Landau Hydrodynamics with Dissipation and non-conformal equation of state

Deeptak Biswas

The Institute of Mathematical Sciences

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In collaboration with Kishan Deka, Amaresh Jaiswal, Sutanu Roy

Emergent Topics in Relativistic Hydrodynamics, Chirality, Vorticity and Magnetic field

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Motivation

Hydrodynamics and spectra

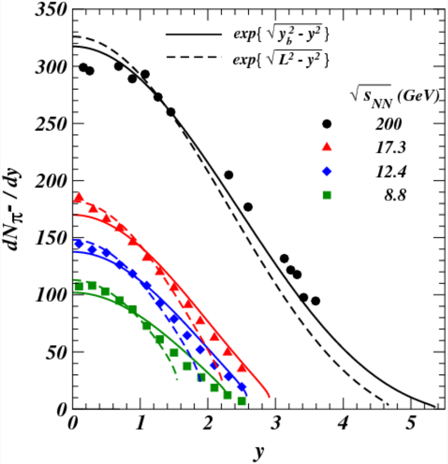
- Landau first formulated ideal hydrodynamics in a non-boost invariant framework. Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953)

- C.Y Wong modified the Landau solution introducing beam-rapidity. PRC 78, 054902 (2008)

- s/n conserved for an ideal evolution and rapidity spectra dN/dy is proportional to entropy production.

- These formulations have been performed with an equation of state in the conformal limit $P = \epsilon/3$.

Spectra?



- The proposed rapidity spectra dN/dy has qualitative agreement with data.
- In the SPS energy range the rapidity spectra look like a gaussian but can not be explained suitably with the available solutions.
- Realistic scenario demands consideration of dissipation and a general equation of state.

Framework of Hydrodynamics

Coarse gaining with Hydrodynamics

- Hydrodynamics is an effective theory to describe the evolution of the medium with macroscopic quantities like ϵ, P, n_B, etc.

- Microscopic interactions $\rightarrow$ Equation of state.

- The evolution is derived from the conservation of energy momentum and number density.

$$\partial_\mu T^{\mu\nu} = 0 \; ; \partial_\mu n^\mu = 0$$

Solving for evolution

- Energy-momentum tensor in Landau frame,

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - (P - \zeta\theta)\, \Delta^{\mu\nu} + 2\eta\sigma^{\mu\nu} \text{ and } P = c_S^2 \epsilon$$

- u^μ is the fluid four-velocity and η and ζ are the coefficient of shear and bulk viscosity.

- Shear tensor, $\sigma^{\mu\nu} \equiv \frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu\right) - \frac{1}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$

- Projection operator $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ and derivative operator $\nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu$, are orthogonal to u^μ

- Space-time evolution is described by solving conservation equations with an equation of state.

Solving viscous Landau Hydrodynamics

construction of equations

- In Landau prescription, rapid longitudinal expansion is followed by slower transverse expansion. We can treat them independently.

- For longitudinal part, we have solved $\partial_\mu T^{\mu\nu} = 0$ for only (t, z) with velocity profile $(u^0, u^1, u^2, u^3) = (\cosh y, 0, 0, \sinh y)$.

- y is the longitudinal rapidity.

- We shall solve these equations,

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{03}}{\partial z} = 0, \;\; \frac{\partial T^{30}}{\partial t} + \frac{\partial T^{33}}{\partial z} = 0$$

Light cone co-ordinates

$$
\begin{aligned}
T^{00} &= \epsilon(u^0)^2 + c_s^2\epsilon(u^3)^2 - (\zeta + 4\eta/3)(u^3)^2\nabla u \\
T^{33} &= \epsilon(u^3)^2 + c_s^2\epsilon(u^0)^2 - (\zeta + 4\eta/3)(u^0)^2\nabla u \\
T^{03} &= T^{30} = \epsilon u^0 u^3(1 + c_s^2) - (\zeta + 4\eta/3)(u^0)^2\nabla u
\end{aligned}
$$

- In terms of the light-cone variables, $t_\pm \equiv t \pm z$,

$$
\frac{\partial}{\partial t_+}\left[c_+\epsilon - \xi\,\nabla u\right]e^{2y} + \frac{\partial}{\partial t_-}\left[c_-\epsilon + \xi\,\nabla u\right] = 0
$$

$$
\frac{\partial}{\partial t_+}\left[c_-\epsilon + \xi\,\nabla u\right] + \frac{\partial}{\partial t_-}\left[c_+\epsilon - \xi\,\nabla u\right]e^{-2y} = 0
$$

- Here we have shortened the notations, using $c_\pm \equiv 1 \pm c_s^2$, and $\xi \equiv \zeta + 4\eta/3$

Solving equations

- We have introduced another set of variables, $y_\pm \equiv \ln(t_\pm/\Delta)$, here $\Delta = 2R/\gamma$ is longitudinally Lorentz contracted diameter of each colliding nuclei.

- The Landau flow relates fluid rapidity in terms of the space-time rapidity as,

$$e^{2y} = f\, e^{2\eta_s} = f\, \frac{t_+}{t_-} = f e^{y_+ - y_-} \qquad f = 1 \textit{ for boost invariant flow}$$

- Landau solved the flow profile for ideal Hydrodynamics, $f = \sqrt{y_+/y_-}$.

- We shall assume the flow profile to be the same as ideal and f to be a slow-varying function.

Inputs for solving

• We have used the newly defined coordinates $y_\pm$ and simplified the two evolution equations by addition/subtraction.

• Under these co-ordinates the velocity gradient becomes, $\nabla u = \frac{1}{\Delta} e^{-(y_- + y_+)/2}$

• Considering the symmetry of the colliding system, we solve the equation which remains invariant under the interchange of $y_\pm$.

$$f \frac{\partial \epsilon}{\partial y_+} + \frac{\partial \epsilon}{\partial y_-} + \frac{1+f}{2} \left[c_+ \epsilon - \frac{\xi}{\Delta} e^{-(y_+ + y_-)/2} \right] = 0$$

• For a constant ξ/s, $\xi = \alpha\, \epsilon^{1/(1+c_s^2)} = \alpha\, \epsilon^{1/c_+}$, α is constant.

$$f \frac{\partial \epsilon}{\partial y_+} + \frac{\partial \epsilon}{\partial y_-} = \frac{1+f}{2} \left[\frac{\alpha}{\Delta} \epsilon^{\frac{1}{c_+}} e^{-\frac{1}{2}(y_+ + y_-)} - c_+ \epsilon \right]$$

Solution for ideal Landau

- Before proceeding to the total solution, we have solved a simpler equation for $\alpha = 0$ for comparison.

- The ideal solution is,

$$\epsilon_{id} = \epsilon_0 \, \exp\left[-\frac{c_+^2}{1+c_s^2}(y_+ + y_-) + \frac{c_+ c_-}{2c_s^2}\sqrt{y_+ y_-} \right]$$

- This solution matches with the conformal solution $c_S^2 = 1/3$.

PRC 78, 054902 (2008)

- The normalization factor ϵ_0 is related to the intial energy density.

Analytical solution

• We have solved the dissipative equation analytically to obtain the final form of ϵ,

$$\epsilon = \left[g(\alpha)\, \epsilon_{id}^{c_s^2/c_+} - \frac{c_s^2\, \alpha}{c_+ c_- \Delta}\, e^{-(y_+ + y_-)/2} \right]^{\frac{c_+}{c_s^2}}$$

where $g(\alpha)$ is an arbitrary function of α such that $g(0) = 1$.

• We have used the non-conformal solution of Bjorken Hydrodynamics to find the form of $g(\alpha)$.

• Matching with the solution from Bjorken hydrodynamics, we get

$$g(\alpha) = 1 + \frac{\alpha c_s^2}{c_- c_+ \tau_0 \epsilon_0^{c_+/c_s^2}}$$

• At $\alpha = 0$ limit, we recover the ideal solution and it matches with the conformal solution for $c_S^2 = 1/3$.

Definition of freeze-out

• In hydrodynamics, Freeze out is defined as the point onwards which fluid description is not applicable.

• Within the approximation of a slower transverse expansion with constant acceleration, the freeze-out time does not get any correction due to dissipation.

• Although the non-conformal E.O.S changes the freeze-out time to,

$$t_{\mathrm{FO}} = 2R\sqrt{\frac{1+c_S^2}{c_S^2}}\cosh y.$$

• At the freeze-out hypersurface, $y_{\pm} = y_b' \pm y$,

Here $y_b' \equiv \frac{1}{2} ln[c_+/(4c_S^2)] + y_b$ and $y_b \equiv \ln(\sqrt{s_{NN}}/m_p)$ is the beam rapidity.

Energy density to rapidity spectra

- The exponent term in the ϵ simplifies as, $e^{-(y_+ + y_-)/2} = \frac{\Delta}{\tau}$.

- At freeze-out this term is negligible as the τ_{FO} is larger.

- The only contribution to the energy density solution comes from the overall normalization factor $g(\alpha)$.

- The entropy density does not get any direct correction from the dissipative term in the relativistic Navier-Stokes equation, so $s \sim \epsilon^{1/c_+}$

Final rapidity spectra

• The s/n is approximately conserved here, n can be related to the entropy density.

• With these assumptions, one can evaluate the rapidity spectra at freeze-out.

$$\frac{dN}{dy} \sim \exp\left(\frac{c_-}{2c_s^2}\sqrt{{y'_b}^2 - y^2}\right).$$

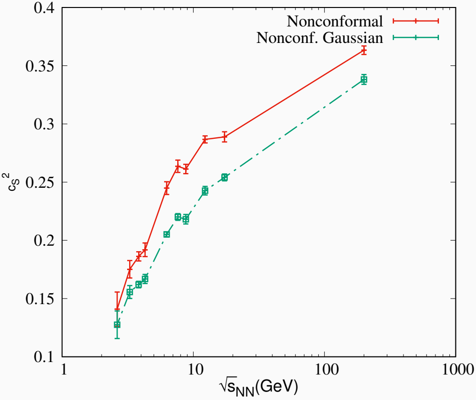
• This solution shows a better agreement with the data.

• The above solution can be simplified to a Gaussian form to match the form of Landau result of Gaussian rapidity.

$$\frac{dN}{dy} \sim exp\left(-\frac{c_-}{4c_s^2 y'_b} y^2\right).$$

• We have fitted rapidity data of pions at PHENIX, SPS, and AGS to find out the value of the speed of sound (c_S^2).

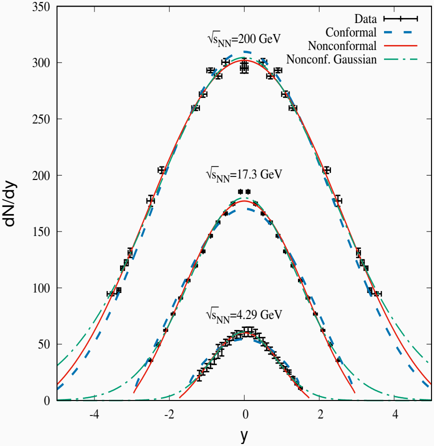
Results



- The extracted value of c_S^2 is seen to increase with energy.
- Gazdzicki et.al proposed a minimum in this variation. Acta Phys. Polon. B 42, 307 (2011)
- We have not obtained a signature for minima, this difference originates from the definition of y_b'.
- We have used a constant velocity of sound to derive the analytical expression, hence the extracted c_S^2 is an approximate time-averaged value for that particular $\sqrt{s_{NN}}$.

Spectra dN/dy

- Non-conformal solution has a better agreement with the spectra, especially for lower AGS and SPS.



Summary

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• We have found the exact analytical solution for viscous landau hydrodynamics in the non-conformal limit.

• In the constant time freeze-out prescription, correction from the shear and bulk viscosity in the spectra is negligible.

• Predicted rapidity spectra show better agreement with data.

• Extracted speed of sound at PHENIX has good agreement with the conformal limit of $1/3$ and decreases with $\sqrt{s_{NN}}$.

• This result can be helpful to determine the c_s^2 of QCD matter at finite density and lower T, especially for the upcoming FAIR experiment.