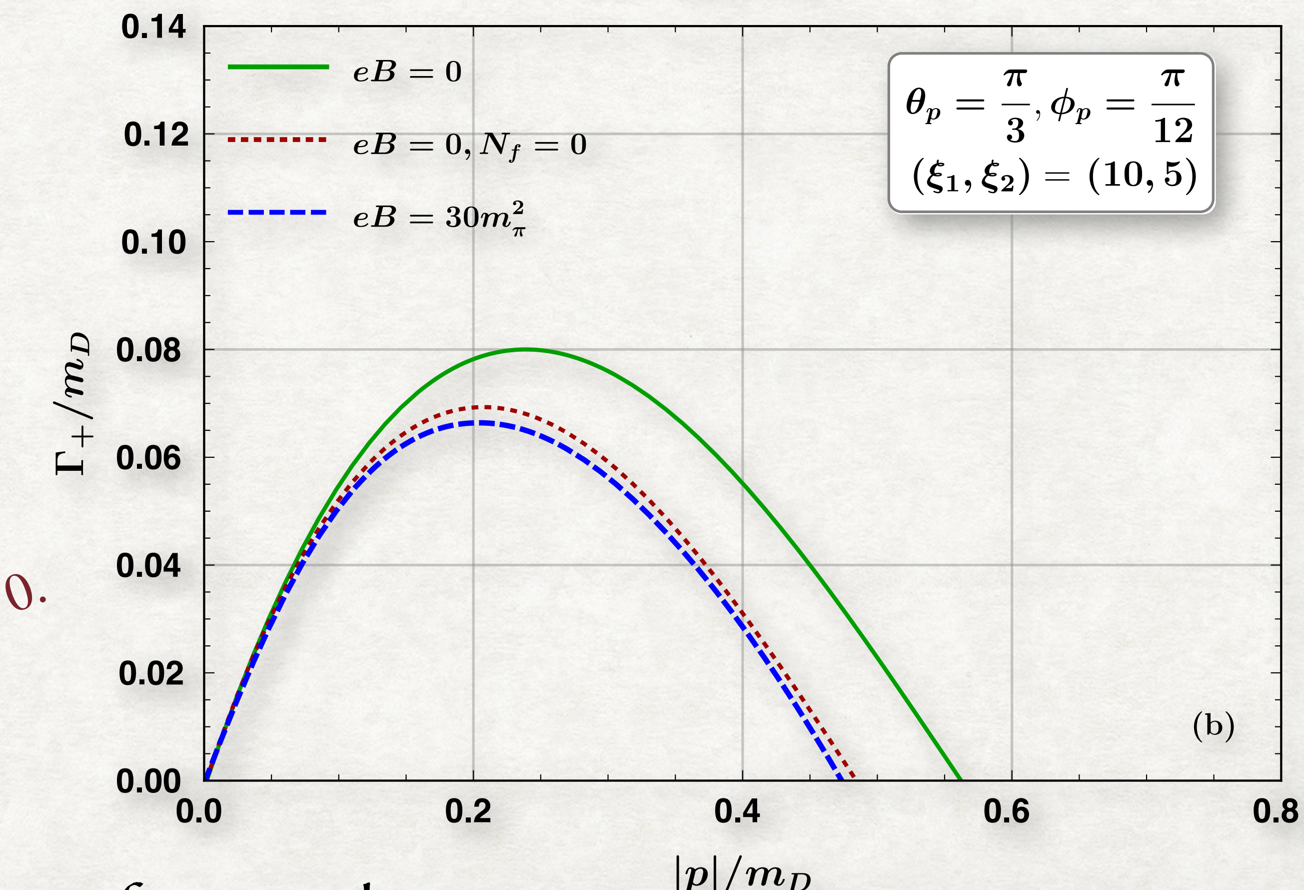
Green and Cyan: isotropic case



B.karmakar, R. Ghosh, A. Mukherjee *Phys.Rev.D* 106 (2022) 11, 116006







$$(\omega = i\Gamma_{\Omega\_})^2 - p^2 - \Omega\_(\omega = i\Gamma_{\Omega\_}, p, \theta_p, \phi_p) = 0.$$

Growth rate of Ω_+ mode

Summary

- The collective modes of gluon in the presence of momentum space anisotropy along with a constant background magnetic field have been studied using the hard-thermal loop (HTL) perturbation theory.
- Unstable modes are studied.
- No unstable gluon mode exists in an isotropic medium even in the presence of a background magnetic field. It is the momentum space anisotropy that gives rise to the instability.
- The external magnetic field has a significant influence on the growth rate of the unstable modes.
- In particular, the amplitude as well as the critical momentum corresponding to the growth rate of the unstable mode is significantly reduced in presence of strong magnetic background.

THANK YOU