Second order hydrodynamics based on effective kinetic theory and electromagnetic signals from QGP

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February 05, 2023

Emergent Topics in relativistic Hydrodynamics, Chirality, Vorticity and Magnetic field

Puri

- Results are presented in

  Second order hydrodynamics based on effective kinetic theory and electromagnetic signals from QGP,
  Lakshmi J. Naik and V. Sreekanth
  J. Phys. G. 50 025102 (2023)

Introduction

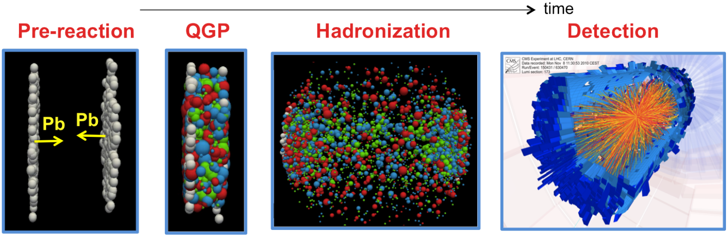

Figure: Schematic sketch of relativistic heavy ion collisions.[http://wl33.web.rice.edu/research.html]

- Quark-Gluon Plasma (QGP) has extreme low value of η/s $(= 1/4\pi)$. [KSS, Phys. Rev. Lett. 94, 111601 (2005)]
- The expansion of QGP can be modelled using relativistic viscous hydrodynamics.

Introduction

- **First-order Navier Stokes theory** $\longrightarrow$ acausal behaviour [W. A. Hiscock and L. Lindblom, Annals Phys. 151 466-96 (1983)]
- **Second order theories** $\longrightarrow$ no unique prescription to derive existing evolution equations for the dissipative quantities and there exist several successful formalisms.
- Recently, a new second order viscous hydrodynamics was developed within the effective fugacity quasiparticle model (EQPM) for the hot QCD medium [S. Bhadury et al., JPhysG 48, 105104 (2021)]

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- Recently, a new second order viscous hydrodynamics was developed within the effective fugacity quasiparticle model (EQPM) for the hot QCD medium [S. Bhadury et al., JPhysG 48, 105104 (2021)]

  $\rightarrow$ consistent analysis of evolution and particle spectra within this formalism

Effective fugacity Quasi-Particle Model

- Model initiates with an ansatz that the lattice QCD EoS can be interpreted in terms of non-interacting quasi-particles with effective fugacity parameters, $z_{q,g}$ encoding the interaction effects.

[V. Chandra and V. Ravishankar, EPJC 64, 63-72 (2009); PRD 84, 074013 (2011)]

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- Equilibrium momentum distribution functions of the quasi-particles are given by

$$f_k^0 = \frac{z_k \exp[-\beta(u_\mu p_k^\mu)]}{1 \pm z_k \exp[-\beta(u_\mu p_k^\mu)]},$$

$k \equiv (q, g)$ represent the quarks and gluons.

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Effective fugacity Quasi-Particle Model

- In this model, quasi-particle 4−momenta is given by the dispersion relation

$$\tilde{p}^{\mu}_{g,q} = p^{\mu}_{g,q} + \delta\omega_{g,q} u^{\mu}; \qquad \delta\omega_{g,q} = T^2 \partial_T \ln(z_{g,q})$$

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- Determination of equilibrium distribution function is achieved by fixing the temperature dependencies of z_k from lattice QCD EoS [M. Cheng et al., Phys. Rev. D 77, 014511 (2008); S. Borsanyi et al., Phys. Lett. B 730, 99104 (2014)]

Form of viscous correction

- Effect of viscosity is studied by applying small perturbation to the equilibrium distribution

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- Effective Boltzmann equation within the framework of EQPM [S. Mitra and V. Chandra, PRD 97, 034032 (2018)]

$$\tilde{p}_k^{\mu} \partial_{\mu} f_k^0(x, \tilde{p}_k) + F_k^{\mu} \partial_{\mu}^{(p)} f_k^0 = -\frac{\delta f_k}{\tau_R} \omega_k,$$

where τ_R is the relaxation time and $F_k^{\mu} = -\partial_{\nu}(\delta\omega_k u^{\nu} u^{\mu})$.

Form of viscous correction

δf is obtained from an iterative Chapman-Enskog like solution of the Boltzmann equation in RTA [S. Bhadury et al., JPhysG 48, 105104 (2021)]

$$\delta f_q = \tau_R \left[\tilde{p}_q^{\mu} \partial_\mu \beta + \frac{\beta\, \tilde{p}_q^{\mu}\, \tilde{p}_q^{\nu}}{u\cdot \tilde{p}_q} \partial_\mu u_\nu - \beta \Theta(\delta\omega_q) - \beta \dot{\beta} \left(\frac{\partial(\delta\omega_q)}{\partial \beta} \right) \right] f_q^0 \bar{f}_q^0$$

with $\bar{f}_q^0 = 1 - a\bar{f}_q^0$ and $a = \pm 1$ for quarks/gluons.

Hydrodynamic evolution equations

Shear stress tensor π^{μ} and bulk viscous pressure Π are expressed in terms of δf within EQPM as

$$
\begin{aligned}
\pi^{\mu\nu} &= \sum_k g_k \Delta^{\mu\nu}_{\alpha\beta} \int d\tilde{P}_k \tilde{p}^{\alpha}_k \tilde{p}^{\beta}_k \delta f_k \\
&\quad + \sum_k g_k \delta\omega_k \Delta^{\mu\nu}_{\alpha\beta} \int d\tilde{P}_k \tilde{p}^{\alpha}_k \tilde{p}^{\beta}_k \frac{\delta f_k}{E_k} \\
\Pi &= -\frac{1}{3} \sum_k g_k \Delta_{\alpha\beta} \int d\tilde{P}_k \tilde{p}^{\alpha}_k \tilde{p}^{\beta}_k \delta f_k \\
&\quad - \frac{1}{3} \sum_k g_k \delta\omega_k \Delta_{\alpha\beta} \int d\tilde{P}_k \tilde{p}^{\alpha}_k \tilde{p}^{\beta}_k \frac{\delta f_k}{E_k}
\end{aligned}
$$

$d\tilde{P}_k \equiv \frac{d^3\vec{p_k}}{(2\pi)^3\omega_k}$.

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Hydrodynamic evolution equations

- The evolution equations for shear stress tensor and bulk viscous pressure are obtained as

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\phi^{\langle\mu}\omega^{\nu\rangle\phi} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_\phi^{\langle\mu}\sigma^{\nu\rangle\phi} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu},$$

$$\dot{\Pi} + \frac{\Pi}{\tau_R} = -\beta_\Pi\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}.$$

Here, $\omega^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$ denotes the vorticity tensor.

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  Here, $\omega^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$ denotes the vorticity tensor.
- The second order transport coefficients are obtained in terms of different thermodynamic integrals

[S. Bhadury et al., JPhysG 48, 105104 (2021)]

1D Boost Invariant Flow

Geometry of QGP expansion: Bjorken's prescription to describe the evolution of QGP: [J. D. Bjorken, PRD 27, 140-151 (1983)]

- coordinates are parameterized using proper time $\tau = \sqrt{t^2 - z^2}$ and pseudo-rapidity $\eta_s = \frac{1}{2} \ln \left[\frac{t+z}{t-z} \right]$
- in the local rest frame of the fluid, $u^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s)$

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- in the local rest frame of the fluid, $u^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s)$
- Under these assumptions, hydrodynamic evolution equations become

$$
\begin{aligned}
\frac{d\epsilon}{d\tau} &= -\frac{1}{\tau}\left(\epsilon + P + \Pi - \pi\right), \\
\frac{d\pi}{d\tau} + \frac{\pi}{\tau_\pi} &= \frac{4}{3}\frac{\beta_\pi}{\tau} - \left(\frac{1}{3}\tau_{\pi\pi} + \delta_{\pi\pi}\right)\frac{\pi}{\tau} + \frac{2}{3}\lambda_{\pi\Pi}\frac{\Pi}{\tau}, \\
\frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_\Pi} &= -\frac{\beta_\Pi}{\tau} - \delta_{\Pi\Pi}\frac{\Pi}{\tau} + \lambda_{\Pi\pi}\frac{\pi}{\tau},
\end{aligned}
$$

$\pi = \pi^{00} - \pi^{zz}$.

Hydrodynamic evolution for Bjorken flow

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$$\tau_R = 2(\eta/s)/T, \quad 1.5(\eta/s)/T, \quad (\eta/s)/T$$

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$$\tau_R = 2(\eta/s)/T, \quad 1.5(\eta/s)/T, \quad (\eta/s)/T$$

- We use the lower bound of shear viscosity to entropy ratio: $\eta/s = 1/4\pi$

Evolution: Shear

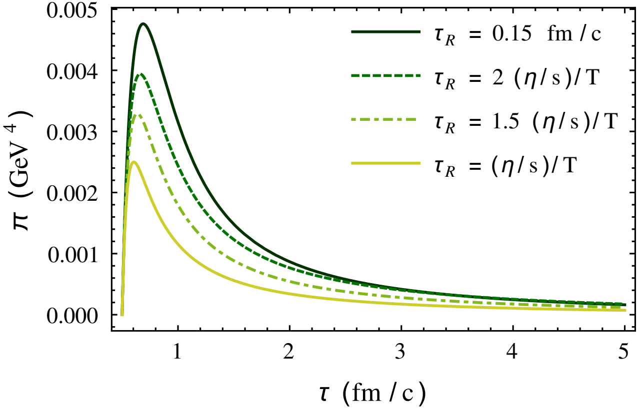

Figure: Proper time evolution of shear stress tensor for different temperature dependent forms of τ_R.

Evolution: Bulk

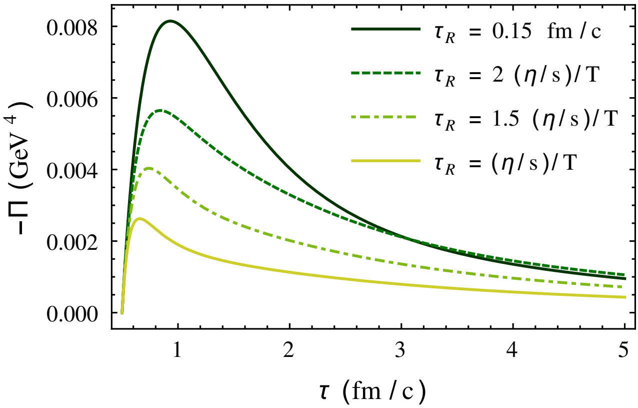

Figure: Proper time evolution of bulk viscous pressure for different temperature dependent forms of τ_R.

Evolution: Pressure anisotropy

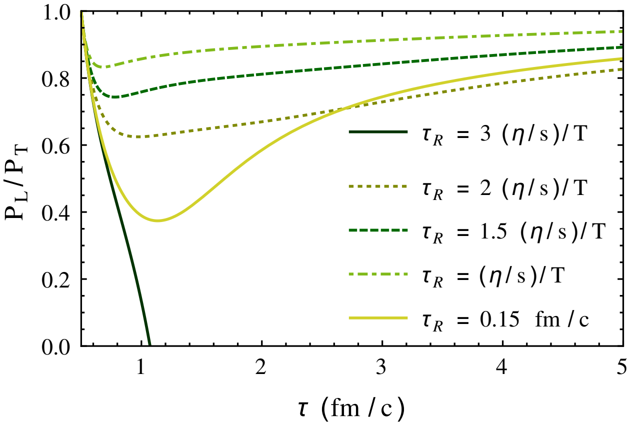

Figure: Proper time evolution of pressure anisotropy P_L/P_T with various temperature dependent relaxation times.

Comparison with a standard formulation

We take the second order hydrodynamics described in :
[A. Muronga, Phys. Rev. C 69, 034903 (2004)]

$$\begin{aligned} \frac{d\pi}{d\tau} &= -\frac{\pi}{\tau_\pi} - \frac{\pi}{2}\left(\frac{1}{\tau} + \frac{1}{\beta_2} T \frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right) + \frac{2}{3}\frac{1}{\beta_2}\frac{1}{\tau}, \\ \frac{d\Pi}{d\tau} &= -\frac{\Pi}{\tau_\Pi} - \frac{1}{2}\frac{\Pi}{\beta_0}\left(\frac{\beta_0}{\tau} + T\frac{d}{d\tau}\left(\frac{\beta_0}{T}\right)\right)\frac{1}{\beta_0}\frac{1}{\tau}, \end{aligned}$$

where β_0 and β_2 are related to the relaxation times as $\tau_\Pi = \zeta\beta_0$ and $\tau_\pi = 2\eta\beta_2$.

Comparison with a standard formulation

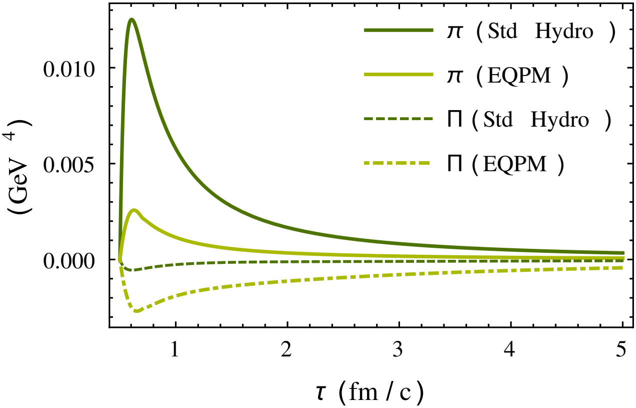

Figure: Comparison of evolution of shear and bulk viscous pressures obtained within EQPM with that obtained using the standard hydrodynamics.

Thermal Dileptons in QGP

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- In QGP the dominant mechanism for the production of thermal dileptons is $q\bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$.

Thermal dilepton production rate

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$$\frac{dN}{d^4x d^4p} = \iint \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3}\frac{M_{\rm eff}^2\, g^2\, \sigma(M_{\rm eff}^2)}{2\omega_1\omega_2} f(\vec{p}_1) f(\vec{p}_2)\delta^4(\tilde{p}-\tilde{p}_1-\tilde{p}_2).$$

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- $M_{\text{eff}}^2 = (\omega_1 + \omega_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$ represents the modified effective mass of the virtual photon in the interacting QCD medium.

Distribution function

- Viscous modified momentum distribution functions :
  $f(\vec{p}) \equiv f_q^0 + f_q^0 \bar{f}_q^0 \delta f_q$, where

$$\begin{aligned} \delta f &= \delta f_\pi + \delta f_\Pi \\ &= \frac{\beta}{2\beta_\pi (u\cdot\tilde{p})}\tilde{p}^\mu \tilde{p}^\nu \pi_{\mu\nu} + \frac{\beta\Pi}{\beta_\Pi}\Big[\xi_1 - \xi_2(u\cdot\tilde{p})\Big], \end{aligned}$$

  where

$$\begin{aligned} \xi_1 &= \beta c_s^2 \frac{\partial \delta\omega_q}{\partial\beta} + \delta\omega_q, \\ \xi_2 &= \left(c_s^2 - \frac{1}{3}\right) + \frac{\delta\omega_q}{3(u\cdot\tilde{p})^2}\left[2(u\cdot\tilde{p}) - \delta\omega_q\right]. \end{aligned}$$

Thermal dilepton production rate

Contribution due to shear and bulk viscosities:

$$\frac{dN^{(\pi)}}{d^4xd^4p} = \frac{dN^{(0)}}{d^4xd^4p}\Bigg\{\frac{\beta}{\beta_\pi}\frac{1}{2|\vec{p}|^5}\Bigg[\frac{(u\cdot\tilde{p})|\vec{p}|}{2}\left(2|\vec{p}|^2-3M_{\text{eff}}^2\right) + \frac{3}{4}M_{\text{eff}}^4\ln\left(\frac{(u\cdot\tilde{p})+|\vec{p}|}{(u\cdot\tilde{p})-|\vec{p}|}\right)\Bigg]\tilde{p}^\mu\tilde{p}^\nu\pi_{\mu\nu}\Bigg\},$$

$$\frac{dN^{(\Pi)}}{d^4xd^4p} = \frac{dN^{(0)}}{d^4xd^4p}\frac{2\beta\Pi}{\beta_\Pi}\Bigg\{\beta c_s^2\frac{\partial\delta\omega_q}{\partial\beta} - \frac{2}{3}\delta\omega_q - \left(c_s^2-\frac{1}{3}\right)\frac{(u\cdot\tilde{p})}{2} + \frac{\delta\omega_q^2}{3}\frac{1}{2|\vec{p}|^5}\Bigg[\frac{(u\cdot\tilde{p})|\vec{p}|}{2}\left(2|\vec{p}|^2-3M_{\text{eff}}^2\right) + \frac{3}{4}M_{\text{eff}}^4\ln\left(\frac{(u\cdot\tilde{p})+|\vec{p}|}{(u\cdot\tilde{p})-|\vec{p}|}\right)\Bigg]\Bigg\}.$$

Thermal dilepton yield

- Dilepton yield within Bjorken expansion is calculated as

$$\frac{dN}{dM^2 d^2p_T dy} = A_\perp \int_{\tau_0}^{\tau_f} d\tau\,\tau \int_{-\infty}^{\infty} d\eta_s\,\chi(T,\eta_s)\left(\frac{1}{2}\frac{dN}{d^4x d^4p}\right),$$

where $\chi(T,\eta_s) = \left[1 + \frac{2}{m_T}\cosh(y-\eta_s)\delta\omega_q\right]$.

- Total dilepton yield,

$$\frac{dN}{dM^2 d^2p_T dy} = \frac{dN^{(0)}}{dM^2 d^2p_T dy} + \frac{dN^{(\pi)}}{dM^2 d^2p_T dy} + \frac{dN^{(\Pi)}}{dM^2 d^2p_T dy}.$$

Dilepton Spectra

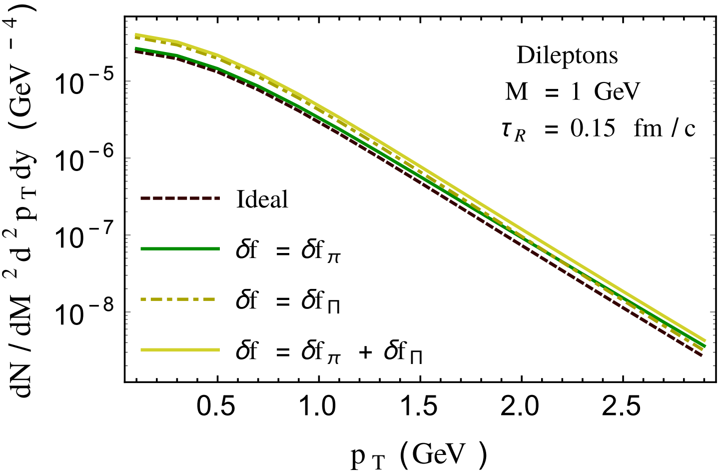

Figure: Thermal dilepton yields in the presence of viscous corrections corresponding to $\tau_R = 0.15$ fm/c and for $M = 1$ GeV.

Dilepton Spectra

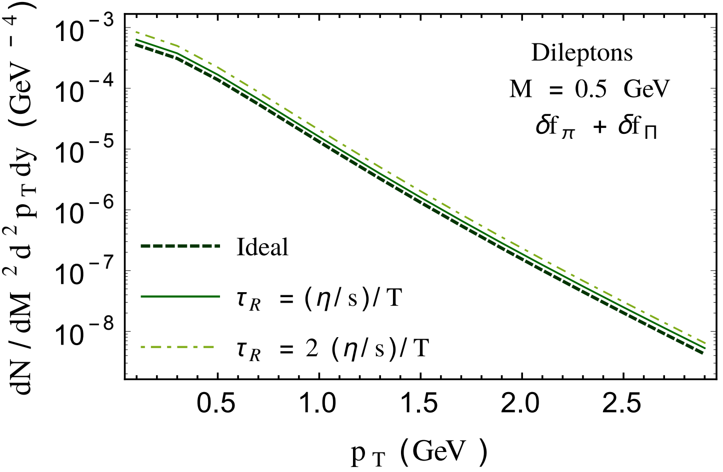

Figure: Dilepton spectra in the presence of viscous corrections by varying τ_R for $M = 0.5$ GeV

Dilepton Spectra

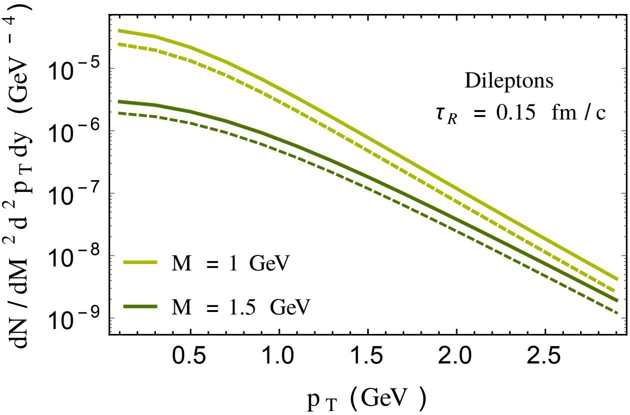

Figure: Comparison of dilepton spectra for different M values with $\tau_R = 0.15$ fm/c. The solid lines represent the total yields and dashed lines correspond to $\delta f = 0$ case.

Conclusions

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- The sensitivity of shear and bulk viscous pressures to the temperature dependence of relaxation time is analyzed within one dimensional boost invariant expansion of QGP
- Thermal particle production rate for QGP is calculated using viscous modified distribution functions
- Particle emission yields are quantified for the longitudinal expansion of QGP with different temperature dependent relaxation times.

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- Dilepton production within this second order hydrodynamic framework need to be studied by considering the tranverse dynamics of QGP expansion

Thank You

Back up

- Relaxation times : $\tau_\pi = b_1(\eta/s)/T$ and $\tau_\Pi = b_2(\zeta/s)/T$.
- $\eta/s = 1/4\pi$ and ζ/s as
  [I. Kanitscheider and K. Skenderis, JHEP 04, 062 (2009)]

$$\frac{\zeta}{s} = 2\frac{\eta}{s}\left(\frac{1}{3} - c_s^2\right) \equiv \kappa(T)\frac{\eta}{s}. \tag{1}$$

- RTA demands $\tau_\pi = \tau_\Pi = \tau_R$.