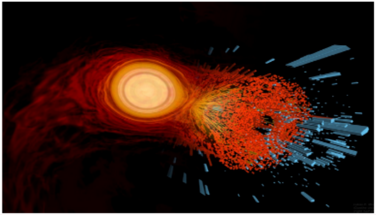

Relativistic dissipative hydrodynamics within extended relaxation time approximation

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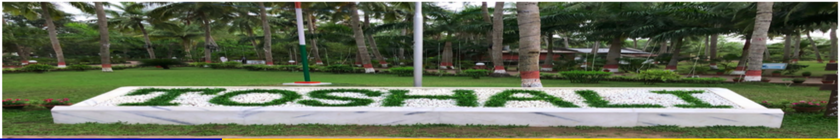

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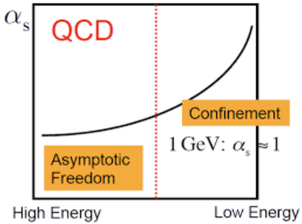
Motivation



Boltzmann Equation

$$p^\mu \partial_\mu f = \mathcal{C}[f] = -\frac{(u\cdot p)}{\tau_R}\delta f = \text{Collision kernel}$$

$$\tau_R \propto \frac{1}{\sigma(\text{cross section})} \propto \frac{1}{\alpha_s(\text{coupling strength})} \propto E$$

Take $\tau_R(p) = \chi(p) * (T/p)$

TABLE I. Summary of the functional dependence of the departure from equilibrium on the theory and approximation considered.

Model	Physics	Formula
Relaxation time, $\tau_R \propto p$	Relaxation time grows with particle momentum.	$\chi(p) \propto p^2$
Relaxation time, $\tau_R = \text{const}$	Relaxation time independent of momentum.	$\chi(p) \propto p$
Scalar theory	Randomizing collisions which happen rarely.	$\chi(p) \propto p^2$
QCD soft scatt.	Soft $q \sim gT$ collisions lead to a random walk of hard particles.	$\chi(p) \propto p^2$
QCD hard satt.	Hard $q \sim \sqrt{pT}$ collisions lead to a random walk of hard particles.	$\chi(p) \propto \frac{p^2}{\log(p/T)}$
QCD rad. energy loss	Radiative energy controls the approach to equilibrium. In the LPM regime, $\hat{q}$ controls the radiation rate.	$\chi(p) \propto \frac{p^{3/2}}{\alpha_s\sqrt{\hat{q}}}$

K Dusling,D. Moore,D.Teany,PHYSICAL REVIEW C 81, 034907 (2010)

Introduction

Aim

- Normally RTA is taken to be momentum independent i.e $\tau_R(x)$.
- Our aim is to consider momentum dependent AW RTA i.e $\tau_R(x, p)$ where $\tau_p = (\frac{u \cdot p}{T})^{\ell}$

Issue

- Use of Extended RTA in Boltzmann equation leads to the violation of energy momentum conservation.
- $u_\mu u_\nu \delta T^{\mu\nu} = \int \mathrm{dP} p^\mu p^\nu \delta f^{(1)} \neq 0$
- Choice of landau frame for hydro need not correspond to the thermal equilibrium system

Constraints

- Two conservation laws must be obeyed for a dissipative fluid system.
- Energy-momentum conservation $(\partial_\mu T^{\mu\nu} = 0)$and Particle current conservation $(\partial_\mu N^\mu = 0)$ respectively.
- First moment of Boltzmann equation must vanish to satisfy the energy- momentum tensor conservation.

$$\int \mathrm{dP} p^\mu p^\nu \partial_\mu f = 0$$

Our Approach

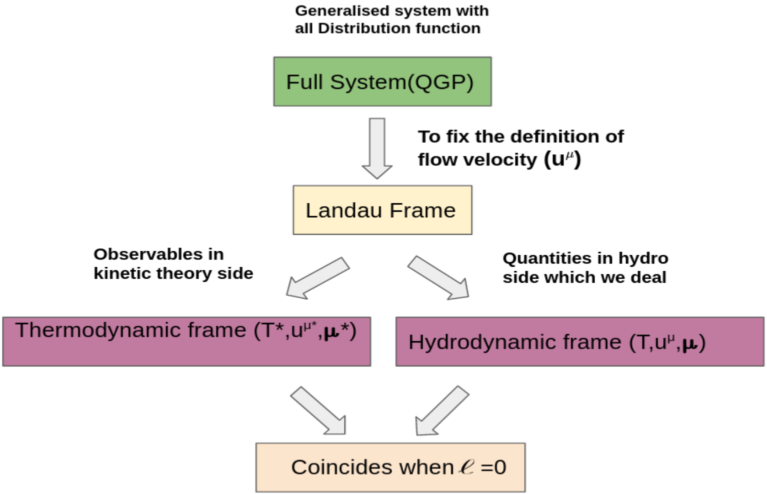
- Nonzero quantity of energy conservation can be compensated by the difference in by defining 2 different frames.
- The differences will be calculated using landau frame and matching condition.

$$u_\mu T^{\mu\nu} = \epsilon u^\nu, u_\mu u_\nu T^{\mu\nu} = \epsilon_0, u_\mu N^\mu = n_0$$

Phys. Rev. C 89, 014901 (2014), arXiv:1304.3753 [nucl-th]

What's special?

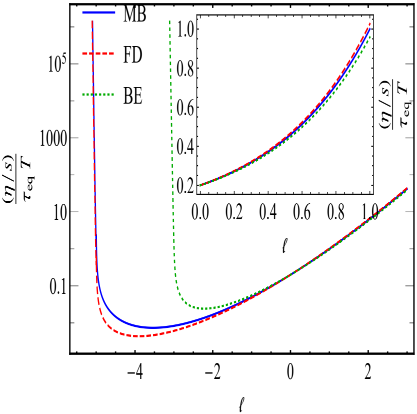
- An Approach by changing the form of AW RTA is already available to deal ERTA, where,conservation equations are satisfied by compromising the simple form of RTA. Phys. Rev. Lett. 127,042301 (2021), arXiv:2103.07489 [nucl-th].
- In our case we kept RTA as usual, but compromise by satisfying the conservation equation order by order in gradient expansion.



Flow chart to Introduce different frames

Theoretical view

- $\mathbf{u}^\mu$ is defined in hydro LRF and $u^{\mu *}$ is defined in thermodynamic LRF with $u_\mu u^\mu = 1$ and $u^*_\mu u^{\mu *} = 1$.
- $u^*_\mu \equiv u_\mu + \delta u_\mu \quad T^* \equiv T + \delta T \quad \mu^* \equiv \mu + \delta\mu \quad f^*_{\rm eq} \equiv f_{\rm eq} + \delta f^*$
- Boltzmann transport equation with Extended RTA is given by $p^\mu \partial_\mu f = \frac{-(u\cdot p)}{\tau_{\rm R}(x,p)}(f - f^*_{\rm eq}(u^*_\mu, T^*, \mu^*))$, $f^*_{\rm eq} = (e^{-\beta^*(u^*\cdot p)-\alpha^*} \pm a)^{-1}$ a=0,1,-1 for MB,FD,BE
- A order-by-order gradient expansion is followed here.
- Dissipative function in hydro (δf) is considered up to $\mathcal{O}(1)$.
- $f = f_{\rm eq} + \delta f$ then $f - f^*_{\rm eq} = f_{\rm eq} + \delta f - f^*_{\rm eq} = f_{\rm eq} + \delta f_{(1)} - f_{\rm eq} - \delta f^* = \delta f_{(1)} - \delta f^*$
- $$\delta f^* = f_{\rm eq} + \left(\frac{\partial f^*_{eq}}{\partial u^{\mu *}}\right)_{(u^\mu, T, \mu)} \delta u^\mu + \left(\frac{\partial f^*_{eq}}{\partial T*}\right)_{(u^\mu, T, \mu)} \delta T + \left(\frac{\partial f^*_{eq}}{\partial \mu *}\right)_{(u^\mu, T, \mu)} \delta\mu - f_{\rm eq}$$
  $$= -\frac{\delta u\cdot p}{T} + \frac{(u\cdot p)\delta T}{T^2} + \frac{\delta\mu}{T}$$
- $\tau_{\rm R}(x,p) = \tau_{\rm eq}(x)\tau_p(p) \quad \text{where} \quad \tau_p = \left(\frac{u\cdot p}{T}\right)^\ell$



Dependence of $(\eta/s\tau_{eq}T)$ on l for MB,FD and BE statistics.Inset plot shows the same within QGP range($0 < \ell < 1$).

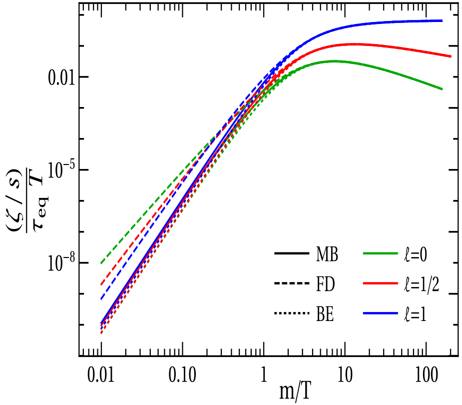
Analytically

$$\eta = \frac{\tau_{\rm eq}\, g\, T^4\, \Gamma(5+\ell)}{15\,\pi^2}\left[\frac{\mathrm{Li}_{4+\ell}(-a)}{-a}\right], \tag{1}$$

For this case, $\eta/(s\tau_{\rm eq}T)$ is just a function of ℓ and is given by

$$\frac{\eta}{s\tau_{\rm eq}\,T} = \frac{\Gamma(5+\ell)}{120}\left[\frac{\mathrm{Li}_{4+\ell}(-a)}{\mathrm{Li}_4(-a)}\right]. \tag{2}$$

Result-2(massive and chargeless case)



Variation of $\zeta/(s\tau_{\rm eq} T)$ with m/T for Maxwell-Boltzmann (MB), Fermi-Dirac (FD) and Bose-Einstein (BE) distributions for three different values of ℓ.

- The non-monotonous behaviour has qualitative agreement with Denicol's results.
- Non negative value of ζ/s confirms, not to violate the second law of thermodynamics.
- Quantum statistics ignore ± 1 in distribution function for high mass limit and approach to MB statistics.

Small and Large m/T behaviour

small-z expansion to study the scaling behaviour of ζ/η with the conformality measure, $1/3 - c_s^2$ and that leads to

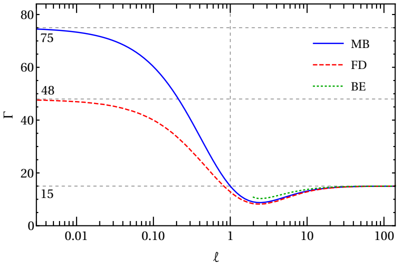
$$\frac{1}{3} - c_s^2 = \begin{cases} \frac{z^2}{36} + \mathcal{O}(z^3) & \text{MB}, \\ \frac{5z^2}{21\pi^2} + \mathcal{O}(z^3) & \text{FD}, \\ \frac{5z^2}{12\pi^2} + \mathcal{O}(z^3) & \text{BE}. \end{cases} \quad (3)$$

For MB and FD statistics, the quantity ζ/η in small-z limit has the leading behavior as

$$\frac{\zeta}{\eta} = \Gamma\left(\frac{1}{3} - c_s^2\right)^2, \quad (4)$$

where $\Gamma \equiv \lim_{z\to 0} \frac{\zeta/\eta}{\left(\frac{1}{3} - c_s^2\right)^2}$.

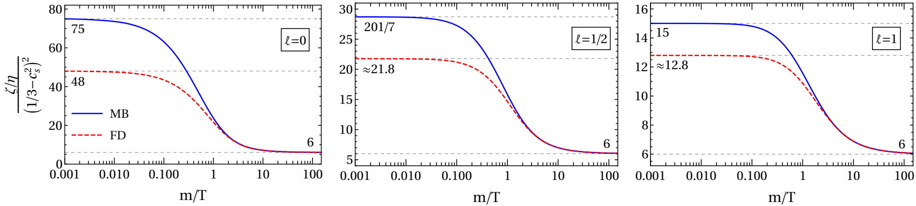
$$\Gamma_{\rm MB} = \frac{15\left(\ell^3 + 6\ell^2 - 13\ell + 30\right)}{(\ell+1)(\ell+2)(\ell+3)}, \quad (5)$$



Behavior of Γ, defined below Eq. (4), with ℓ MB,FD and BE equilibrium statistics.

Integration for massive particle

$$I_{n+l,q} = \frac{gT^{n+l+2}z^{n+l+2}}{2\pi^2(2q+1)!!}(-1)^q\sum_{r=1}^{\infty}(-a)^{r-1}\int_0^{\infty}d\theta \quad (6)$$
$$\times (\cosh\theta)^{n+l-2q}(\sinh\theta)^{2q+2}\,\exp(-rz\cosh\theta),$$
$$J_{n+l,q} = \frac{gT^{n+l+2}z^{n+l+2}}{2\pi^2(2q+1)!!}(-1)^q\sum_{r=1}^{\infty}r(-a)^{r-1}\int_0^{\infty}d\theta \quad (7)$$
$$\times (\cosh\theta)^{n+l-2q}(\sinh\theta)^{2q+2}\,\exp(-rz\cosh\theta).$$



Variation of $\frac{\zeta/\eta}{\left(1/3-c_s^2\right)^2}$ with m/T for MB and FD statistics, for $l = 0,\ 1/2$ and 1.

For large m/T limit,

$$\frac{1}{3} - c_s^2 = \frac{1}{3} - \frac{1}{z} + \frac{3}{z^2} + \mathcal{O}\left(\frac{1}{z^3}\right) \quad \text{(Independent of stat)}$$

$\mathcal{P}/\mathcal{E} \to 0$ as $m/T \to \infty$ and hence c_s^2 vanishes

$$\frac{\zeta}{\eta} = \frac{2}{3} - \frac{4}{z} + \frac{26 + (\ell - 6)\ell}{z^2} + \mathcal{O}\left(\frac{1}{z^3}\right). \tag{8}$$

$$\implies \frac{\zeta}{\eta} = 2\left(\frac{1}{3} - c_s^2\right) \tag{9}$$

For massless and charged particle system $(m = 0 \,\&\, \mu \neq 0)$
Thermal conductivity(κ_q) and Charge conductivity (κ_n) are related

$$\kappa_q = \kappa_n \left(\frac{\mathcal{E} + \mathcal{P}}{nT} \right)^2 . \tag{10}$$

For MB statistics

$$\frac{\kappa_n}{\eta} = \Lambda_{\rm MB} \frac{1}{T},$$

where $\Lambda_{\rm MB}(\ell)$ is given by

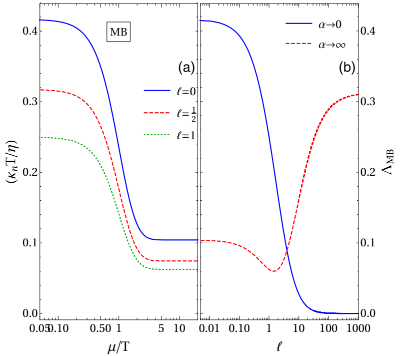
$$\Lambda_{\rm MB} = \begin{cases} \frac{5}{(4+\ell)(3+\ell)} & \text{for } \alpha \to 0\,, \\ \frac{5(\ell^2-\ell+4)}{16\,(4+\ell)(3+\ell)} & \text{for } \alpha \to \infty\,. \end{cases} \tag{11}$$

For FD statistics

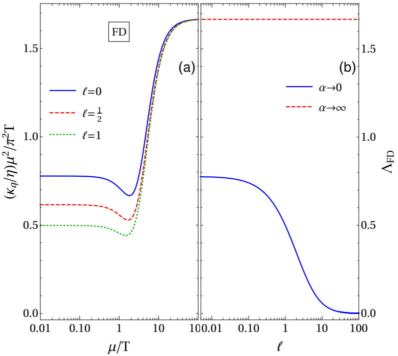
$$\frac{\kappa_q}{\eta} = \Lambda_{\rm FD} \frac{\pi^2 T}{\mu^2},$$

where $\Lambda_{\rm FD}(\ell)$ is given by

$$\Lambda_{\rm FD} = \begin{cases} \frac{196\,\pi^2 \left(2^{1+\ell}-1\right) \zeta(2+\ell)}{45\,\left(2^{3+\ell}-1\right)(4+\ell)(3+\ell)\zeta(4+\ell)} & \text{for } \alpha \to 0\,, \\ \frac{5}{3} & \text{for } \alpha \to \infty\,. \end{cases} \tag{12}$$



The ratio of charge conductivity to shear viscosity multiplied with T for MB vs μ/T (left).Λ_{MB} in low and high μ/T limit(right).



The ratio of thermal conductivity to shear viscosity multiplied with $\mu^2/\pi^2 T$ for FD vs μ/T (left).Λ_{FD} in low and high μ/T limit(right).

Summary and future work

- A successful and well defined frame is developed to consider momentum dependent RTA. D. Dash, S. Bhadury, S. Jaiswal, and A. Jaiswal, Phys. Lett. B 831, 137202 (2022), 2112.14581.
- Ratios of the transport coefficient up to first order are studied.
- New and interesting features of transport coefficients for different statistics are revealed.
- So many other questions are still needed to address (e.g other functional form of momentum dependent τ_R, ζ/η behaviour for $-\ell$, study in other frame of reference etc).
- ERTA for second order relativistic hydrodynamics is my next approach to proceed.

Thank You

Additional references

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